Resumption of jet mass in dijet process at the LHC

Ze Long Liu

Peking University

In collaboration with Chong Sheng Li, Jian Wang and Yan Wang

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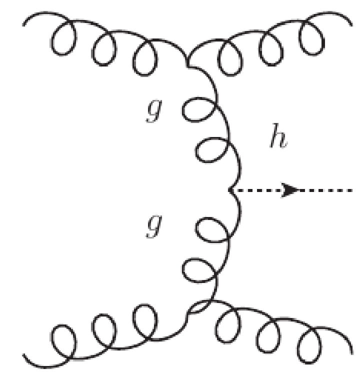
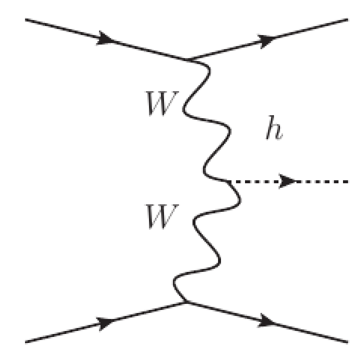
Out Line

- **Motivation**
  - Jet Substructure
  - MC tools vs. Analytical calculation
- **Factorization and Analytical Calculation in SCET**
  - Introduction to SCET
  - Factorization
  - Effect of different Jet Algorithms
  - Hard, Jet & Soft Function
- **Numerical Results**

Jet Substructure

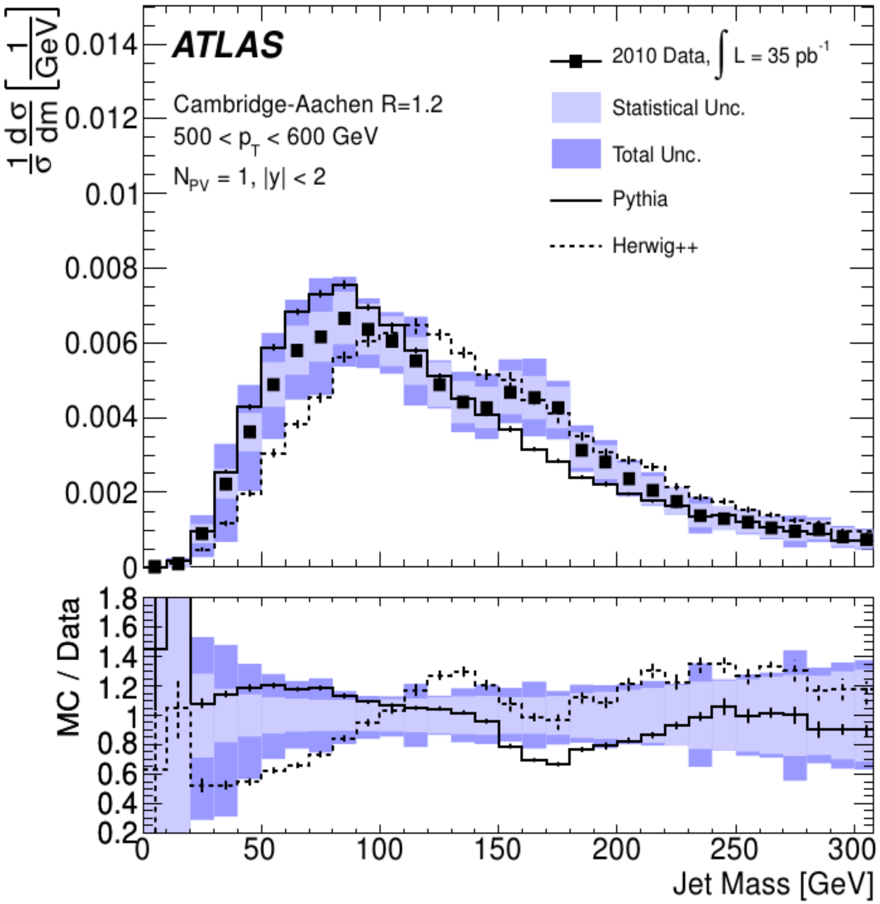
- Understanding the substructure of jets is crucial for LHC phenomenology
- It is important for new physics searches
  - **distinguish jets coming from decays of boosted resonances from QCD jets**
- Jet shapes enable us to look at energy distributions inside a jet



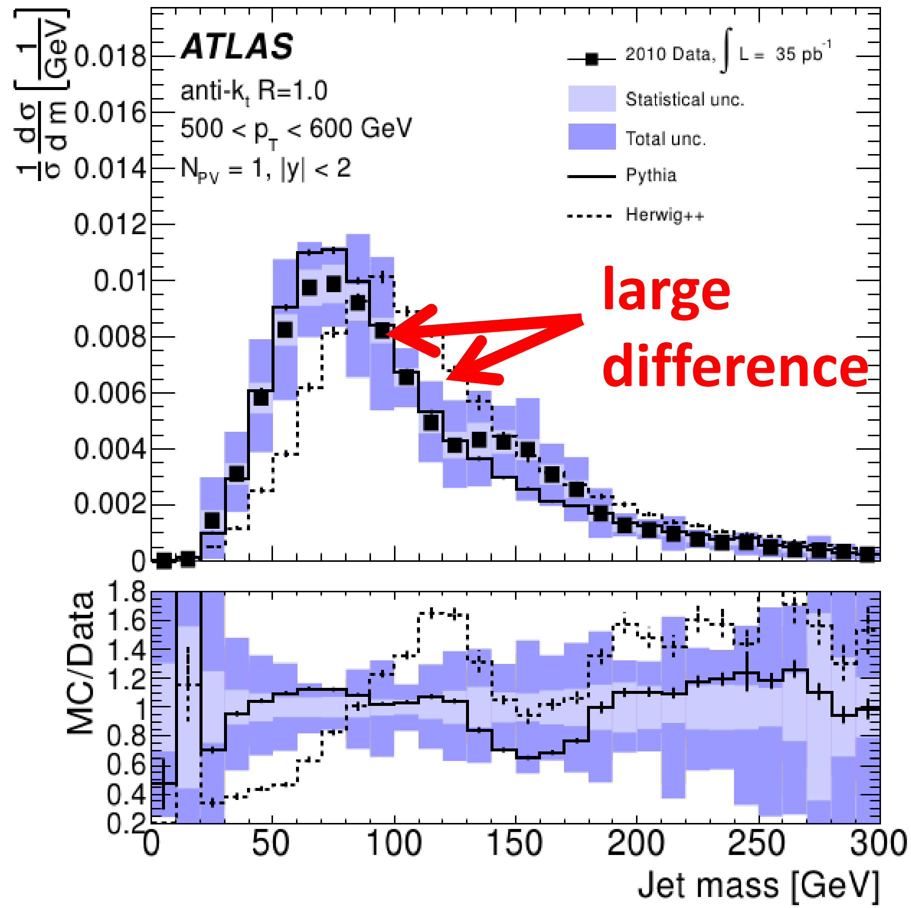


Jet Mass Spectrum in Experiment

Cambridge-Aachen R=1.2



Anti-kT R=1.0



JHEP 1205, 128

MC vs Analytical Approach

- MC simulations using parton showers
  - **provide fully differential events on which any observable can be measured**
  - **interfaced with hadronization to give a realistic description**
  - **formally LL (although contain many sub-leading terms)**
- Analytical Calculation
  - **feasible for a limited number of observables**
  - **well defined and improvable accuracy, which often exceeds the MC one**
  - **they can help development and validation of MC tools**

The two approaches are complementary !

Analytical calcuation

- e^+e^- Colliders
  - angularities in multi-jet events
    Ellis et al. JHEP1011,101 & PLB689,82-89,2010
  - m_J with a jet veto
    R. Kelley, M. D. Schwartz & H. X. Zhu
  - .....
- Hadron Colliders
  - m_J in Higgs + 1 jet & γ+1 jet
    Stewart et al. Schwartz et al.
  - Jet Energy Profile - $\Psi(r)$ H.-n. Li, Z. Li & C.-P Yuan
  - m_J in Z+ 1 jet & dijet at NLL M. Spannowsky et al. JHEP 1210, 126

Soft-Collinear Effective Theory- SCET

Light cone unit vectors:

$$n^\mu = (1,0,0,1) \qquad \bar{n}^\mu = (1,0,0,-1)$$

Vector projection:

$$p^\mu = (n \cdot p)\frac{\bar{n}^\mu}{2} + (\bar{n} \cdot p)\frac{n^\mu}{2} + p_\perp^\mu = p_+^\mu + p_-^\mu + p_\perp^\mu$$

Modes	Field	$p^\mu \sim (+,-,\perp)$	p^2
Hard	---	(Q,Q,Q)	Q^2
Collinear	$A_{n,q}, \xi_{n,p}$	$(\lambda^2/Q, Q, \lambda)$	λ^2
Anti-collinear	$A_{\bar{n},q}, \xi_{\bar{n},p}$	$(Q, \lambda^2/Q, \lambda)$	λ^2
Soft	$A_{s,q}, q_s$	$(\lambda, \lambda, \lambda)$	λ^2

Introduction to SCET

For the processes with >1 scales:

$$\sigma \sim f_a \otimes f_b \otimes H \otimes J \otimes S$$

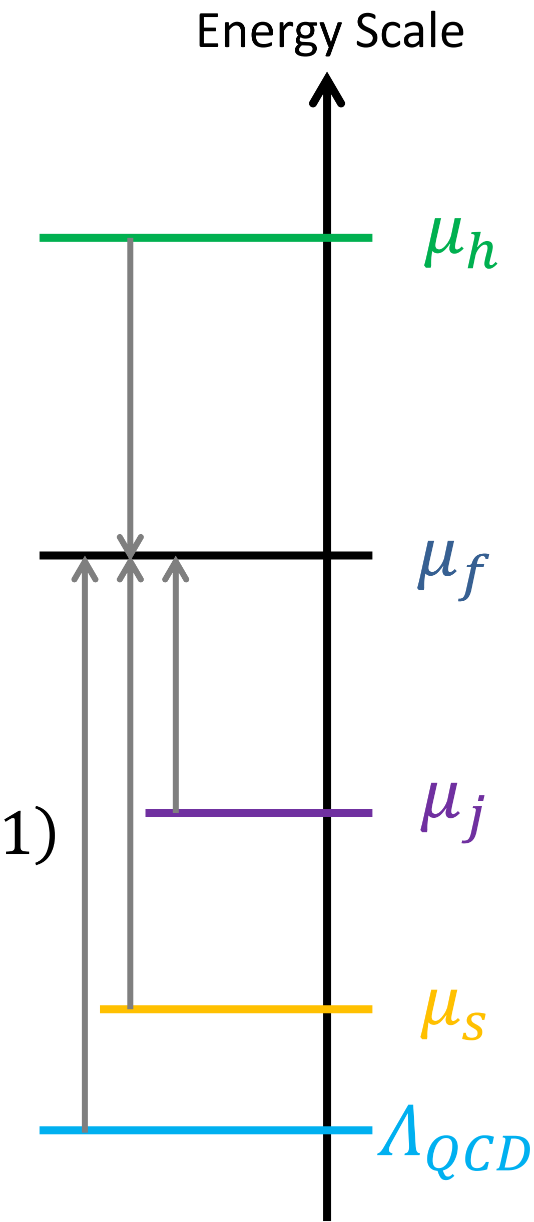
General structure of Sudakov logs :

$$C(\mu, Q) = 1 + a_s(L^2 + L + 1) + a_s^2(L^4 + L^3 + L^2 + L + 1) + a_s^3(L^6 + L^5 + L^4 + L^3 + L^2 + L + 1)$$

$$a_s = \frac{\alpha_s}{4\pi}$$

$$L = \log\left(\frac{\mu}{Q}\right)$$

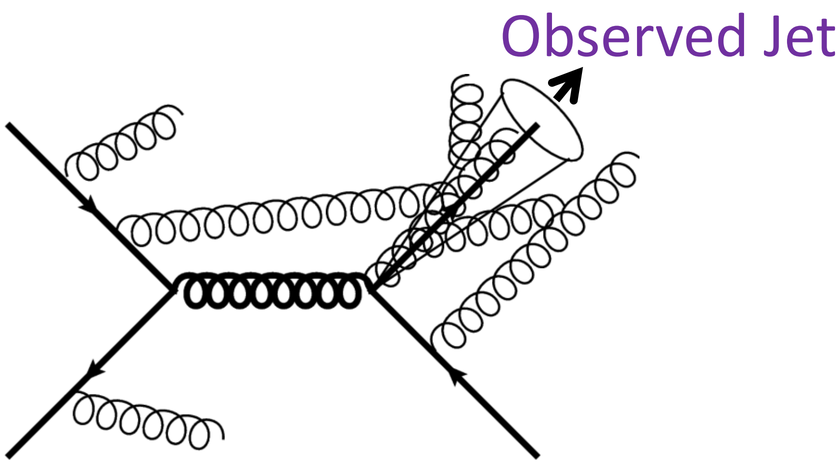
LL NLL NNLL



Jet mass for dijet process

Large logarithms from fixed-order:

$$\frac{d\sigma}{dm_R^2} = \frac{1}{m_R^2}\left(\alpha_s A \ln\frac{m_R^2}{p_T^2} + \alpha_s^2 B \ln^3\frac{m_R^2}{p_T^2} + \cdots\right)$$



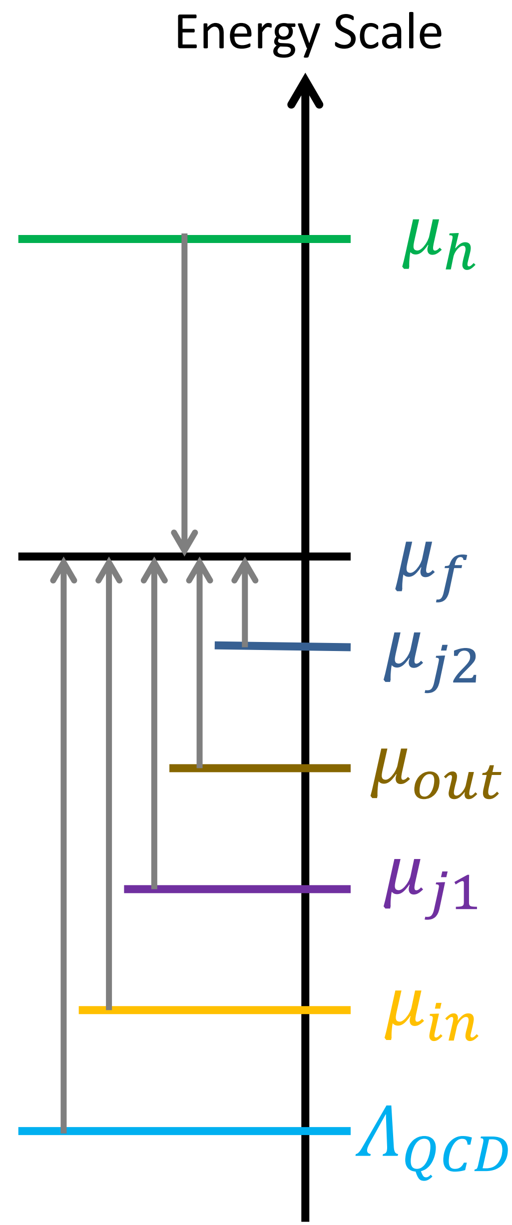
$\mu_h \sim p_T$

$\mu_{j1} \sim m_R$

$\mu_{in} \sim m_R^2/p_T$

$\mu_{j2} \sim \sqrt{s_4}$

$\mu_{out} \sim s_4/p_T$



$$d\sigma = f_a \otimes f_b \otimes H \otimes S \otimes J_{obs} \otimes J$$

f_a, f_b: Λ_{QCD}; H: μ_h; J_{obs}: μ_{j1}; J: μ_{j2}

$$S \Rightarrow S_{in} \otimes S_{out}$$

S_{in}: μ_{in}; S_{out}: μ_{out}

Factorization

$$\mathcal{O}_{I\Gamma}^{\mathrm{QCD}} = (\bar{q}_4^{a_4}\gamma_\mu \Gamma q_2^{a_2})(\bar{q}_3^{a_3}\gamma^\mu \Gamma' q_1^{a_1})(c_I)_{\{a\}}$$

⇩ Match to SECT

hard

$$\mathcal{C}_I^\Gamma(n\cdot P_1, \bar{n}\cdot P_2, n_J\cdot p_{J_1}, \bar{n}_J\cdot p_{J_2}) \sum_{\{a\}} (c_I)_{\{a\}} [O^c(x)]_\Gamma^{b_1b_2b_3b_4} [O^s(x)]^{\{a\},\{b\}}$$

$$[O^c(x)]_\Gamma^{b_1b_2b_3b_4} = \bar{\chi}_{\bar{n}_J}^{b_4}(x)\gamma_\mu\Gamma\chi_{\bar{n}}^{b_2}(x)\bar{\chi}_{n_J}^{b_3}(x)\gamma^\mu\Gamma'\chi_n^{b_1}(x)$$ **collinear**

$$[O^s(x)]^{\{a\},\{b\}} = [Y_{\bar{n}_J}^\dagger(x)]^{b_4a_4}[Y_{\bar{n}}(x)]^{a_2b_2}[Y_{n_J}^\dagger(x)]^{b_3a_3}[Y_n(x)]^{a_1b_1}$$ **soft**

$$|\mathcal{M}^\Gamma(x)\rangle = \langle X|O_\Gamma^c(x)\boldsymbol{O}^s(x)|N_1N_2\rangle|C^\Gamma\rangle$$ scattering amplitude

$$\frac{d\sigma}{dp_T dy dm_J^2} = \frac{1}{2s}\sum_X\sum_\Gamma \int d^4x \langle\mathcal{M}^\Gamma(x)|\widehat{\mathcal{M}}(m_J^2, p_T, y, R)|\mathcal{M}^\Gamma(0)\rangle$$

Factorization

$$\sum_X \langle \mathcal{M}(x)|\widehat{\mathcal{M}}(m_J^2, p_T, y, R)|\mathcal{M}(0)\rangle = \frac{1}{N_{\rm init}} \sum_\Gamma \langle N_1(P_1)|\bar{\chi}_n(x)\frac{\not{\bar{n}}}{2}\chi_n(0)|N_1(P_1)\rangle$$

$$\times \langle N_2(P_2)|\bar{\chi}_{\bar{n}}(x)\frac{\not{n}}{2}\chi_{\bar{n}}(0)|N_2(P_2)\rangle$$

PDF

Jet func.

$$\times \langle 0|\bar{\chi}_{n_J}(0)\frac{\not{\bar{n}}_J}{2}\chi_{n_J}(x)|0\rangle \langle 0|\bar{\chi}_{\bar{n}_J}(0)\frac{\not{n}_J}{2}\chi_{\bar{n}_J}(x)|0\rangle$$

$$\times \sum_{X_s} \langle C^\Gamma|\langle 0|\boldsymbol{O}^{s\dagger}(x)|X_s\rangle\langle X_s|\boldsymbol{O}^s(0)|0\rangle|C^\Gamma\rangle$$

$$\times \mathcal{M}(m_J^2, p_T, y, R, \{p_c\}, \{k_s\}),$$

Phase space constraint

$$\sum_\Gamma \langle C^\Gamma|\langle 0|\boldsymbol{O}^{s\dagger}(x)|X_s\rangle\langle X_s|\boldsymbol{O}^s(0)|0\rangle|C^\Gamma\rangle = \sum_\Gamma \sum_{IJ} H_{JI}\, S_{IJ}\,.$$

Hard func.

$$H_{IJ} = \sum_\Gamma \langle C^\Gamma|c_I\rangle\langle c_J|C^\Gamma\rangle$$

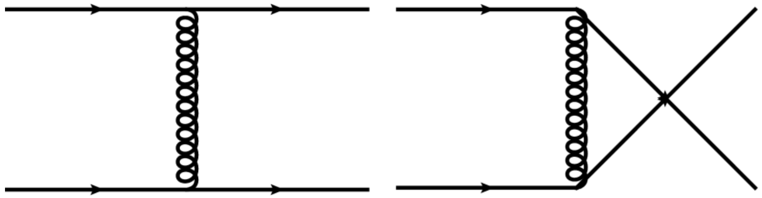
Soft func.

$$S_{IJ} = \langle c_I|\langle 0|\boldsymbol{O}^{s\dagger}(x)|X_s\rangle\langle X_s|\boldsymbol{O}^s(0)|0\rangle|c_J\rangle$$

Color Structure

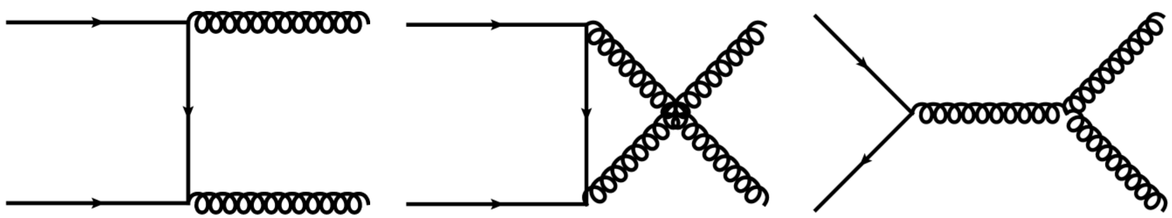
R. Kelley & M. D. Schwartz, PRD83, 045022

$q_i + q_j \rightarrow q_i + q_j$



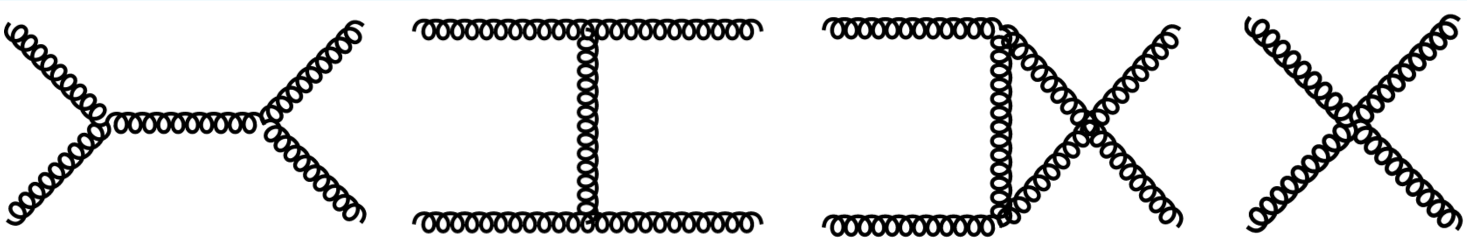
$$|c_1\rangle = t^c_{i_3,i_1} t^c_{i_4,i_2} \,, \qquad |c_2\rangle = \delta_{i_3,i_1}\delta_{i_4,i_2}$$

$q_i + q_i \rightarrow g + g$



$$|c_1\rangle = (t^{a_1}t^{a_2})_{i_3,i_4} \,, \qquad |c_2\rangle = (t^{a_2}t^{a_1})_{i_3,i_4} \,, \qquad |c_3\rangle = \delta^{a_1,a_2}\delta_{i_3,i_4}$$

$g + g \rightarrow g + g$



$$|c_1\rangle = \mathrm{Tr}\left(t^{a_1}t^{a_2}t^{a_3}t^{a_4}\right) \,, \qquad |c_2\rangle = \mathrm{Tr}\left(t^{a_1}t^{a_2}t^{a_4}t^{a_3}\right) \,, \qquad |c_3\rangle = \mathrm{Tr}\left(t^{a_1}t^{a_4}t^{a_3}t^{a_2}\right) \,,$$

$$|c_4\rangle = \mathrm{Tr}\left(t^{a_1}t^{a_4}t^{a_2}t^{a_3}\right) \,, \qquad |c_5\rangle = \mathrm{Tr}\left(t^{a_1}t^{a_3}t^{a_4}t^{a_2}\right) \,, \qquad |c_6\rangle = \mathrm{Tr}\left(t^{a_1}t^{a_3}t^{a_2}t^{a_4}\right) \,,$$

$$|c_7\rangle = \mathrm{Tr}\left(t^{a_1}t^{a_4}\right)\mathrm{Tr}\left(t^{a_2}t^{a_3}\right) \,, \qquad |c_8\rangle = \mathrm{Tr}\left(t^{a_1}t^{a_2}\right)\mathrm{Tr}\left(t^{a_3}t^{a_4}\right) \,, \qquad |c_9\rangle = \mathrm{Tr}\left(t^{a_1}t^{a_3}\right)\mathrm{Tr}\left(t^{a_2}t^{a_4}\right)$$

Hard Function

$$H_{IJ} = \sum_\Gamma \mathcal{C}_I^\Gamma \mathcal{C}_J^{\Gamma\star} \qquad \frac{d}{d\ln\mu}\mathcal{C}_I^\Gamma(\mu) = \Gamma_{IJ}^H \mathcal{C}_J^\Gamma(\mu)$$

R. Kelley & M. D. Schwartz
PRD83, 045022

$$\Gamma_{IJ}^H(\hat{s},\hat{t}_1,\hat{u}_1\mu) = \left(\gamma_{\text{cusp}}\frac{c_H}{2}\ln\frac{-\hat{t}_1}{\mu^2} + \gamma_H - \frac{\beta(\alpha_s)}{\alpha_s}\right)\delta_{IJ} + \gamma_{\text{cusp}} M_{IJ}(s,t,u)$$

$$M_{IJ}(\hat{s},\hat{t}_1,\hat{u}_1) = \begin{pmatrix} 4C_F U - C_A(T+U) & 2U \\ \frac{C_F}{C_A}U & 0 \end{pmatrix}$$

off-diagonal

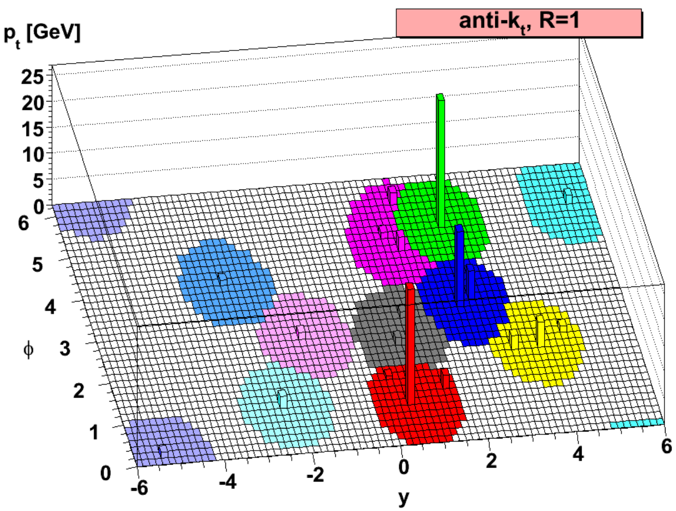
diagonalize

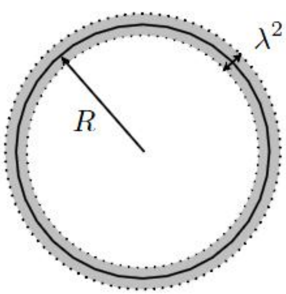
$$\left(\tilde{F}\cdot M \cdot \tilde{F}^{-1}\right)_{KK'} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

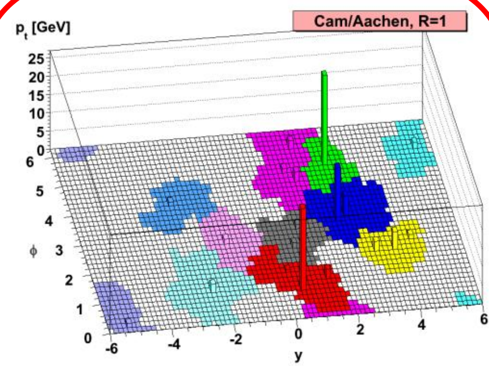
RG equation in diagonalized basis:

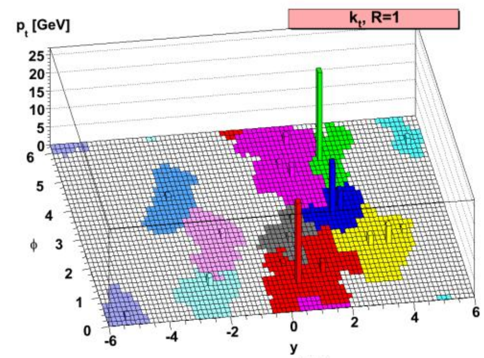
$$\frac{d}{d\ln\mu}\hat{H}_{KK'}(\mu) = \left[\gamma_{\text{cusp}}\left(c_H \ln\left|\frac{\hat{t}_1}{\mu^2}\right| + \lambda_K + \lambda_{K'}^\star\right) + 2\gamma_H - \frac{2\beta(\alpha_s)}{\alpha_s}\right]\hat{H}_{KK'}(\mu)$$

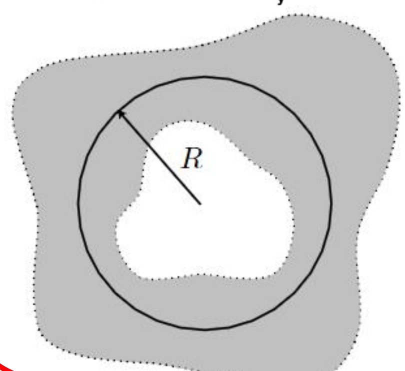
Jet Algorithm











$$d_{ij} = \min(k_{ti}^{2p}, k_{tj}^{2p}) \frac{\Delta_{ij}^2}{R^2}$$

$$d_{iB} = k_{ti}^{2p}$$

$$\Delta_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

p= -1 : anti-kT

p= 0 : CA

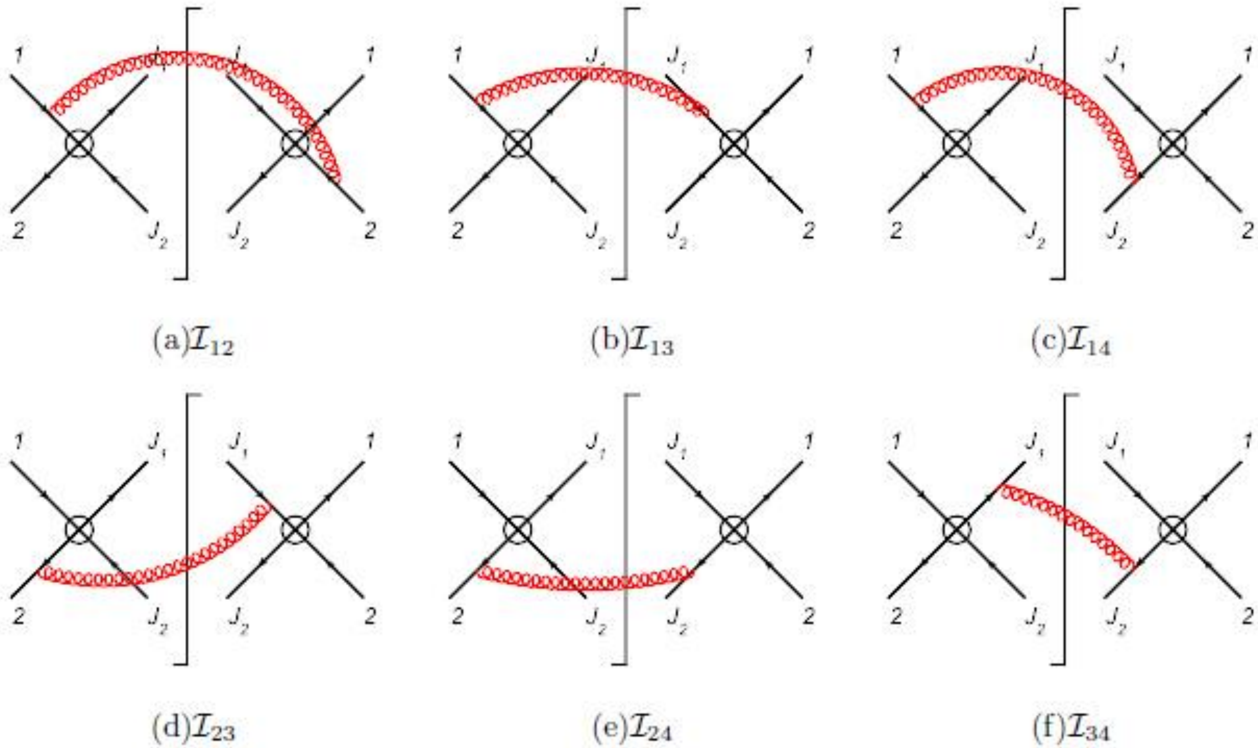
p= 1 : kT

⇨ **boundary clustering change the jet boundary by O(1)**

R. Kelley et al., JHEP 1209,117

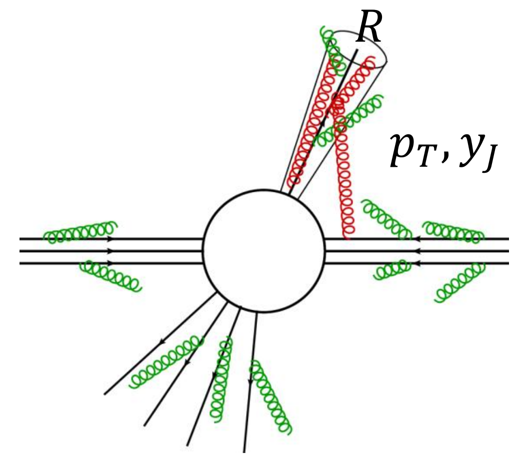
Soft Function

$$\boldsymbol{S}(k_{\text{in}}, k_{\text{out}}, \beta, r, \mu) = \sum_{i,j}^{i \neq j} \boldsymbol{w}_{ij} \mathcal{I}_{ij}(k_{\text{in}}, k_{\text{out}}, \beta, r, \mu)$$



(a) $\mathcal{I}_{12}$ (b) $\mathcal{I}_{13}$ (c) $\mathcal{I}_{14}$

(d) $\mathcal{I}_{23}$ (e) $\mathcal{I}_{24}$ (f) $\mathcal{I}_{34}$



constrained by jet algorithm

$$\mathcal{I}_{ij}(k_{\text{in}}, k_{\text{out}}, y_J, R, \mu) = -\frac{4\pi\alpha_s}{(2\pi)^{d-1}} \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{\epsilon} \int d^d q \, \delta(q^2)\theta(q_0) \boxed{\mathcal{M}_R(k_{\text{in}}, k_{\text{out}}, R, q)} \frac{n_i \cdot n_j}{(n_i \cdot q)(n_j \cdot q)}$$

Refactorization of Soft Function

The soft gluon in/out the cone correspond to different scales: $\mu_{in} \sim m_R^2/p_T$ & $\mu_{out} \sim s_4/p_T$

R. Kelley, M. D. Schwartz & H. X. Zhu PRD87, 014010

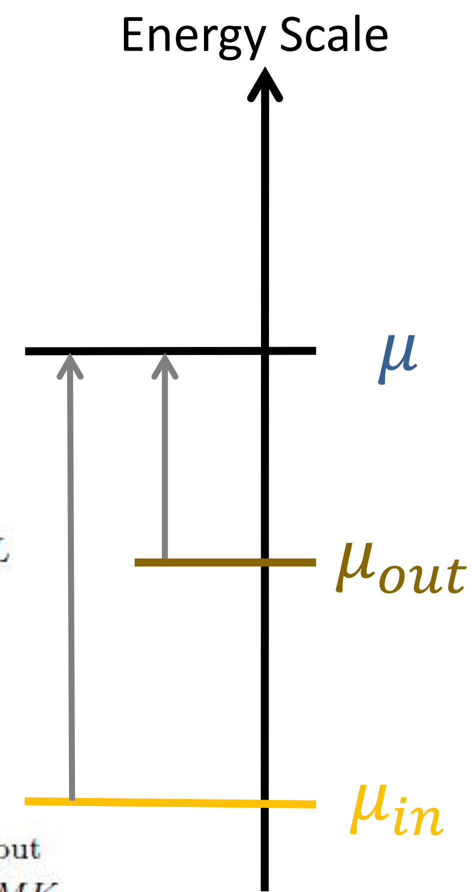
$$\hat{S}_{K'K}(k_{\rm in}, k_{\rm out}, \mu) = \boxed{\hat{S}^{\rm in}_{K'L}(k_{\rm in}, \mu_{\rm in}, \mu)} \left(\hat{S}^{(0)}\right)^{-1}_{LM} \boxed{\hat{S}^{\rm out}_{MK}(k_{\rm out}, \mu_{\rm out}, \mu)}$$

$$\hat{\tilde{s}}^{\rm in}_{K'L}(\kappa_{\rm in}, \mu) = \hat{\tilde{s}}^{(0)}_{K'L} + \sum_{i \neq j} [w_{ij}]_{K'L} \, \tilde{I}^{\rm in}_{ij}(\kappa_{\rm in}, \mu)$$

$$\frac{d}{d\ln\mu} \hat{\tilde{s}}^{\rm in}_{K'L}(L_{\rm in}, \mu) = \left[-2\tilde{B}^{\rm in}_{K'L}\gamma_{\rm cusp}\, L_{\rm in} - \tilde{C}^{\rm in}_{K'L}\gamma_{\rm cusp} - \tilde{\gamma}^{\rm in}_{K'L}\right] \hat{\tilde{s}}^{\rm in}_{K'L}$$

$$\hat{\tilde{s}}^{\rm out}_{MK}(\kappa_{\rm out}, \mu) = \hat{\tilde{s}}^{(0)}_{MK} + \sum_{i \neq j} [w_{ij}]_{MK} \, \tilde{I}^{\rm out}_{ij}(\kappa_{\rm out}, \mu)$$

$$\frac{d}{d\ln\mu} \hat{\tilde{s}}^{\rm out}_{MK}(L_{\rm out}, \mu) = \left[-2\tilde{B}^{\rm out}_{MK}\gamma_{\rm cusp}\, L_{\rm out} - \tilde{C}^{\rm out}_{MK}\gamma_{\rm cusp} - \tilde{\gamma}^{\rm out}_{MK}\right] \hat{\tilde{s}}^{\rm out}_{MK}$$



RG invariance

$$\frac{d\tilde{f}_{q/N}(\tau,\mu)}{d\ln\mu} = \left[2C_F\gamma_{\rm cusp}\ln(\tau) + 2\gamma^{f_q}\right]\tilde{f}_{q/N}(\tau,\mu)$$

$$\frac{d}{d\ln\mu}\hat{H}_{KK'}(\mu) = \left[\gamma_{\rm cusp}\left(c_H\ln\left|\frac{\hat{t}_1}{\mu^2}\right| + \lambda_K + \lambda^\star_{K'}\right) + 2\gamma_H - \frac{2\beta(\alpha_s)}{\alpha_s}\right]\hat{H}_{KK'}(\mu)$$

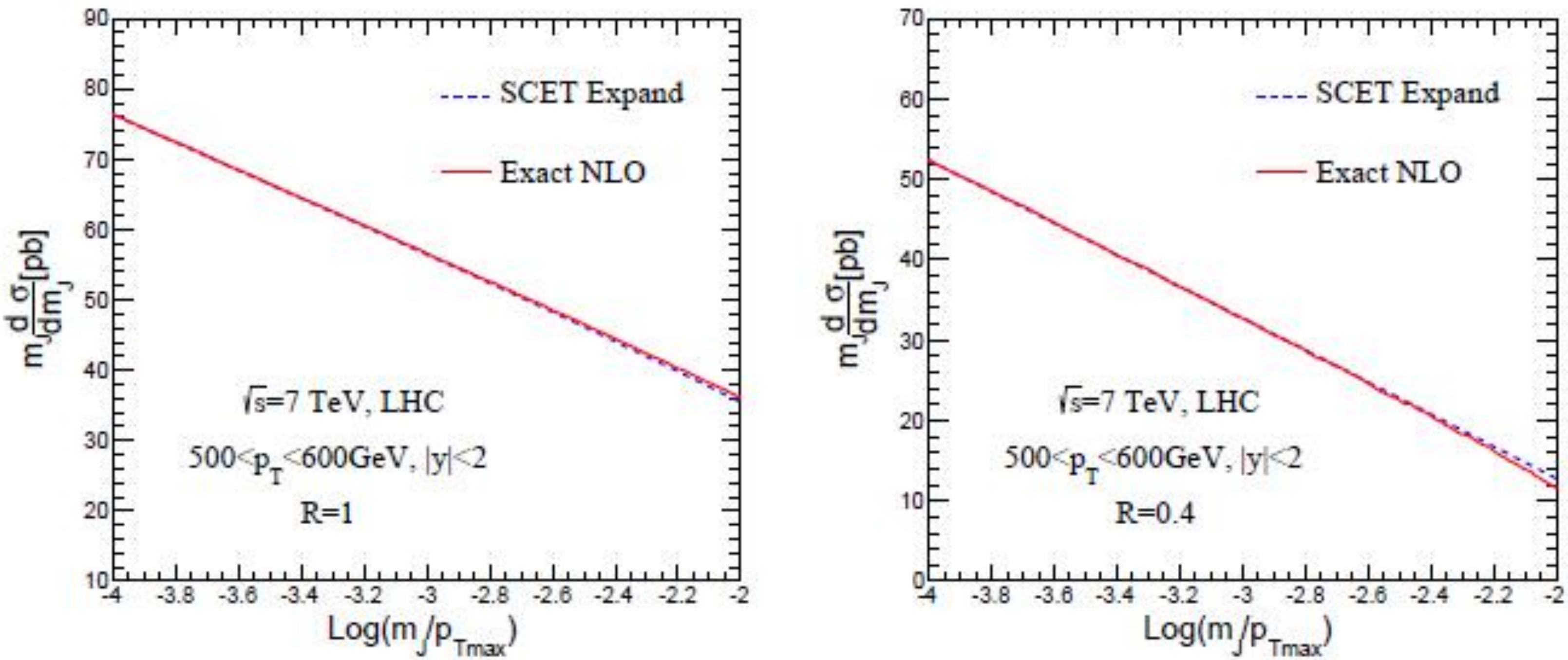
$$\frac{d}{d\ln\mu}\tilde{j}_g(Q^2,\mu) = \left[-2C_A\gamma_{\rm cusp}\ln\left(\frac{Q^2}{\mu^2}\right) - 2\gamma^{J_g}\right]\tilde{j}_g(Q^2,\mu)$$

$$\frac{d}{d\ln\mu}\hat{\tilde{s}}_{K'K} = \Big\{\gamma_{\rm cusp}\left[2C_{i_1}L(\hat{u}_1) + (2C_{i_2} - c_H)L(\hat{t}_1) - \lambda_K - \lambda^\star_{K'}\right] - 2\,\gamma_{\rm cusp}\left(C_{i_1} + C_{i_2} - C_{j_1} - C_{j_2}\right)\ln\frac{Q^2}{\mu^2} - 2\gamma^S\Big\}\hat{\tilde{s}}_{K'K}$$

$$\frac{d}{d\ln\mu}\left[H_{IJ}(p_T,v,\mu)\tilde{s}_{JI}\left(\frac{Q^2}{2E_J^*},\frac{Q^2}{2E_J^*},\mu\right)\tilde{f}_{i_1/N_1}\left(\frac{Q^2}{p_T^2}\bar{v},\mu\right)\tilde{f}_{i_2/N_2}\left(\frac{Q^2}{p_T^2}v,\mu\right)\tilde{j}_1(Q^2,\mu)\tilde{j}_2(Q^2,\mu)\right] = 0$$

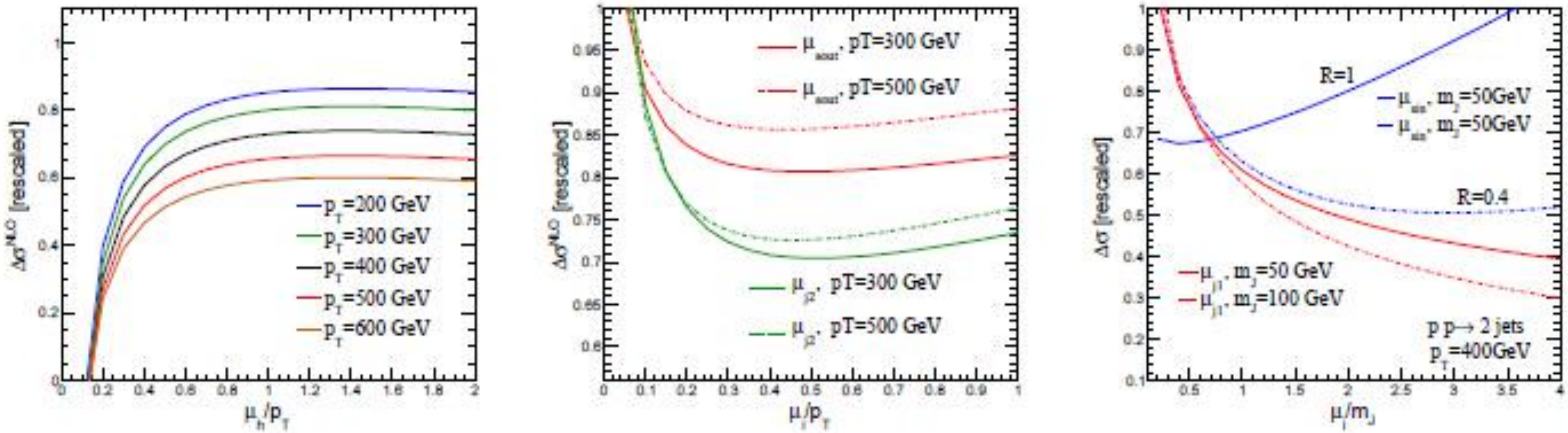
The RG invariance has been checked at $O(\alpha_s)$

Jet Mass Spectrum at Fixed-Order



The validity of soft function is checked !

Scale Choices

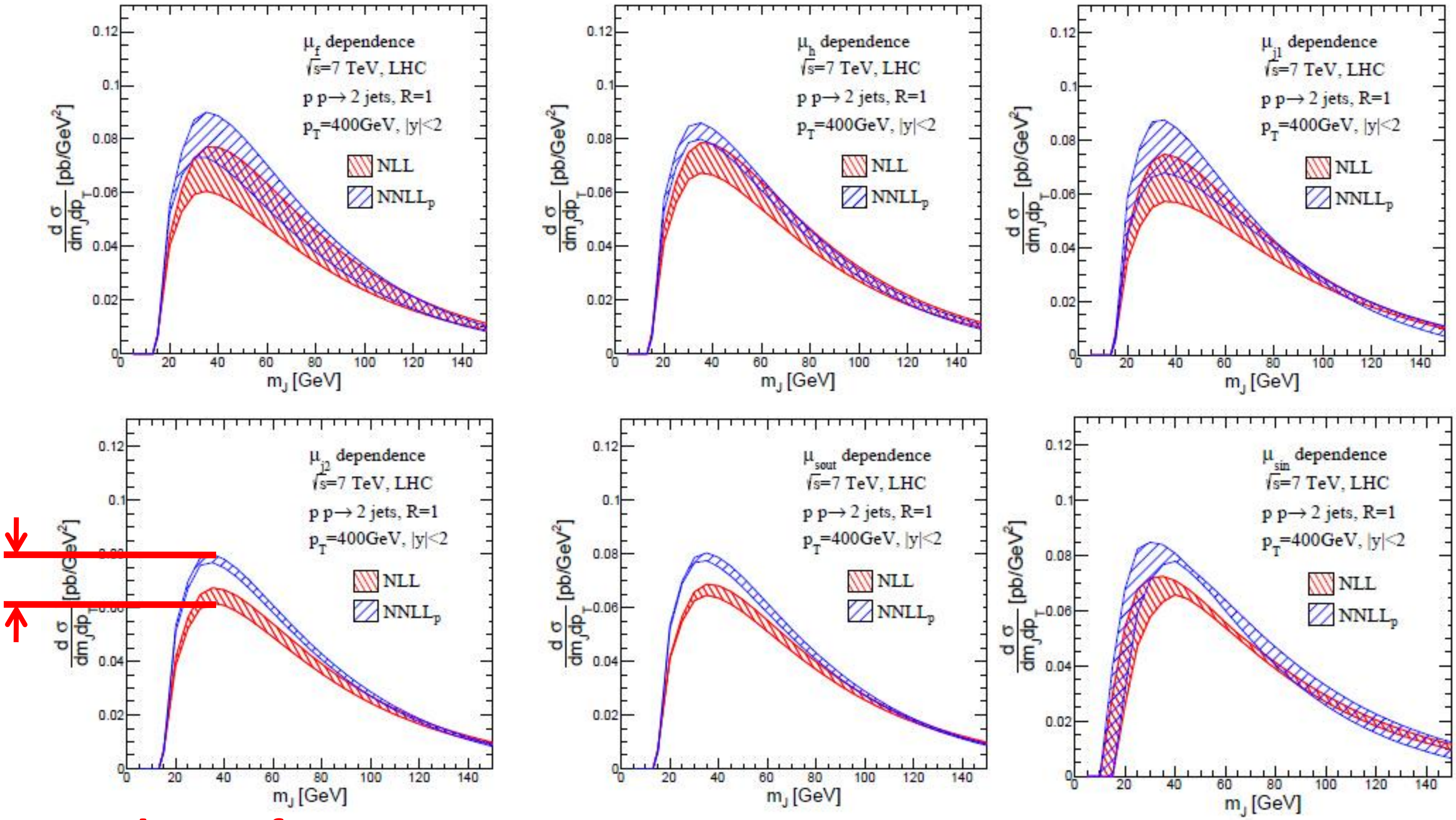


(a) $\boldsymbol{H}$ (b) $\boldsymbol{S}_{\text{out}}$ and $J^{\text{rec.}}$ (c) $\boldsymbol{S}_{\text{in}}$ and $J^{\text{obs.}}$

$$\mu_h = 1.4\, p_T\,, \quad \mu_{s_{\text{out}}} = 0.2\, p_T + 80\, \text{GeV}\,, \quad \mu_{j2} = 0.5\, p_T\,.$$

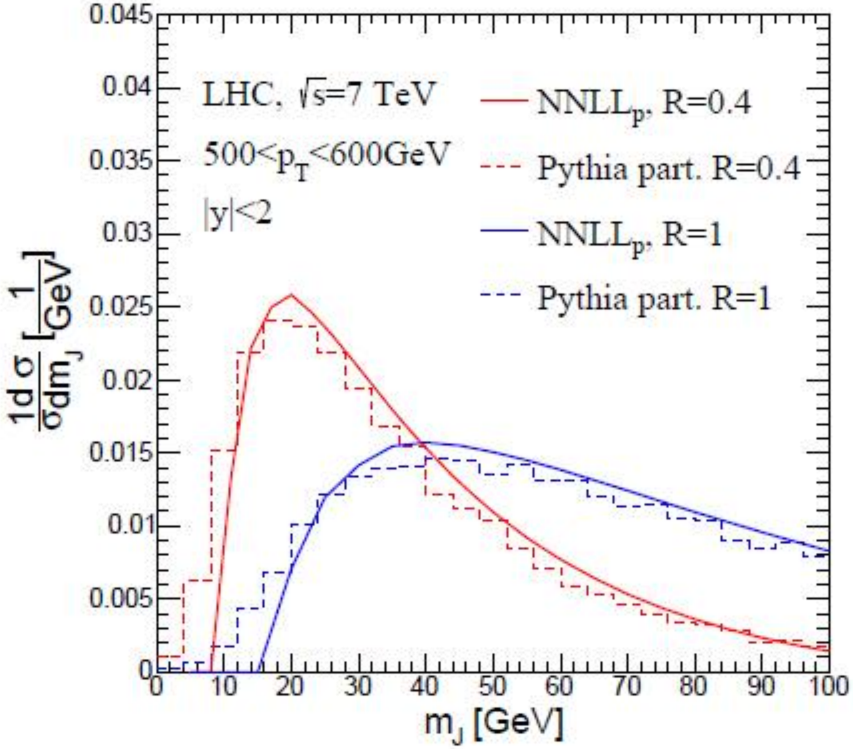
$$\mu_{s_{\text{in}}} = \frac{{\mu_*}^2}{c_R} \frac{p_T^*}{p_T}\,,\ \mu_* = 1.67 m_J^{1.47}\ (m_J \text{ in GeV})$$

Scale uncertainties

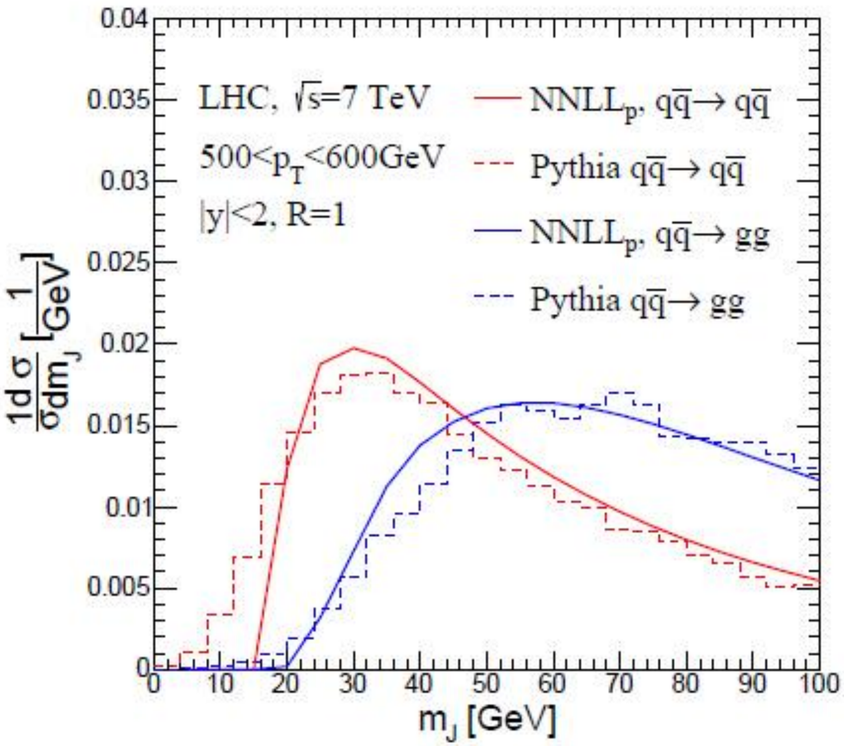


Enhanced 23% from NLL to NNLL_p

Discussion



gluon jet vs. quark jet



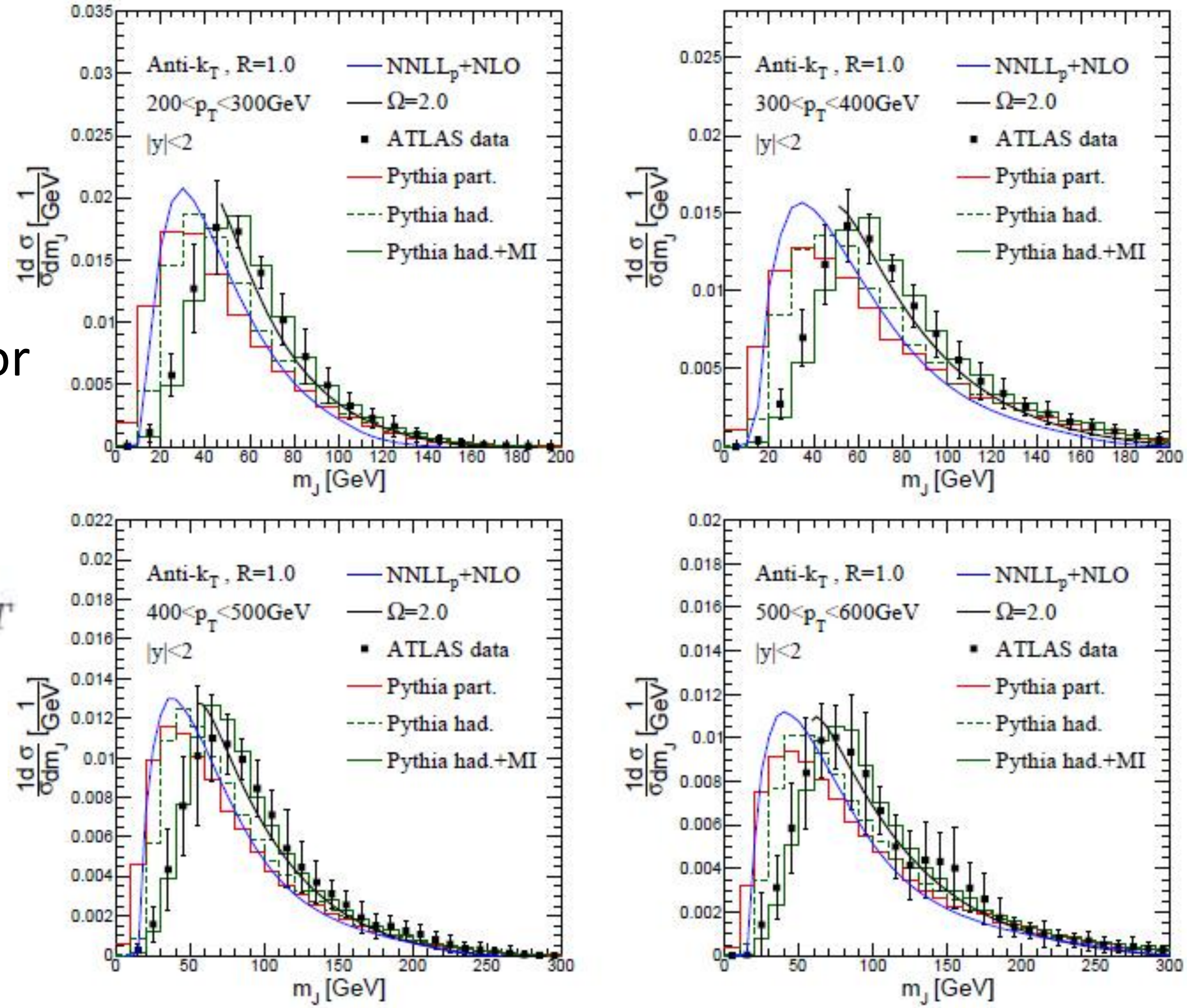
R-dependence

More large-angle soft radiation are contained with larger R

Predictions of jet mass spectrum

A shift accounting for non-perturbative effects

$$m_J^2 \to m_J^2 + 2\Omega R\, p_T$$



Summary

- Factorization formula has been derived with SCET
- Calculate the soft function with anti-kT algorithm
- RG invariance has been checked
- A significant enhancement for NLL to NNLL_p
- Jet mass spectrum is sensitive with non-perturbative effects

Thank you!