# Vector boson production in hadron-hadron scattering

*(Drell-Yan-like processes)*

Pavel Nadolsky

Department of Physics
Southern Methodist University (Dallas, TX)

Lecture 4
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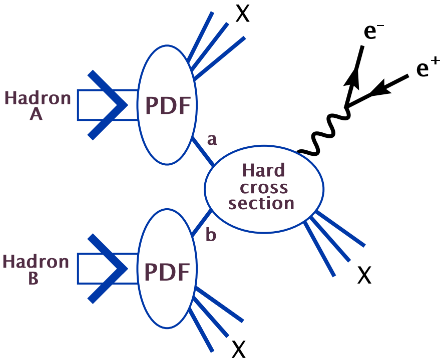
# DY-like processes: $A(p_A)B(p_B) \to (V(q) \to v_1 v_2 ...)X$

DY-like processes play a special role at the LHC and other hadron-hadron colliders, refer to resonant production of QCD-neutral heavy final states

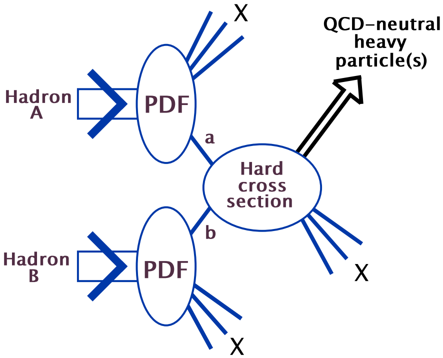



Drell-Yan production of lepton pairs

# DY-like processes: $A(p_A)B(p_B) \to (V(q) \to v_1 v_2 ...)X$

## Notations

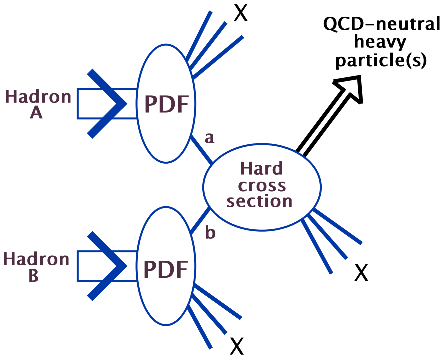
- $A,B$ – initial-state hadrons ($p$, $\bar{p}$, $n$, nuclei, $\pi$, ...)
- $V$ – a final-state QCD-neutral system (a vector boson or boson pair with mass $Q \gg \Lambda_{QCD}$)
- $v_1$, $v_2$– observed particles from decay of $V$ (e.g., leptons)



# DY-like processes: $A(p_A)B(p_B) \to (V(q) \to v_1 v_2 \ldots)X$

## DY-like processes are ubiquitous

- $AB \to (\gamma^*, Z \to \ell^+\ell^-)X$ (with $\ell = e, \mu$)
- $AB \to (W \to \ell\nu_\ell)X$
- $AB \to VVX$ (with $V = \gamma, W, Z, \ldots$)
- $AB \to$ Higgs $+ X$
- $AB \to V_{BSM}X$ (with $V_{BSM} = Z'$, Randall-Sundrum graviton, etc.)



# DY-like processes: $A(p_A)B(p_B) \to (V(q) \to v_1 v_2 ...)X$

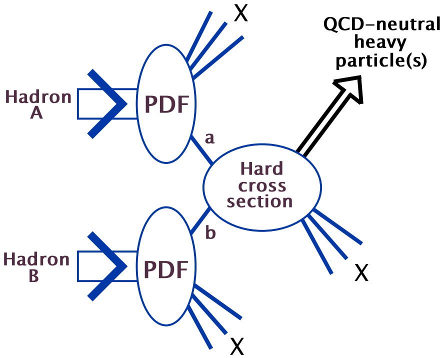
## DY-like processes are "simple"

- $V$ does not interact with final-state hadrons, which are summed over in cross sections

  $\Rightarrow$ no dependence on final-state nonperturbative functions

- QCD factorization is **proved** to all orders in $\alpha_s$ for a number of DY observables

  ▶ (In many other processes, factorization is only a **plausible conjecture**)



# DY-like processes: $A(p_A)B(p_B) \to (V(q) \to v_1 v_2 ...)X$

## Example: factorization for the total cross section

$$\frac{d\sigma}{dQ^2} = \sum_{a,b} \int_\tau^1 \frac{d\xi}{\xi} f_{a/A}(\xi, Q) f_{b/B}(\frac{\tau}{\xi}, Q) \frac{d\widehat{\sigma}_{ab}}{dQ^2},$$

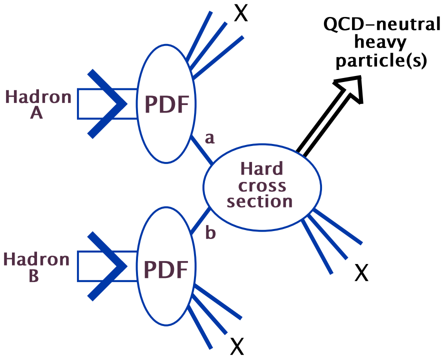
where

$Q$ is the invariant mass of $V$;

$\tau \equiv Q^2/s$;

$\widehat{\sigma}_{ab}$ is the hard-scattering cross section (calculated as a series in the QCD coupling $\alpha_s$);

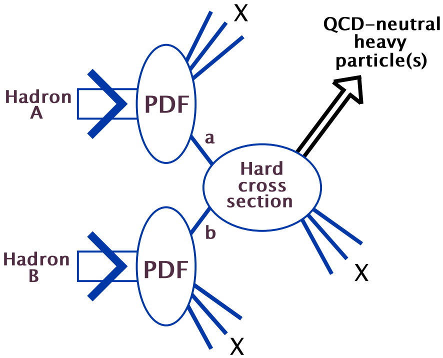
$f_{a/A}(\xi,\mu)$ and $f_{b/B}(\tau/\xi,\mu)$ are parton distribution functions



# DY-like processes: $A(p_A)B(p_B) \to (V(q) \to v_1 v_2 ...)X$

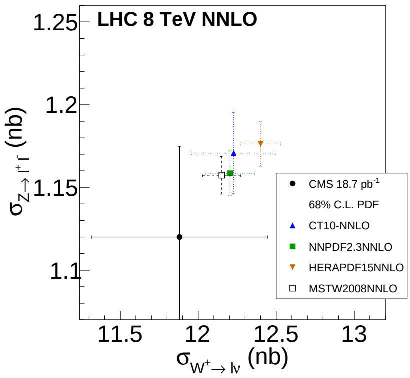
**DY-like processes produced many important discoveries**

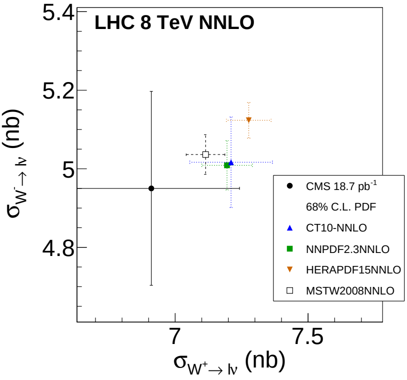
- early confirmation of the parton model
- discovery of heavy quarks (which ones?)
- discovery of massive carriers of weak force ($W$ and $Z$)




Modern DY experiments provide most precise QCD tests at hadron colliders

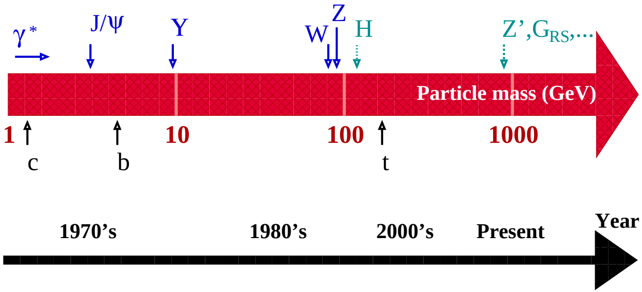
# $W$ and $Z$ cross sections at the LHC




Measurement of $\sigma_W$ and $\sigma_Z$ confirms the validity of perturbative QCD at $\sqrt{s} = 7$ TeV

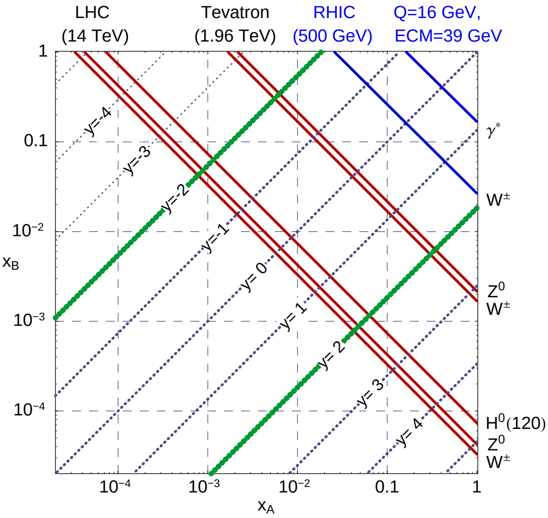
# Final states in DY-like processes




Explore the DY-like processes as a function of $Q \equiv M_{\ell\ell'}$, the invariant mass of the heavy EW state

$$\frac{d\sigma}{dQ^2} = \sum_{a,b} \int_{\tau}^{1} \frac{d\xi}{\xi} f_{a/A}(\xi, Q) f_{b/B}(\frac{\tau}{\xi}, Q) \frac{d\widehat{\sigma}_{ab}}{dQ^2}$$

# Typical parton momentum fractions



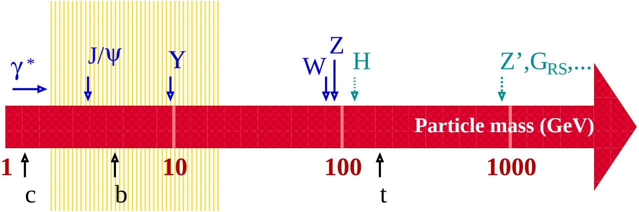

$$x_{A,B} \equiv \frac{Q}{\sqrt{s}} e^{\pm y}$$

Born level: $p_a^\mu = x_A p_A^\mu$, $p_b^\mu = x_B p_B^\mu$

Typical rapidities in the experiment: $|y| \lesssim 2$

- experiments at higher energies are sensitive to PDF's at smaller $x$

# Final states in DY-like processes



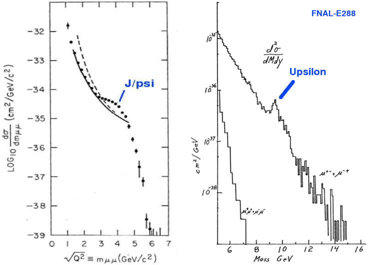

## $pN \xrightarrow{\gamma^*} \ell^+\ell^- X$ at $Q < 20$ GeV

- Continuous $\gamma^*$ cross section
- Multiple quarkonium resonances

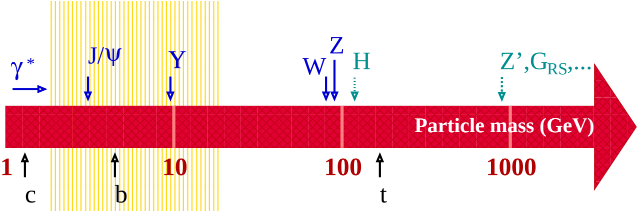
  (studied by non-relativistic QCD, **not** in the PDF fit)

▲ $J/\psi$ ($c\bar{c}$)– found in $e^+e^-$ scattering (1974)

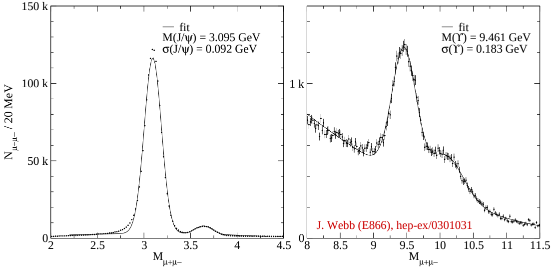
▲ $\Upsilon$ ($b\bar{b}$)– found in $pN \to \mu^+\mu^- X$ (FNAL-E288, 1977)



# Final states in DY-like processes




$J/\psi$, $\Upsilon$ resonances shown with better resolution (FNAL-E866)

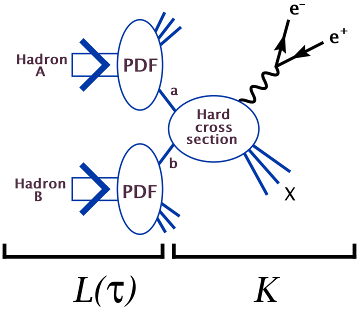


# Scaling of the continuum cross section

## S. Drell, T. M. Yan, 1970

$$s\frac{d\sigma}{dQ^2} \approx \mathcal{L}_{ab}(\tau) \cdot \text{const}$$

- $\mathcal{L}_{ab}(\tau)$ is the "parton luminosity", originally derived from DIS functions; depends only on $\tau$ if the $\ln Q$ dependence is neglected



# Scaling of the continuum cross section

**S. Drell, T. M. Yan, 1970**

$$s\frac{d\sigma}{dQ^2} \approx \mathcal{L}_{ab}(\tau) \cdot \text{const}$$

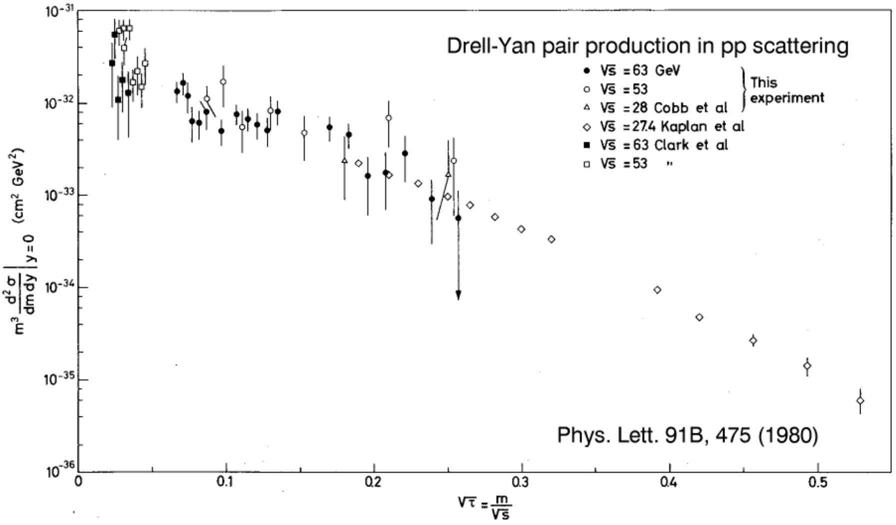
- Compare to the Born cross section:

$$\left(\frac{d\sigma}{dQ^2}\right)_{LO} = \frac{4\pi\alpha_{EM}^2}{3N_c Q^2 s}$$

$$\times \underbrace{\sum_{i=u,d,s,\dots} e_i^2 \int_\tau^1 \frac{d\xi}{\xi}\left[f_{q_i/A}(\xi,Q) f_{\bar{q}_i/B}(\frac{\tau}{\xi},Q) + f_{\bar{q}_i/A}(\xi,Q) f_{q_i/B}(\frac{\tau}{\xi},Q)\right]}_{\mathcal{L}(\tau)},$$

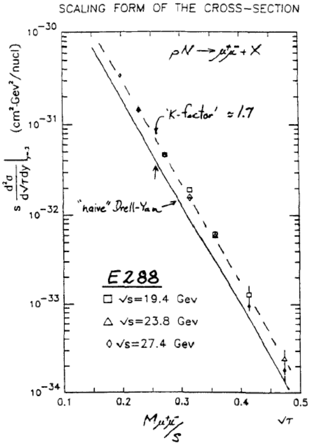
with $N_c = 3$, $\alpha_{EM} \equiv e^2/(4\pi)$, $ee_i$ is the fractional quark charge

# Scaling of the low-$Q$ data



# NLO corrections and the K-factor

Tau-scaling works because radiative corrections to $q\bar{q} \to VX$ are relatively constant at $x \sim 0.1$




## A useful estimate

$$\frac{d\sigma}{dQ^2} \approx \left(\frac{d\sigma}{dQ^2}\right)_{LO}(\tau)\cdot K_{NLO}(Q),$$

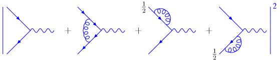
where $K_{NLO} = 1 + \kappa\alpha_s(Q)$ with $\kappa = 3 \pm 1$

(also applies to $W$, $Z$, ... production)

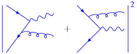
**Exercise: show that $K \approx 1.65$ ($1.35$) at $Q = 5$ ($90$) GeV**

# NLO cross section

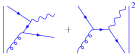
- NLO: $(\alpha_s^{(1)})$ virtual corrections $(q\overline{q'})_{virt}$



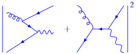
- NLO: $(\alpha_s^{(1)})$ real emission diagrams $(q\overline{q'})_{real}$



- NLO: $(\alpha_s^{(1)})$ real emission diagrams $(qG)_{real}$



- NLO: $(\alpha_s^{(1)})$ real emission diagrams $(G\overline{q'})_{real}$



## Virtual contributions

The dominant contribution to $\sigma_{tot}$, if $x$ is of order 0.1

$$\sigma_{tot}^{NLO} \sim \left[1 + \frac{\alpha_s}{2\pi} C_F \left(1 + \frac{4\pi^2}{3}\right)\right] \sigma_{tot}^{LO}$$
$$\sim \left[1 + 3.005\alpha_s\right] \sigma_{tot}^{LO}$$

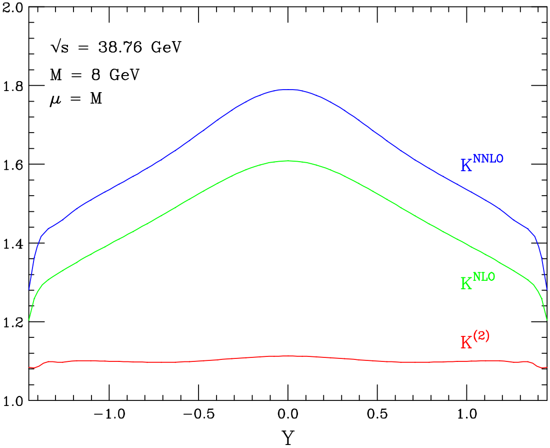
At $x \to 0$ or 1, $ln(x)$ or $ln^p(1-x)/(1-x)_+$ terms are enhanced; the NLO factor is not constant!

## $2 \to 3$ contributions

Generate $Q_T \neq 0$, non-trivial $\theta_*, \varphi_*$ dependence

# NNLO cross sections for low-$Q$ DY process

*Anastasiou, Dixon, Melnikov, Petriello, 2003-05*




$K^{NLO} \approx 1.6$ at $y = 0$

$K^{NLO} \approx 1.4$ at $y = 1$

Compare with $1 + 3\alpha_s(8) \approx 1.56$

$K^{(2)} = \sigma_{NNLO}/\sigma_{NLO}$ – uniform enhancement over NLO by $\sim 8\%$

# Classical measurements in low-$Q$ DY process

1. Sea quark PDFs $\bar{q}_i(x, Q)$ from rapidity $(y)$ distributions **(lecture 2)**

2. Spins of $\gamma^*$ and quarks from angular distributions of decay leptons

# 2. Lepton distributions in the rest frame of $\gamma^*$

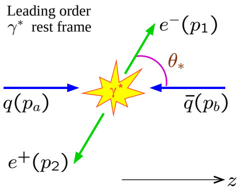
The Born cross section for $q_j \bar{q}_{\bar{k}} \rightarrow V \rightarrow \ell \bar{\ell}'$ is

$$\frac{d\sigma}{dQ^2 dy d\cos\theta_*} \propto$$

$$\times \sum_{j,\bar{k}=u,\bar{u},d,\bar{d},\dots} \Big\{ (f_R^2 + f_L^2)(g_{L,j\bar{k}}^2 + g_{R,j\bar{k}}^2)(1+\cos^2\theta_*)\left[q_j(x_A)\bar{q}_{\bar{k}}(x_B) + \bar{q}_{\bar{k}}(x_A) q_j(x_B)\right]$$

$$+ (f_R^2 - f_L^2)(g_{L,j\bar{k}}^2 - g_{R,j\bar{k}}^2)(2\cos\theta_*)\left[q_j(x_A)\bar{q}_{\bar{k}}(x_B) - \bar{q}_{\bar{k}}(x_A) q_j(x_B)\right] \Big\}$$

- $f_L$, $f_R$ are left-handed and right-handed $V \ell \bar{\ell}'$ couplings
- $g_{L,j\bar{k}}$, $g_{R,j\bar{k}}$ are left-handed and right-handed $V q_j \bar{q}_{\bar{k}}$ couplings




The $E$, $p_x$, $p_y$, $p_z$ components are

$$p_a = \frac{Q}{2}(1,0,0,1)\,;\; p_b = \frac{Q}{2}(1,0,0,-1)\,;$$

$$p_1 = \frac{Q}{2}(1,0,0,\cos\theta_*)\,;\; p_2 = \frac{Q}{2}(1,0,0,-\cos\theta_*)\,;$$

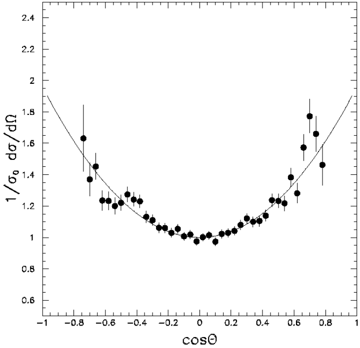
# The Born cross section

$$\frac{d\sigma}{dQ^2 dy d\cos\theta_*} \propto$$

$$\times \sum_{j,\bar{k}=u,\bar{u},d,\bar{d},\dots} \left\{ (f_R^2 + f_L^2)(g_{L,j\bar{k}}^2 + g_{R,j\bar{k}}^2)(1+\cos^2\theta_*)\left[q_j(x_A)\bar{q}_{\bar{k}}(x_B) + \bar{q}_{\bar{k}}(x_A)q_j(x_B)\right]\right.$$

$$\left. +(f_R^2 - f_L^2)(g_{L,j\bar{k}}^2 - g_{R,j\bar{k}}^2)(2\cos\theta_*)\left[q_j(x_A)\bar{q}_{\bar{k}}(x_B) - \bar{q}_{\bar{k}}(x_A)q_j(x_B)\right]\right\}$$

- The $2\cos\theta_*$ term vanishes in the parity-conserving case ($f_L = f_R$ or $g_L = g_R$)
- The $(1+\cos^2\theta_*)$ dependence in the experimental data confirms the vector (spin-1) nature of low-$Q$ Drell-Yan process



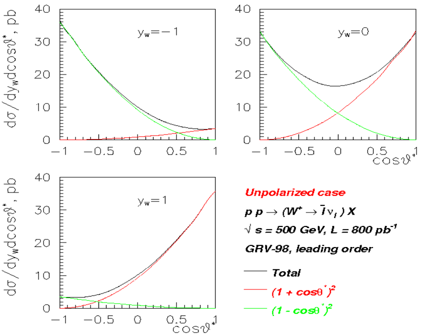
# The Born cross section

$$\frac{d\sigma}{dQ^2 dy d\cos\theta_*} \propto$$

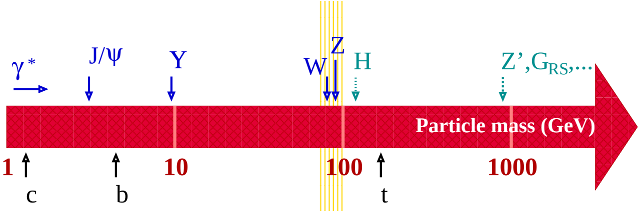
$$\times \sum_{j,\bar{k}=u,\bar{u},d,\bar{d},\dots} \left\{ (f_R^2 + f_L^2)(g_{L,j\bar{k}}^2 + g_{R,j\bar{k}}^2)(1+\cos^2\theta_*)\left[q_j(x_A)\bar{q}_{\bar{k}}(x_B) + \bar{q}_{\bar{k}}(x_A)q_j(x_B)\right]\right.$$

$$\left. +(f_R^2 - f_L^2)(g_{L,j\bar{k}}^2 - g_{R,j\bar{k}}^2)(2\cos\theta_*)\left[q_j(x_A)\bar{q}_{\bar{k}}(x_B) - \bar{q}_{\bar{k}}(x_A)q_j(x_B)\right]\right\}$$

- $W$ boson production: $f_R = g_R = 0$
- $W$ cross section depends on two functions $(1 \pm \cos\theta_*)^2$ weighted by different parton luminosities
- non-trivial correlation between $y$ and $\theta_*$ in the acceptance, etc.

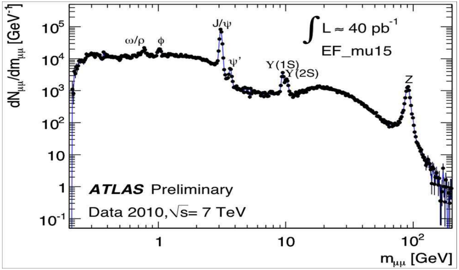


# Final states in DY-like processes




## $W$ and $Z$ boson production

- good convergence of the $\alpha_s$ series
- small backgrounds
- separation of PDF flavors (via the CKM matrix)
- sensitivity to new physics




$Z$ pole and $\gamma^*$ continuum in $\ell^+\ell^-$ production

# Leptonic vs. hadronic decay modes

The $W$ and $Z$ branching ratios Br$_i \equiv \Gamma_i/\Gamma$ are

- Br $[W \to \ell\nu_\ell] \approx 3 \times 11\%$, Br $[W \to$ jets$] \approx 68\%$
- Br $[Z \to \ell^+\ell^-] = 3 \times 3.36\%$, Br $[Z \to \nu_\ell\bar{\nu}_\ell] = 3 \times 6.67\%$, Br $[Z \to$ jets$] \approx 70\%$

At $\sqrt{s}$ of a few TeV, hadronic $W, Z$ decays are hard to observe because of the large background from QCD jets

The most viable decay modes are

- $Z \to e^+e^-$, $Z \to \mu^+\mu^-$
- $W \to e + \nu_e$, $W \to \mu + \nu_\mu$, with neutrinos identified by missing transverse energy $\not{E}_T$

# $W$ and $Z$ observables

- **Total cross sections**

$$\sigma_Z = \int \frac{d\sigma\left(pp \to (Z \to e^+e^-)X\right)}{d\vec{p}_{e^+}\,d\vec{p}_{e^-}} d\vec{p}_{e^+}\,d\vec{p}_{e^-}$$

- Rapidity distributions and asymmetries

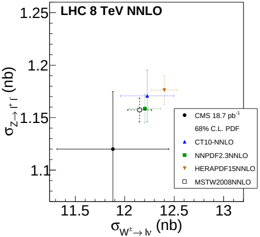
$$\frac{d\sigma_{W,Z}}{dQ^2dy}, \text{ etc.}$$

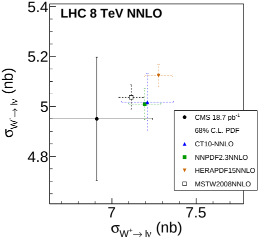
- $W$ boson mass $M_W$

- Transverse momentum and related distributions

$$\frac{d\sigma_{W,Z}}{dQ_T^2}, \frac{d\sigma_{W,Z}}{d(p_T^e)^2}, \frac{d\sigma_{W,Z}}{d\left(M_T^{\ell\nu}\right)^2}$$

# Total $W$ and $Z$ cross sections




Provide tests of perturbative QCD and collider luminosity with accuracy 3-5%

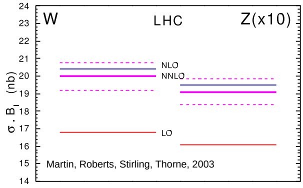
Require understanding of

- $\mathcal{O}(\alpha_s^2)$, or NNLO, QCD corrections
- $\mathcal{O}(\alpha)$, or NLO, EW corrections
- PDF uncertainties
- Experimental acceptance
- QCD and EW showering (all-orders resummations)

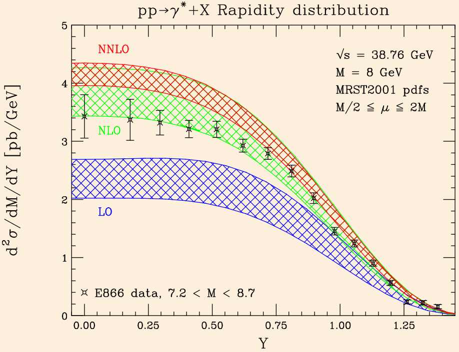
## NNLO total section $\sigma_{tot}(AB \to W, Z)$

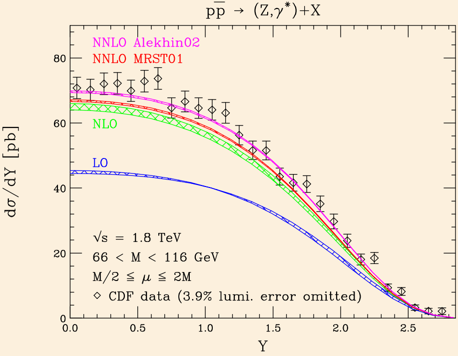
*(Hamberg, van Neerven, Matsuura; Harlander, Kilgore)*

- Scale dependence of order 1%
- NNLO $K$-factor is about 1.04 at the Tevatron and 0.98 at the LHC *(MRST'03)*

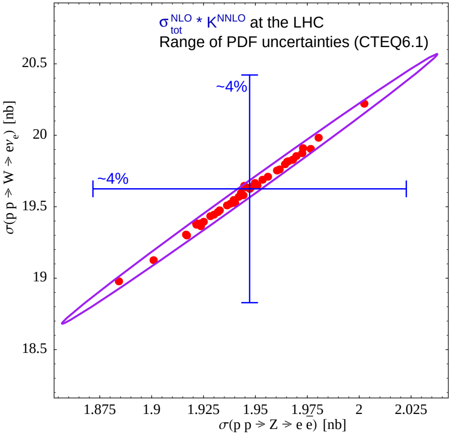


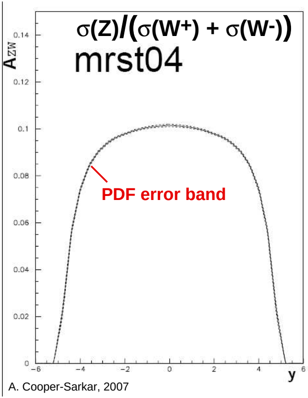

## NNLO differential cross sections *(Anastasiou, Dixon, Melnikov, Petriello, 2003-05)*



# Ratios of $W$ and $Z$ cross sections




Radiative contributions, PDF dependence have similar structure in $W$, $Z$, and alike cross sections; cancel well in Xsection ratios

# $W$ and $Z$ observables

- Total cross sections

$$\sigma_Z = \int \frac{d\sigma\left(pp \to \left(Z \to e^+e^-\right)X\right)}{d\vec{p}_{e^+}d\vec{p}_{e^-}} d\vec{p}_{e^+}d\vec{p}_{e^-}$$

- **Rapidity distributions and asymmetries**

$$\frac{d\sigma_{W,Z}}{dQ^2dy}, \text{ etc.}$$

- $W$ boson mass $M_W$

- Transverse momentum and related distributions

$$\frac{d\sigma_{W,Z}}{dQ_T^2}, \frac{d\sigma_{W,Z}}{d(p_T^e)^2}, \frac{d\sigma_{W,Z}}{d\left(M_T^{\ell\nu}\right)^2}$$

# Charged lepton asymmetry at the Tevatron

$y_e$ and $\eta \approx y_e$ are rapidity and pseudorapidity of an electron from $W$ decay

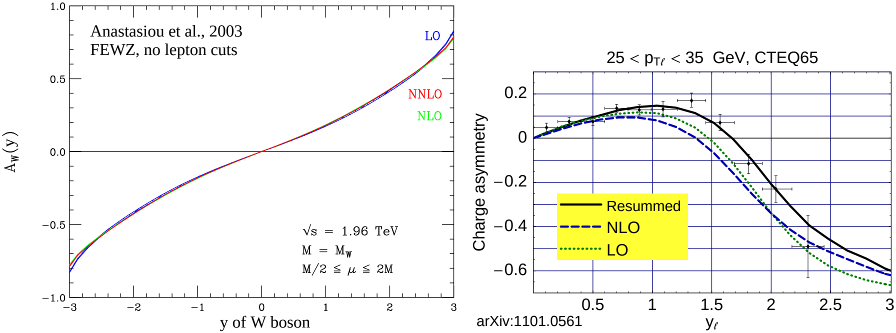
$$A_{ch}(y_e) \equiv \frac{\frac{d\sigma^{W^+}}{dy_e} - \frac{d\sigma^{W^-}}{dy_e}}{\frac{d\sigma^{W^+}}{dy_e} + \frac{d\sigma^{W^-}}{dy_e}}$$

- related to the boson Born-level asymmetry when $y_e$ is large

$$A_{ch}(y) \stackrel{y \to y_{max}}{\longrightarrow} \frac{r(x_B) - r(x_A)}{r(x_B) + r(x_A)},\ r(x) \equiv \frac{d(x, M_W)}{u(x, M_W)}$$

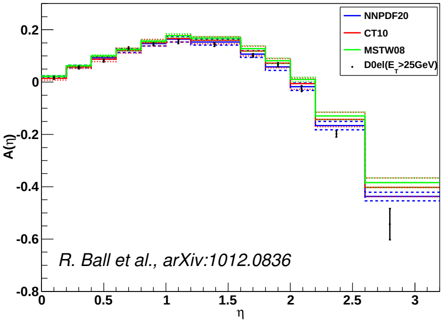
- constrains the PDF ratio $d(x, M_W)/u(x, M_W)$ at $x \to 1$
- In experimental analyses, a selection cut $p_{Te} > p_{Te}^{min}$ is imposed

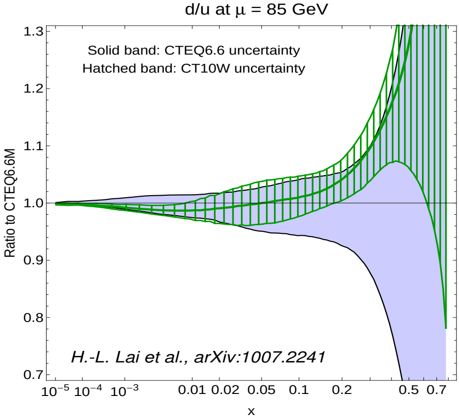
# Charge asymmetry in $p_T^e$ bins (CDF Run-2)




- Without $p_{Te}$ cuts, $A_{ch}(y_e)$ is not sensitive to radiative contributions
- With $p_{Te}$ cuts, $A_{ch}(y_e)$ is sensitive to small-$Q_T$ resummation

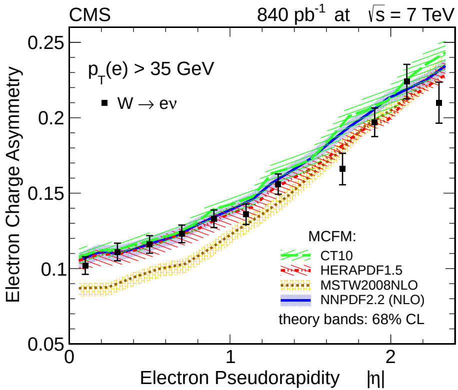
# Impact of the Tevatron $A_{ch}$ data on PDFs




- The $A_{ch}$ data distinguish between the PDF models, reduce the PDF uncertainty
- Very precise data! ⇒ Many subtleties in their analysis

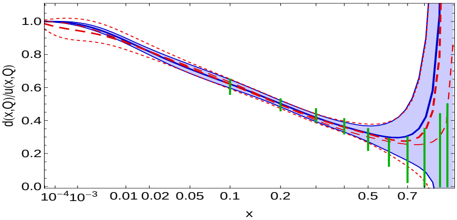
# Charge asymmetry at the LHC



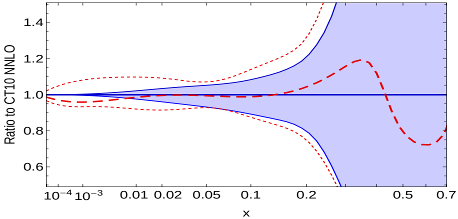

Sensitive both to $d/u$ at $x > 0.1$ **and** $\bar{u}/\bar{d}$ at $x \sim 0.01$ (not constrained well by other experiments)

# $d/u$ and $\bar{d}/\bar{u}$: CT1X NNLO (prelim.) vs. CT10 NNLO and CJ 12 analysis of large-x DIS

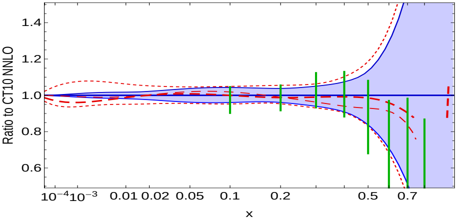
PRELIMINARY; Q=10 GeV
CT10 NNLO (blue), CT1X NNLO (red); CJ12 (green)




PRELIMINARY; dbar(x,Q)/ubar(x,Q) at Q= 10 GeV
CT10 NNLO (blue), CT1X NNLO (red); CJ12 (green)




PRELIMINARY; d(x,Q)/u(x,Q); Q=10 GeV
CT10 NNLO (blue), CT1X NNLO (red); CJ12 (green)




CT1X PDF uncertainty is larger at $x \to 0$ and $1$, is compatible with the $d/u$ band from the CJ12 analysis *(Owens et al., 1212.1702)* of large-$x$ DIS for PDFs+nuclear+higher-twist corrections

# $W$ and $Z$ observables

- Total cross sections

$$\sigma_Z = \int \frac{d\sigma\left(pp \to (Z \to e^+e^-)X\right)}{d\vec{p}_{e+}\, d\vec{p}_{e-}} d\vec{p}_{e+}\, d\vec{p}_{e-}$$

- Rapidity distributions and asymmetries

$$\frac{d\sigma_{W,Z}}{dQ^2 dy}, \text{ etc.}$$

- $W$ **boson mass** $M_W$

- Transverse momentum and related distributions

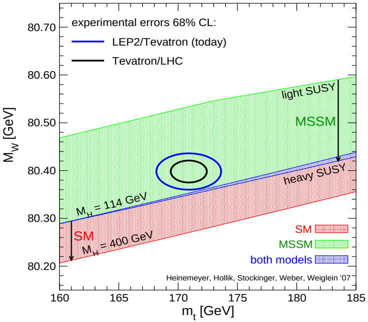
$$\frac{d\sigma_{W,Z}}{dQ_T^2}, \frac{d\sigma_{W,Z}}{d(p_T^e)^2}, \frac{d\sigma_{W,Z}}{d\left(M_T^{\ell\nu}\right)^2}$$

## Constraints on the Higgs sector and $W$ boson mass $M_W$

**Both the Tevatron and LHC measure $M_W$. It provides key constraints on Higgs mass $M_H$ in electroweak fits.**

**In SM:**

$$M_W = 80.3827 - 0.0579 \ln\left(\frac{M_H}{100\text{ GeV}}\right) - 0.008 \ln^2\left(\frac{M_H}{100\text{ GeV}}\right)$$
$$+0.543\left(\left(\frac{m_t}{175\text{ GeV}}\right)^2 - 1\right) - 0.517\left(\frac{\Delta\alpha^{(5)}_{had}(M_Z)}{0.0280} - 1\right) - 0.085\left(\frac{\alpha_s(M_Z)}{0.118} - 1\right)$$



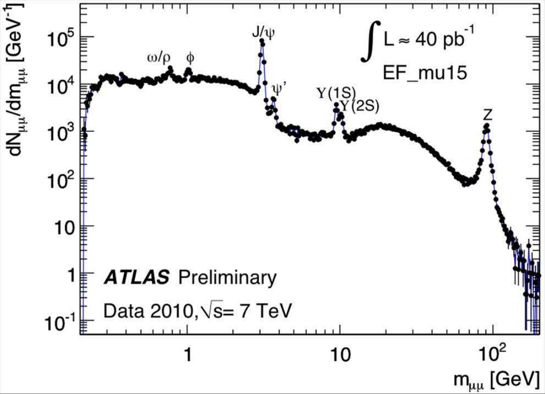

**SM band:** $114 \leq M_H \leq 400$ GeV
**SUSY band:** random scan

To know $M_H$ to better than $\pm 50$ GeV (50%) from the fit, $M_W$ must be measured to better than $\pm 0.030$ GeV (0.03%) – the accuracy that is already reached!

# Question to the audience

In $p\bar{p} \to (Z \to \mu^+\mu^-)X$, the value of $M_Z$ is found from the resonance in $d\sigma/dM_{\mu^+\mu^-}$

But in $p\bar{p} \to (W \to \ell\nu)X$, $d\sigma/dM_{\ell\nu}$ is not observed, because the $\nu$'s longitudinal momentum $p_{\nu 3}$ is not measured!

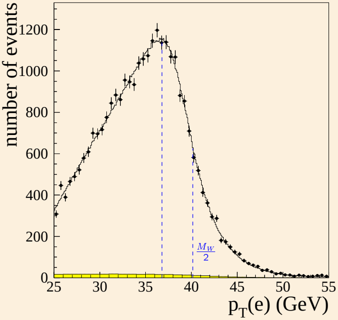



**In this situation, which trick is used to measure $M_W$?**

# Jacobian peaks in distributions of decay leptons

Certain distributions contain a quasi-resonance (the Jacobian peak) that indicates the value of $M_W$

## Electron's transverse momentum $p_T^e$



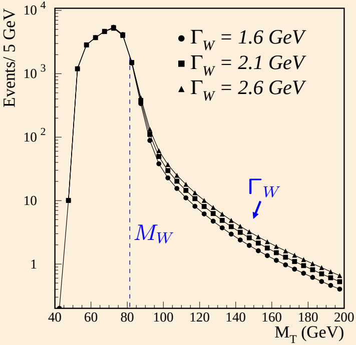

Jacobian peak at $p_T^e = M_W/2 \approx 40$ GeV

# Jacobian peaks in distributions of decay leptons

Certain distributions contain a quasi-resonance (the Jacobian peak) that indicates the value of $M_W$

## Leptonic transverse mass $M_T^{e\nu}$

*(Smith, van Neerven, Vermaseren, 1983)*




$$M_T^{e\nu} \equiv 2(p_T^e p_T^\nu - \vec{p}_T^{\,e} \cdot \vec{p}_T^{\,\nu})$$

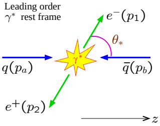
Jacobian peak at $M_T^{e\nu} = M_W$

# The origin of the Jacobian peak

In the $W$ rest frame, for $Q = M_W$:

$$p_T^e = |\vec{p}_1| \sin\theta_* = \frac{M_W}{2} \sin\theta_*$$

$$\frac{d\sigma}{d\cos\theta_*} = \sum_j F_j(Q, Q_T, y) a_j(\theta_*, \varphi_*)$$




$a_1 = 1 + \cos^2\theta_*$, $a_2 = 2\cos\theta_*$, etc. (smooth functions)

$$\frac{d\sigma}{dp_T^e} = \underbrace{\left|\frac{d\cos\theta_*}{dp_T^e}\right|}_{\text{Jacobian}} \frac{d\sigma}{d\cos\theta_*} = \frac{1}{\sqrt{1-\left(\frac{2p_T^e}{M_W}\right)^2}} \frac{4p_T^e}{M_W^2} \frac{d\sigma}{d\cos\theta_*}$$

$$\frac{d\sigma}{dp_T^e} \to \infty \text{ if } p_T^e \to M_W/2 \text{ (!)}$$

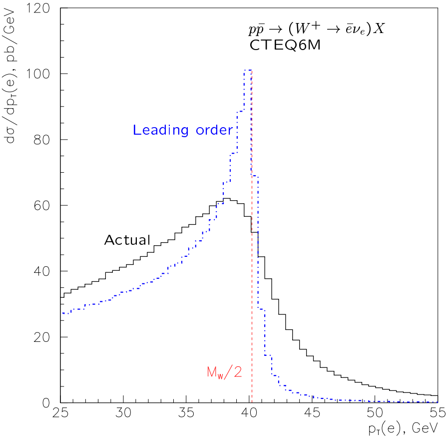
# The origin of the Jacobian peak

If $Q_T = 0$: $(p_T^e)_{\text{lab frame}} = (p_T^e)_{\text{CS frame}}$

(the boost from the CS frame to the lab frame is along the $z-$axis)

Corrections to $d\sigma/dp_T^e$ are of order

- $\mathcal{O}(\Gamma_W^2/M_W^2)$ due to the non-zero $W$ width $\Gamma_W$ ($Q \neq M_W$)
- $\mathcal{O}(Q_T/Q)$ due to the boost $\Rightarrow$ sensitivity to the shape of $d\sigma/dQ_T$ (soft radiation) at $Q_T \ll Q$



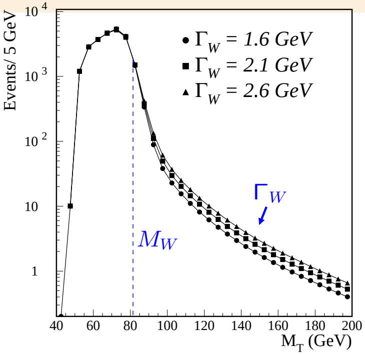

A similar Jacobian peak is present in $d\sigma/dp_T^\nu$

# More on lepton transverse mass

## Exercise

Assuming $Q_T = 0$, verify that there is a Jacobian peak in $d\sigma/dM_T^{e\nu}$ at $M_T^{e\nu} = M_W$

- Corrections to $d\sigma/dM_T^{e\nu}$ are of order $\mathcal{O}(Q_T^2/Q^2) \Rightarrow$ reduced sensitivity to small-$Q_T$ soft contributions
- $d\sigma/dM_T^{e\nu}$, $d\sigma/dp_T^e$, and $d\sigma/dp_T^\nu$ are commonly used to measure $M_W$. $\Gamma_W$ is found from $d\sigma/dM_T^{e\nu}$ at large $M_T^{e\nu}$



# Multi-scale factorization (resummation)

So far, we discussed QCD factorization for observables dependent on one hard scale $Q$

However, to achieve a small error of order $10$ MeV in $W$ mass measurement, one must accurately predict transverse momentum ($Q_T$) distributions of $W$ bosons. This prediction depends on two momentum scales: $Q \approx M_W \approx 80$ GeV, and $Q_T$.

Since $Q_T \ll Q$ in the majority of $W$ production events, the one-scale factorized cross section does not converge because of large logarithms $\ln^p(Q/Q_T)$, $p > 0$

One must use another formalism (**transverse-momentum dependent (TMD) factorization**) to obtain a converging prediction through **resummation** of $\ln^p(Q/Q_T)$ terms to all orders in $\alpha_s$

# Factorization for one-scale cross sections

Scale dependence of the renormalized QCD charge $g(\mu)$ and fermion masses $m_f(\mu)$:

$$\mu\frac{dg(\mu)}{d\mu} = \beta(g(\mu)), \qquad \mu\frac{dm_f(\mu)}{d\mu} = -\gamma_m(g(\mu))m_f(\mu)$$

The RG equations predict that $\alpha_s(\mu) \to 0$ and $m_f(\mu) \to 0$ as $\mu \to \infty$

These features are employed to prove factorization for inclusive Drell-Yan cross sections *(Bodwin, PRD 31, 2616 (1985); Collins, Soper, Sterman, NPB 261, 104 (1985); B308, 833 (1988))*:

$$\frac{d\sigma(Q,\{m_f\})}{d\tau} = \sum_{a,b}\int_{x_A}^{1} d\xi_A \int_{x_B}^{1} d\xi_B \frac{d\widehat{\sigma}\left(\frac{Q}{\mu}, \frac{\tau}{\xi_A\xi_B}, \{m_f = 0\}\right)}{d\tau} f_{a/A}(\xi_A,\mu) f_{b/B}(\xi_B,\mu)$$
$$+\,\mathcal{O}\left(\{m_f^2/\mu^2\}\right)$$

assuming $\mu \sim Q \sim \sqrt{s} \gg \{m_f\}, \Lambda_{QCD}$

# Factorization for one-scale cross sections

$$\frac{d\sigma(Q,\{m_f\})}{d\tau} = \sum_{a,b}\int_{x_A}^{1} d\xi_A \int_{x_B}^{1} d\xi_B \frac{d\widehat{\sigma}\left(\frac{Q}{\mu}, \frac{\tau}{\xi_A\xi_B}, \{m_f = 0\}\right)}{d\tau} f_{a/A}(\xi_A,\mu) f_{b/B}(\xi_B,\mu) + \mathcal{O}\left(\{m_f^2/\mu^2\}\right)$$

- The hard cross section $\widehat{\sigma}$ is infrared-safe: $\lim_{\{m_f\to 0\}}\widehat{\sigma}(\{m_f\})$ is finite and can be computed as a series in $\alpha_s(\mu)$
- Collinear logarithms are subtracted from $\widehat{\sigma}$ and resummed in $f(\xi,\mu)$ using DGLAP equations
- Soft-gluon singularities in $\widehat{\sigma}$ vanish when the sum of all Feynman diagrams is integrated over all phase space (Kinoshita-Lee-Nauenberg theorem)

# Factorization for $Q_T$ distributions (two scales)

- Differential distributions may still contain integrable soft singularities of the type $\alpha_s^k \ln^m(Q^2/p_i \cdot p_j)$, e.g., $L \equiv \ln(Q^2/Q_T^2) \gg 1$:

$$
\begin{aligned}
\left.\frac{d\sigma}{dQ^2\,dy\,dQ_T^2}\right|_{Q_T\to 0} \quad &\approx \quad \frac{1}{Q_T^2}\Big\{ \\
& \quad\ \alpha_S\,(L+1) \\
& + \quad \alpha_S^2\left(L^3+L^2+L+1\right) \\
& + \quad \alpha_S^3\left(L^5+L^4+L^3+L^2+L+1\right) \\
& + \quad \ldots\Big\}.
\end{aligned}
$$

The purpose of $Q_T$ resummation is to reorganize this series as

$$
\left.\frac{d\sigma}{dQ^2 dy\, dQ_T^2}\right|_{Q_T\to 0} \approx \frac{1}{Q_T^2}\left\{\alpha_S Z_1 + \alpha_S^2 Z_2 + \ldots\right\},
$$

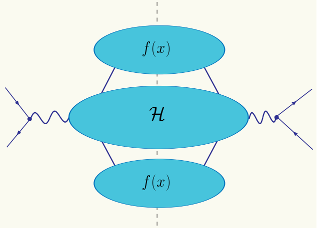
where $\alpha_S^{n+1} Z_{n+1} \ll \alpha_S^n Z_n$:

$$
\begin{aligned}
\alpha_S Z_1 \quad &\sim \quad \alpha_S(L+1) + \alpha_S^2(L^3+L^2) + \alpha_S^3(L^5+L^4) + \ldots \quad &| A_1, B_1, \mathcal{C}_0\ ; \\
\alpha_S^2 Z_2 \quad &\sim \quad \alpha_S^2(L+1) + \alpha_S^3(L^3+L^2) + \ldots \quad &| A_2, B_2, \mathcal{C}_1\ ; \\
\alpha_S^3 Z_3 \quad &\sim \quad \alpha_S^3(L+1) + \ldots \quad &| A_3, B_3, \mathcal{C}_2\ . \\
&\ldots
\end{aligned}
$$

# QCD factorization at large and small $Q_T$

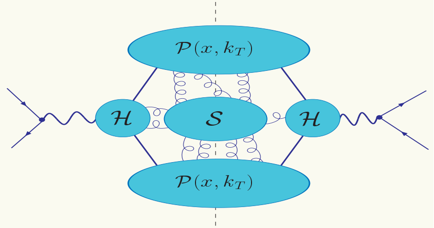
Finite-order (FO) factorization

$$\Lambda^2_{QCD} \ll q_T^2 \sim Q^2$$

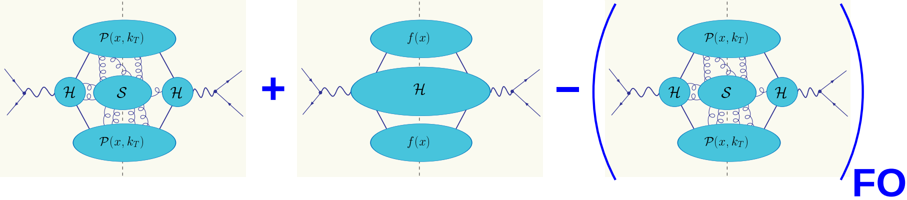



Small-$q_T$ factorization

$$\Lambda^2_{QCD} \ll q_T^2 \ll Q^2$$




Solution for all $q_T$:



# Factorization at $Q_T \ll Q$

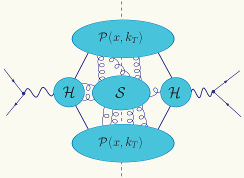
**(Collins, Soper, Sterman, 1985)**

Realized in space of the impact parameter $b$

$$\left.\frac{d\sigma_{AB\to VX}}{dQ^2 dy dq_T^2}\right|_{q_T^2 \ll Q^2} = \sum_{flavors} \int \frac{d^2b}{(2\pi)^2} e^{-i\vec{q}_T\cdot\vec{b}} \widetilde{W}_{ab}(b, Q, x_A, x_B)$$

$$\widetilde{W}_{ab}(b, Q, x_A, x_B) = |\mathcal{H}_{ab}|^2 \; e^{-\mathcal{S}(b,Q)} \overline{\mathcal{P}}_a(x_A, b) \overline{\mathcal{P}}_b(x_B, b)$$

$\mathcal{H}_{ab}$ is the hard vertex, $\mathcal{S}$ is the soft (Sudakov) factor, $\overline{\mathcal{P}}_a(x, b)$ is the unintegrated PDF




For $b \ll 1$ GeV$^{-1}$, $\widetilde{W}_{ab}(b, Q, x_A, x_B)$ is calculable in perturbative QCD; at $Q \sim M_Z$, this region dominates the resummed cross section

# Nonperturbative contributions at large $b$

At $b \gtrsim 1$ GeV$^{-1}$, the leading nonperturbative contribution is approximated as $\exp\left(-a(Q)b^2\right)$, where $a(Q)$ is an effective "nonperturbative parton $\langle k_T^2 \rangle/4$" inside the proton

The RG invariance suggests that

$$a(Q) \approx a_1 + a_2 \ln Q,$$

where $a_{1,2} \sim \Lambda_{QCD}^2$, and $a_2$ is process-independent

The $\ln Q$ growth of $a(Q)$ is indeed observed in the Drell-Yan and $Z$ $p_T$ data

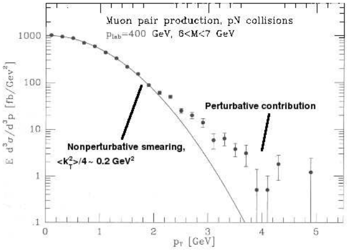
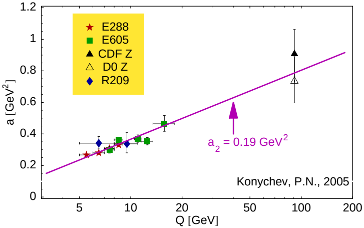
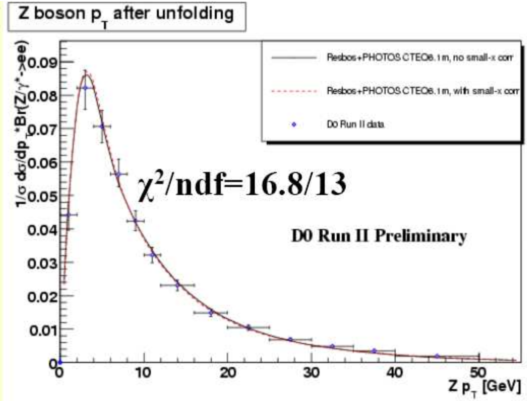



Fig. 9.2. The lepton pair transverse momentum from the CFS collaboration [4]. The curve corresponds to a Gaussian intrinsic $k_T$ distribution for the annihilating



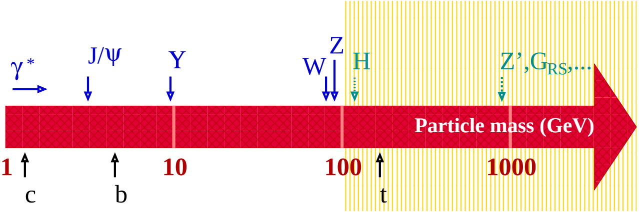
# An example of the resummed cross section

## $Z$ production at the Tevatron vs. resummed NLO (*Balazs, Ladinsky, PN, Yuan*)



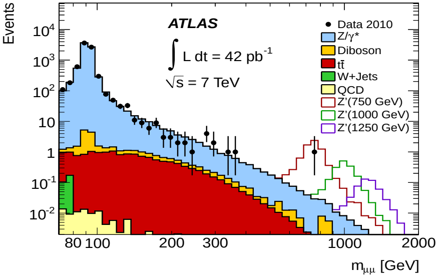

In this case, precise predictions for $d\sigma/dQ_T$ are employed to measure $M_W$ with high accuracy

# Final states in DY-like processes

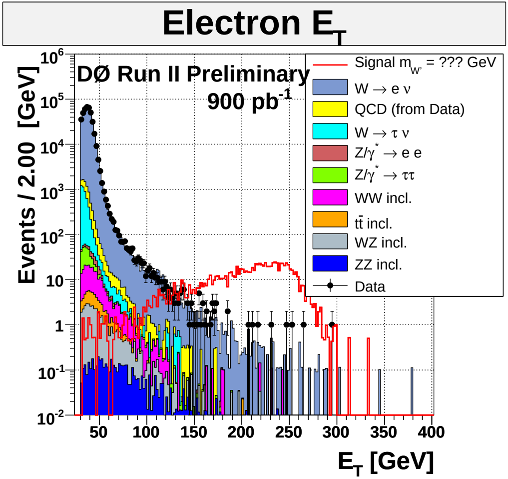



## New physics at $Q > 100$ GeV

- Indirect constraints from electroweak precision measurements
- direct new physics searches

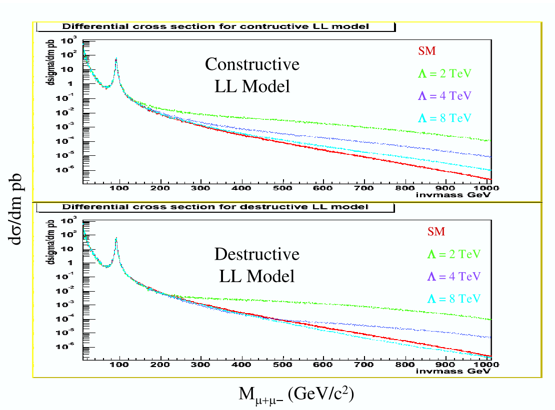


# Search for heavy states at DO



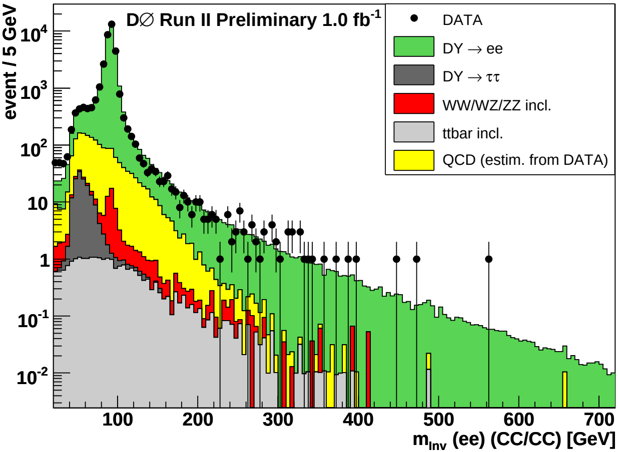

$$W' \to \ell\nu$$

# Search for heavy states at DO



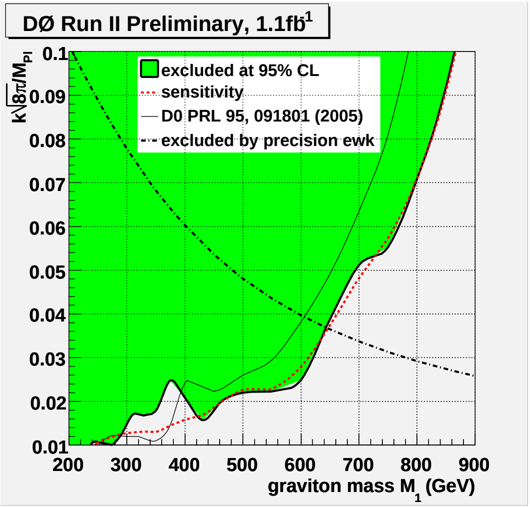

*Leptoquark* $\rightarrow \mu\mu$

# Search for heavy states at DO




*Contact interactions:* $p\bar{p} \to e(e^* \to e\gamma)X$

# Search for heavy states at DO




*Randall-Sundrum graviton* $\to ee, \gamma\gamma$

# Summary

Essential applications of Drell-Yan-like processes

- clean tests of QCD factorization
- studies of the nucleon structure (quark sea, flavor separation,...)
- "standard candle" processes (NNLO,...)
- electroweak precision measurements
- searches for new physics

Many interesting topics were not covered

- Polarized Drell-Yan-like processes (measurements of new nucleon structure functions)
- Connections to $k_T$ factorization
- Various resummations ($Q_T$, small $x$, threshold, heavy-quark....)
- Drell-Yan production in heavy-ion scattering