Electroweak Theory of the Standard Model (II)

Qing-Hong Cao

Peking University

What is the most important property of a particle?

THE MASS!

标准模型的规范对称性

$$\underbrace{SU(3)_{\text{Color}}}_{\text{QCD}} \otimes \underbrace{SU(2)_{\text{Left}} \otimes U(1)_{\text{Hyper charge}}}_{\text{WEAK} \oplus \text{QED}}$$

(Strong Interaction)

(Unification of Weak and Electromagnetic)

对称性自发破缺
(希格斯机制)

$U(1)_{\text{E.M.}}$
量子电动力学
(电磁相互作用)


Brout


Englert


Higgs

(1964)

对称性意味着“力”

电磁相互作用（Abelian gauge symmetry）

规范变换

$$\psi(x) \to e^{iq\alpha(x)}\psi(x)$$

$$A_\mu(x) \to A_\mu(x) - \partial_\mu \alpha(x)$$

$$D_\mu \equiv \partial_\mu + iqA_\mu(x)$$

QED

$$\begin{aligned}\mathcal{L} &= \bar{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi \\ &= \bar{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi - qA_\mu \bar{\psi}\gamma^\mu\psi \\ &= \mathcal{L}_{\text{free}} - J^\mu A_\mu\end{aligned}$$

规范对称性要求光子的质量为零

$$\cancel{\frac{1}{2}m_\gamma^2 A^\mu A_\mu}$$

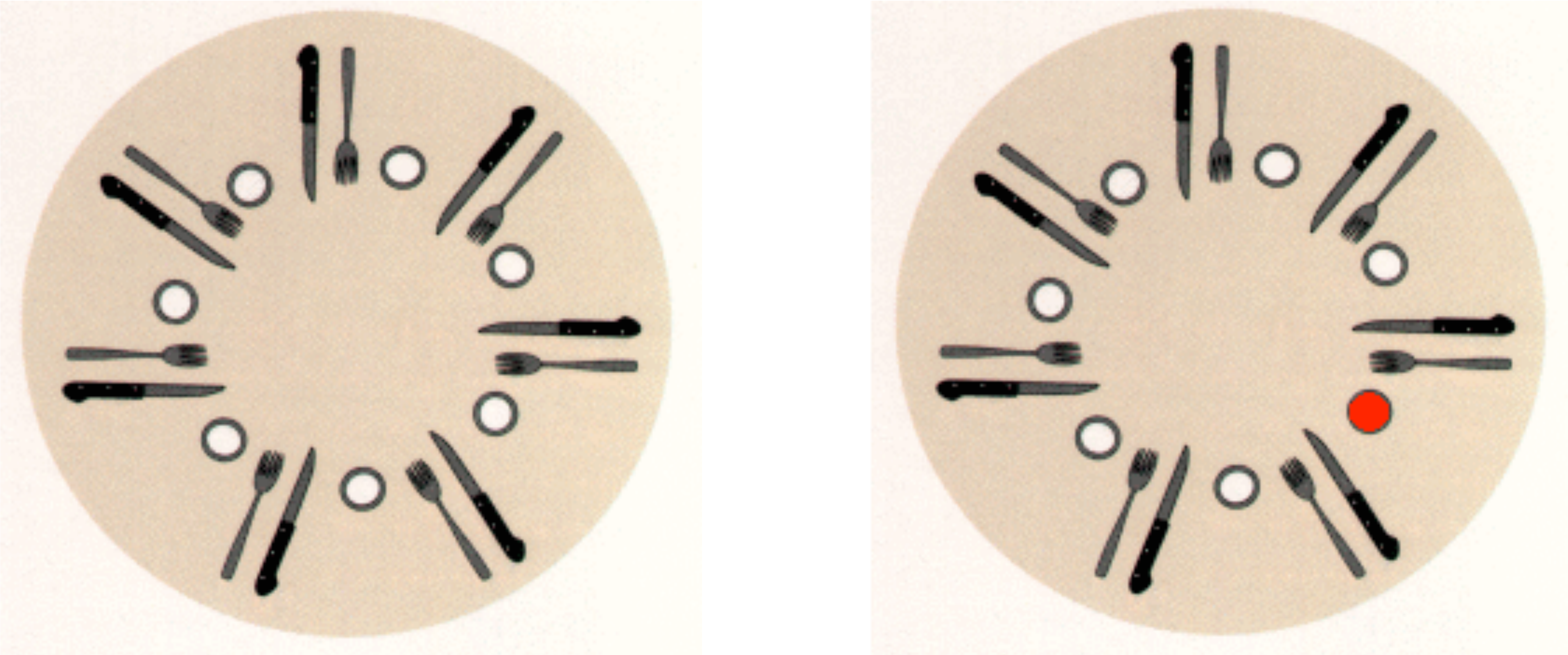
对称性意味着“力”

杨振宁和米尔斯（1954）

定域同位旋对称性意味着有3个无质量的规范波色子和同位旋耦合

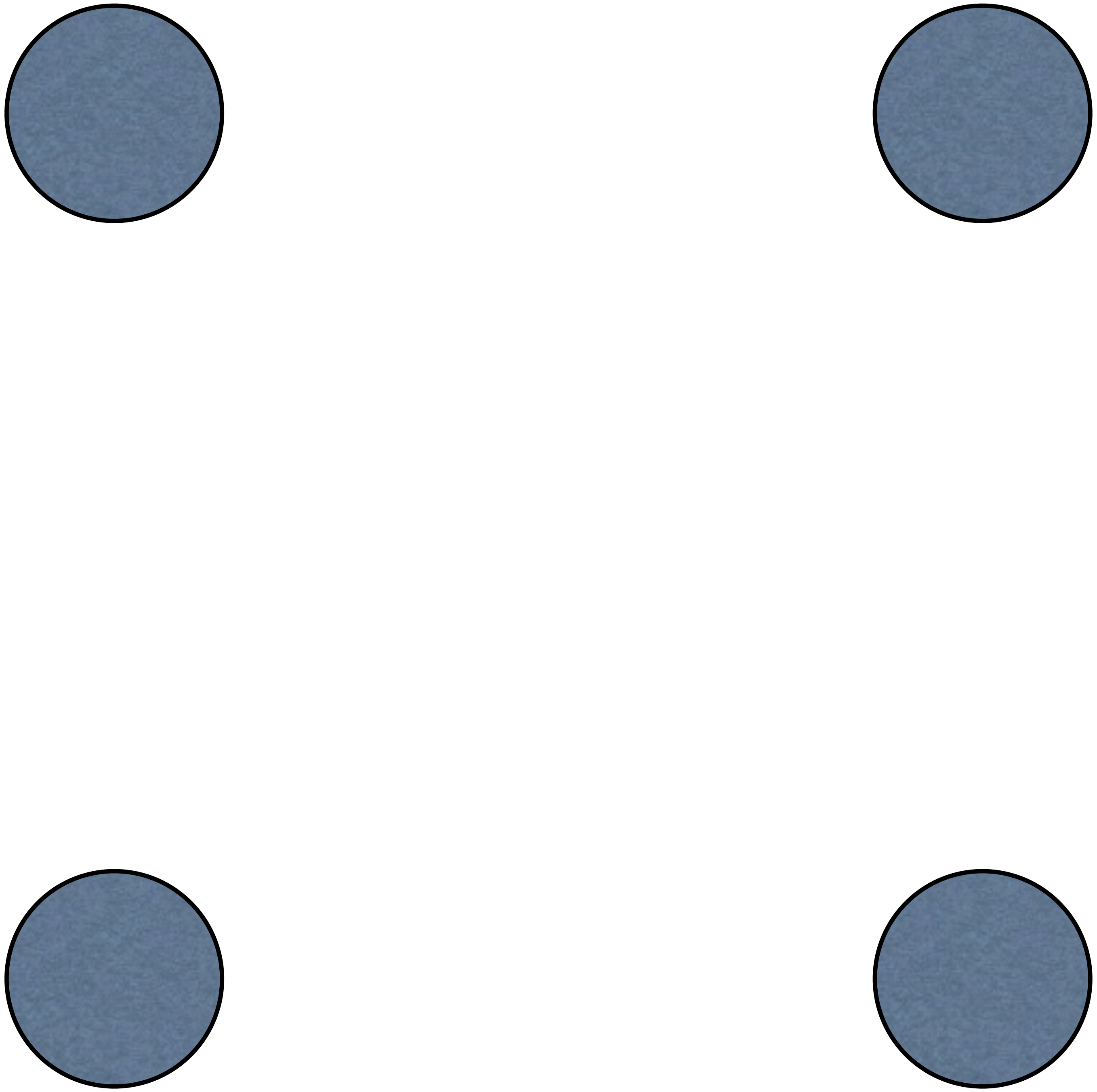
$$\begin{pmatrix} p \\ n \end{pmatrix}$$

对称性自发破缺



对称性自发破缺

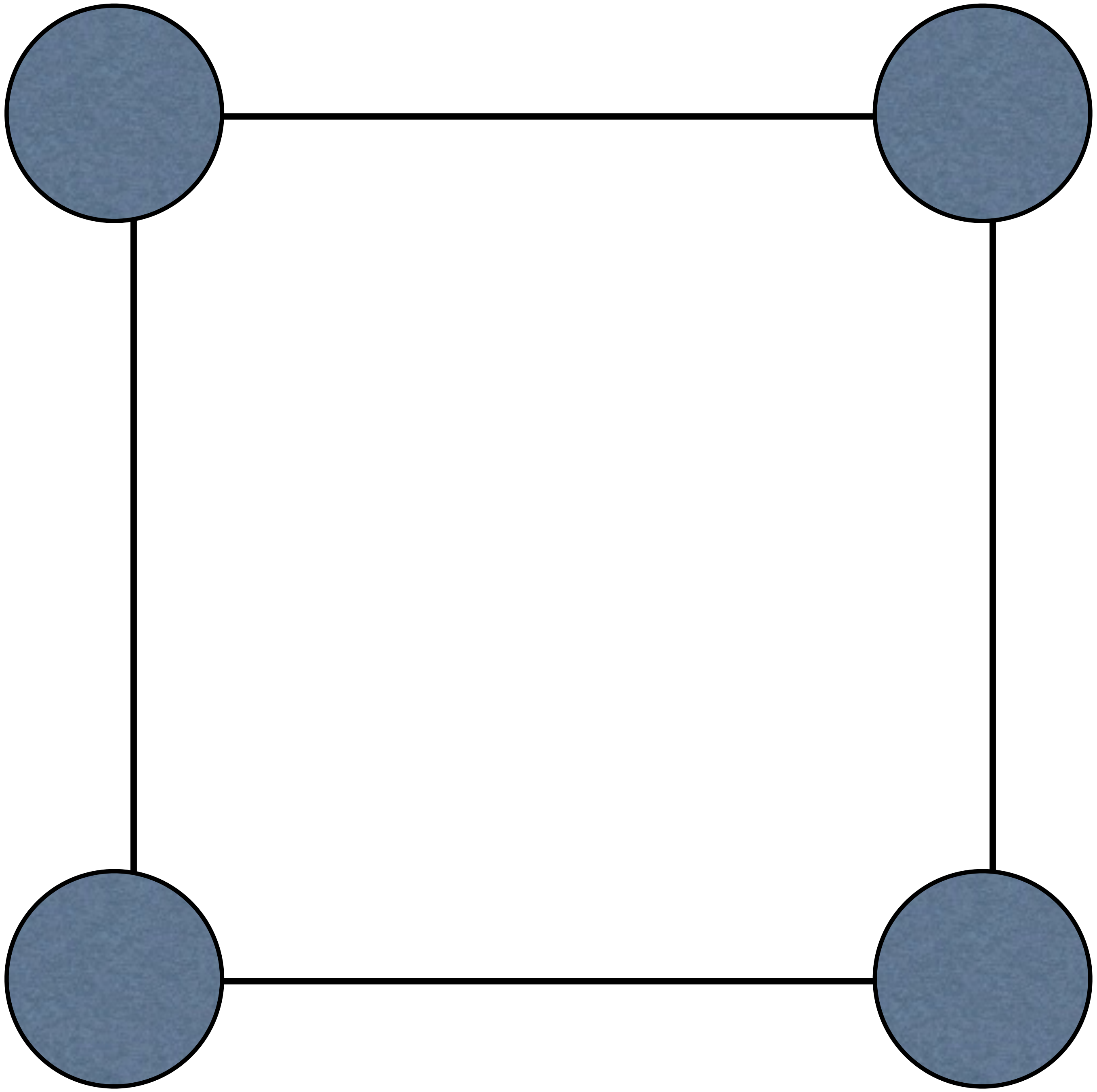
（具有高对称性的系统的解具有较低对称性）



将4个城市
联系起来
所需的
最小路径?

对称性自发破缺

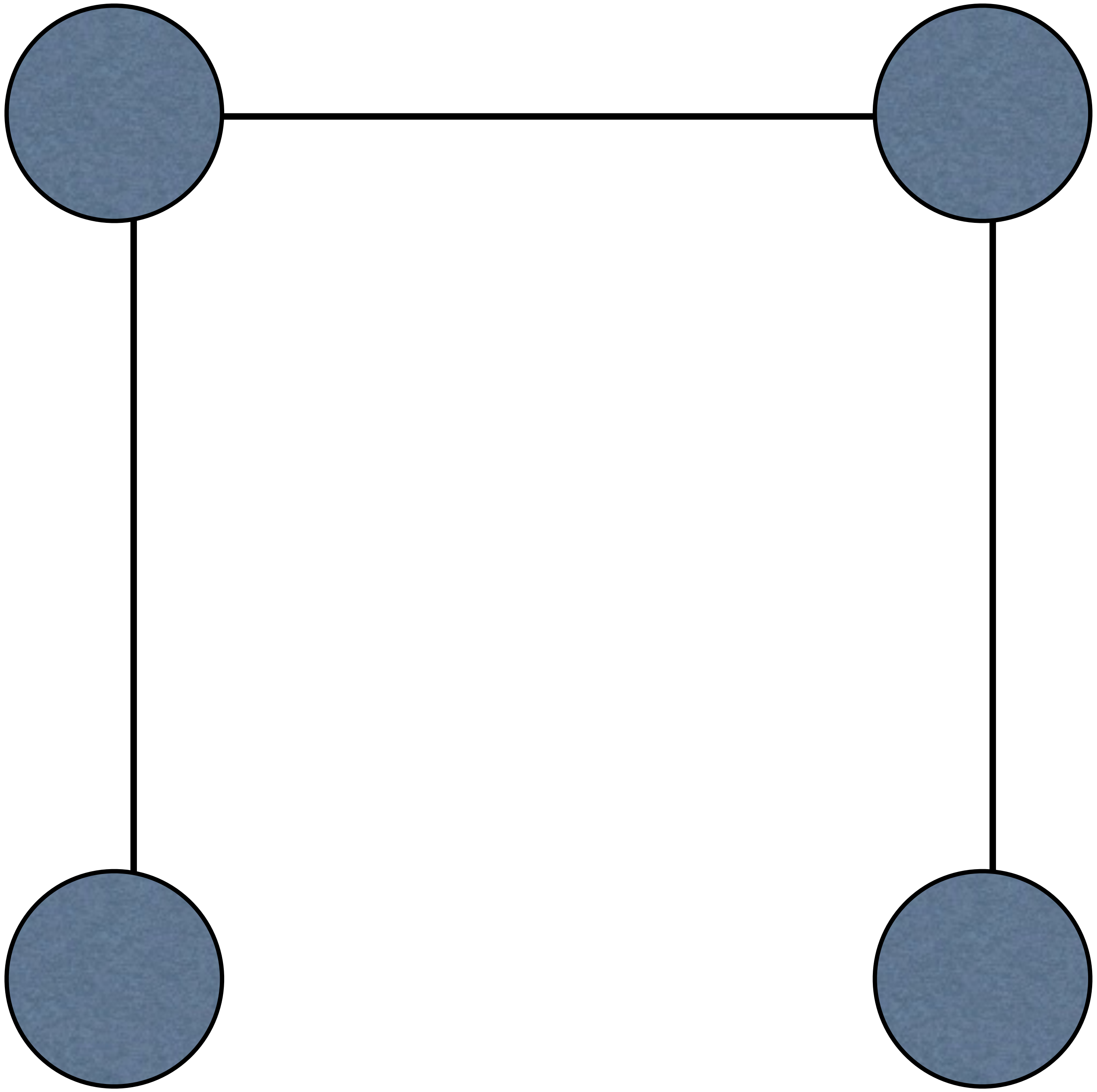
（具有高对称性的系统的解具有较低对称性）



需要花费
4个单位

对称性自发破缺

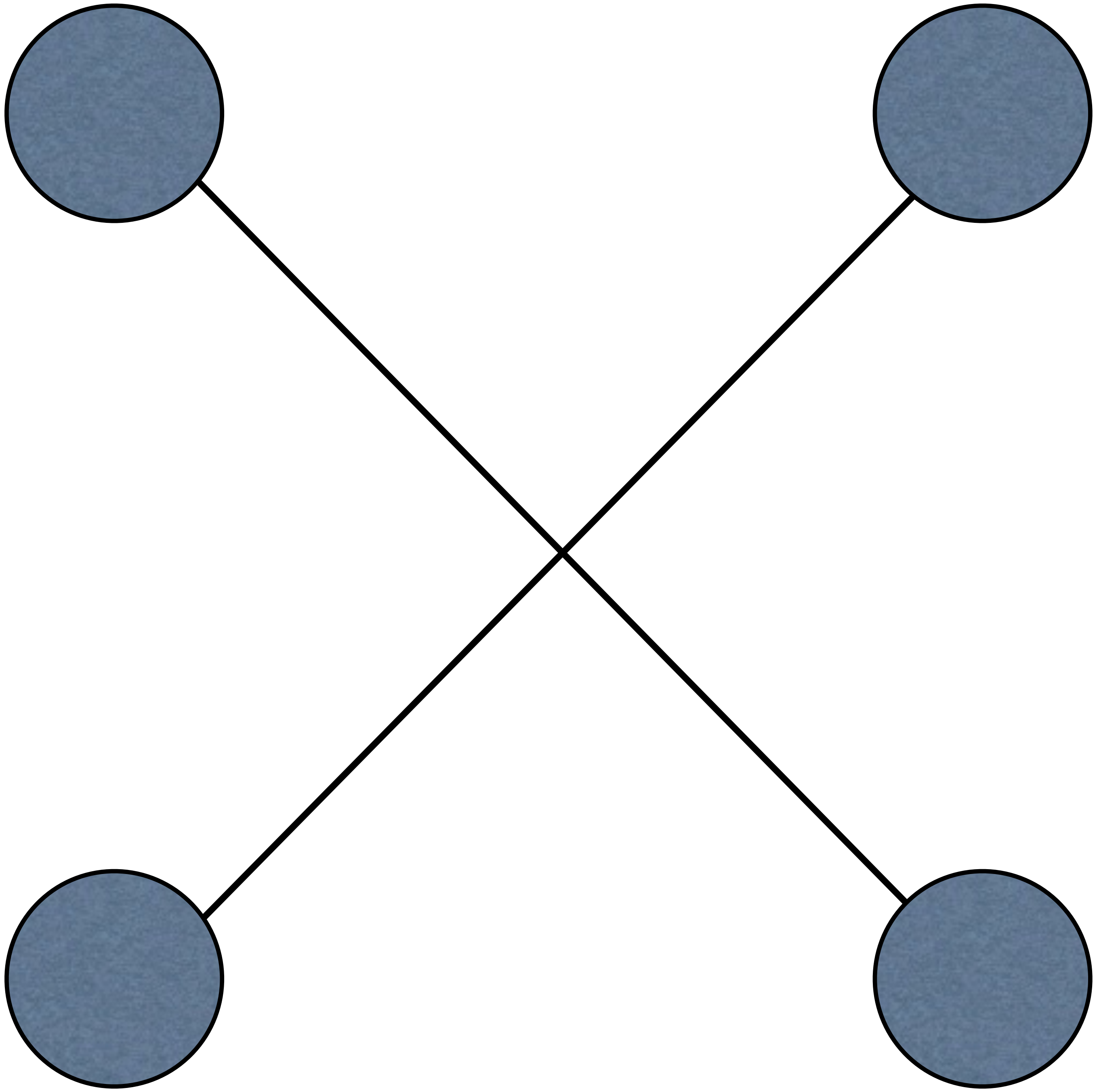
（具有高对称性的系统的解具有较低对称性）



需要花费
3个单位

对称性自发破缺

（具有高对称性的系统的解具有较低对称性）

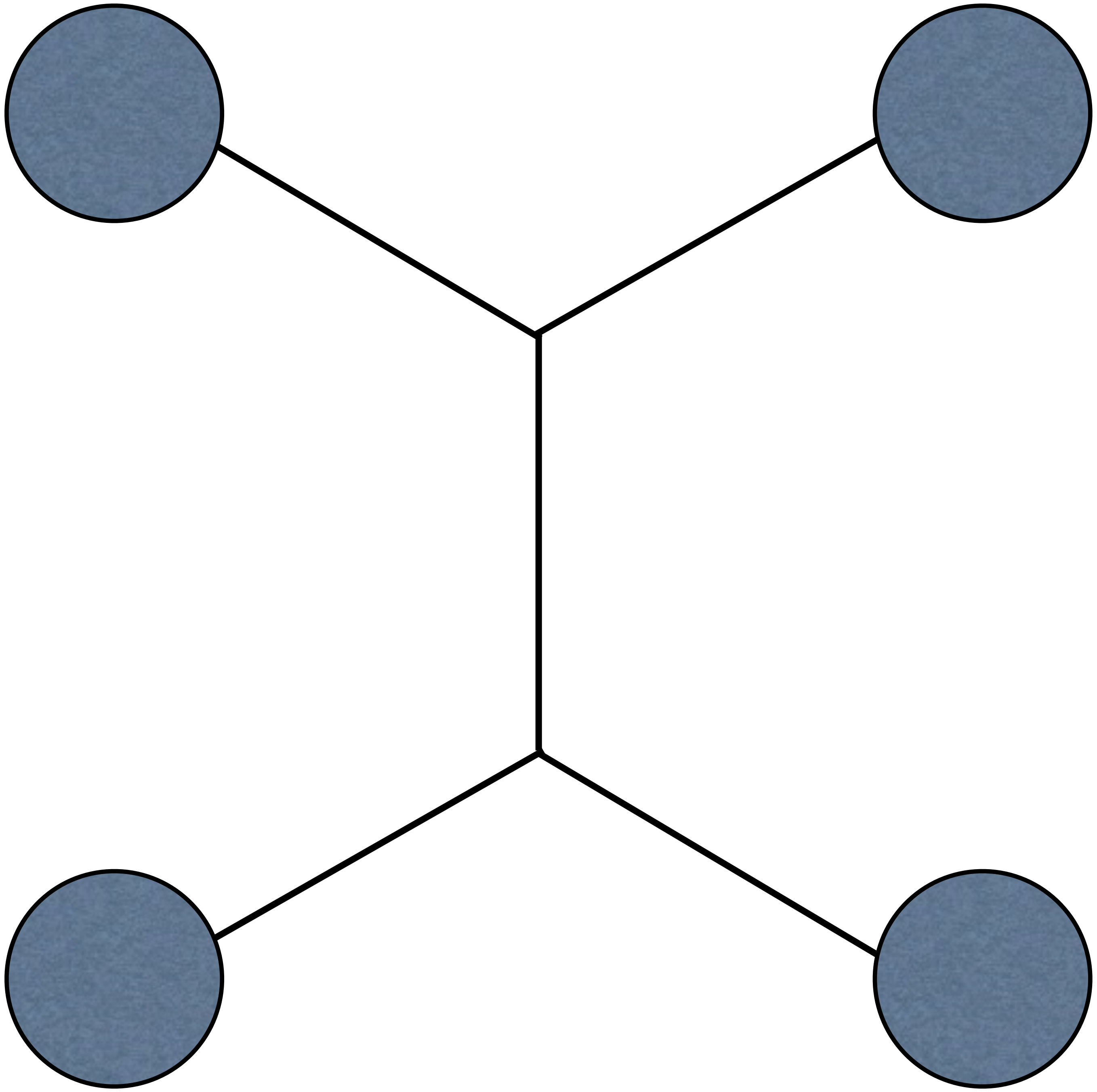


需要花费

$2\sqrt{2}$个单位

对称性自发破缺

（具有高对称性的系统的解具有较低对称性）



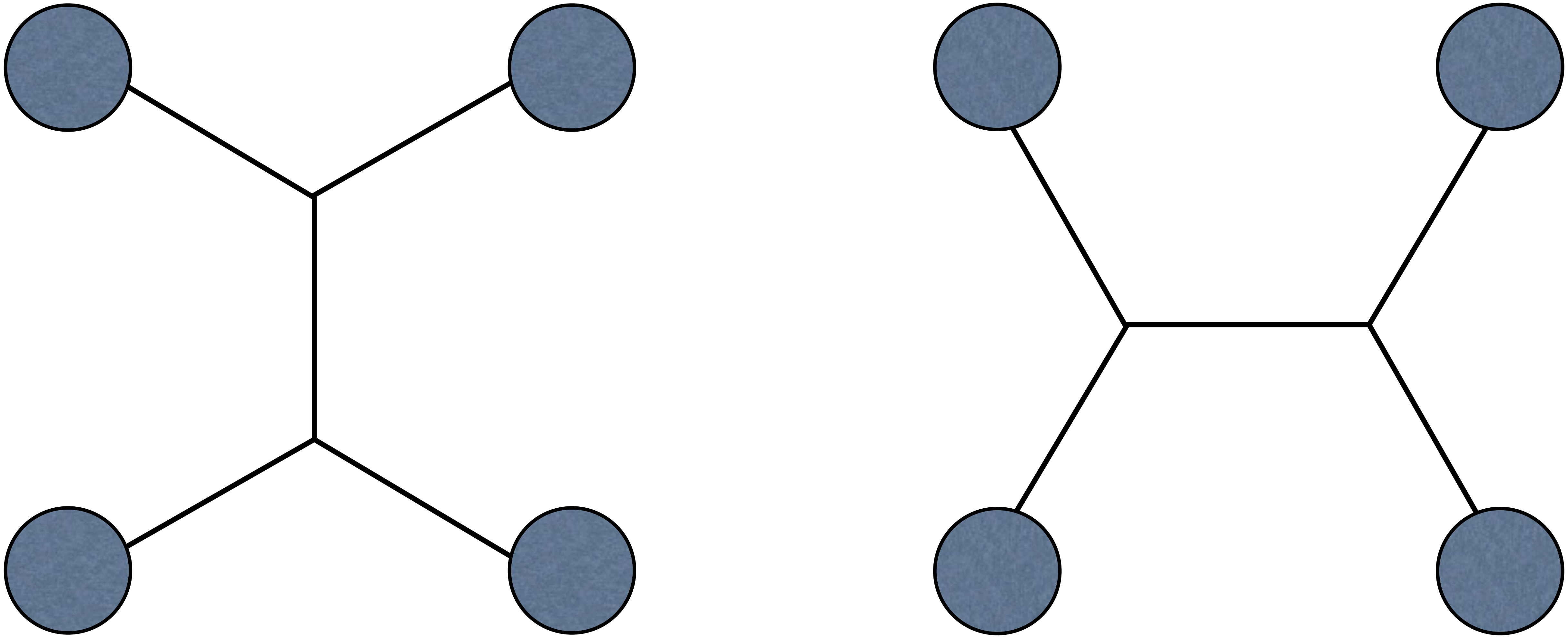
The definition of *Spontaneous Symmetry Breaking*.

需要花费

$1+\sqrt{3}$个单位

对称性自发破缺

（具有高对称性的系统的解具有较低对称性）



两种方案之和还具有原始对称性

Spontaneous Symmetry Breaking

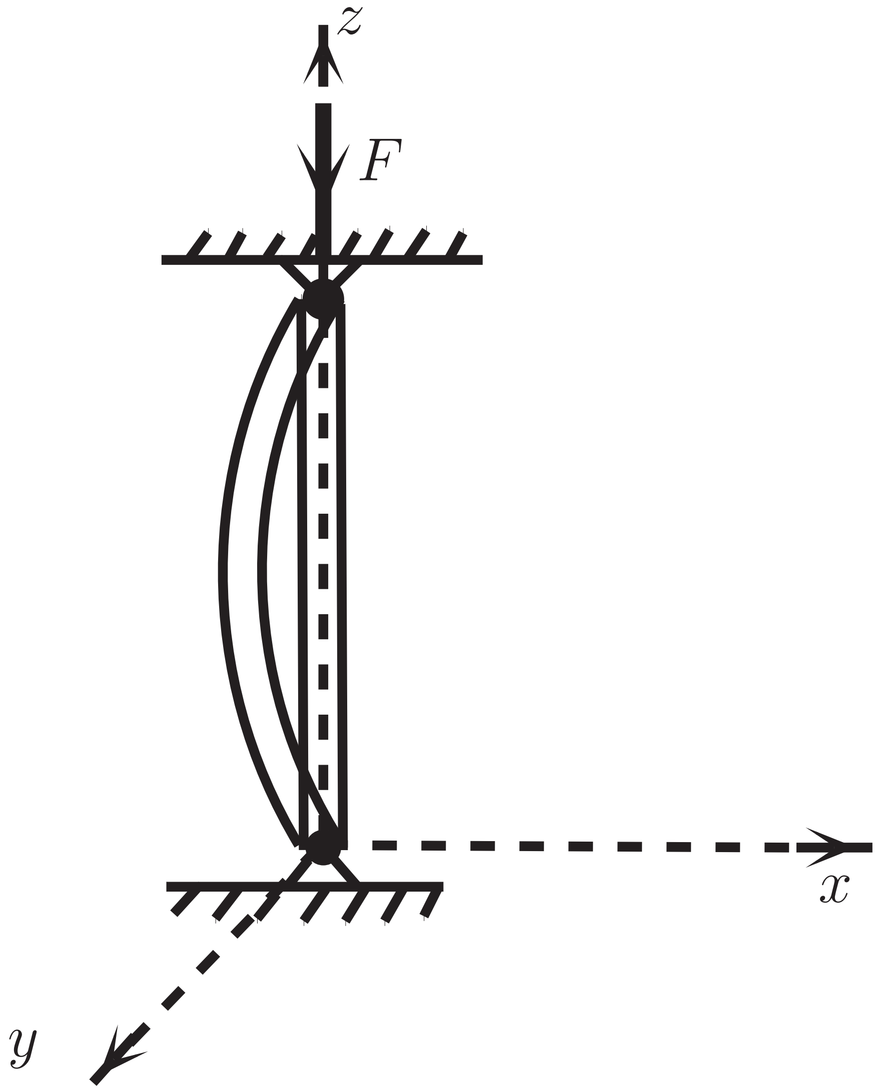
$$IE\frac{d^4X}{dz^4}+F\frac{d^2X}{dz^2}=0$$

$$IE\frac{d^4Y}{dz^4}+F\frac{d^2Y}{dz^2}=0$$

solved
by Euler

$$I=\frac{\pi R^4}{4}$$

E is Young Modulus



Solution:

1) $X=Y=0$

2) $X=C\sin kz$

$kl=n\pi$

$n=1,2,\cdots$

First solution appears when

$$F>F_c=\frac{\pi^2EI}{l^2}$$

Spontaneous Symmetry Breaking

- There exists a critical point, i.e. a critical value of some external quantity which we can vary freely

  (e.g. external force F; temperature in CMP)

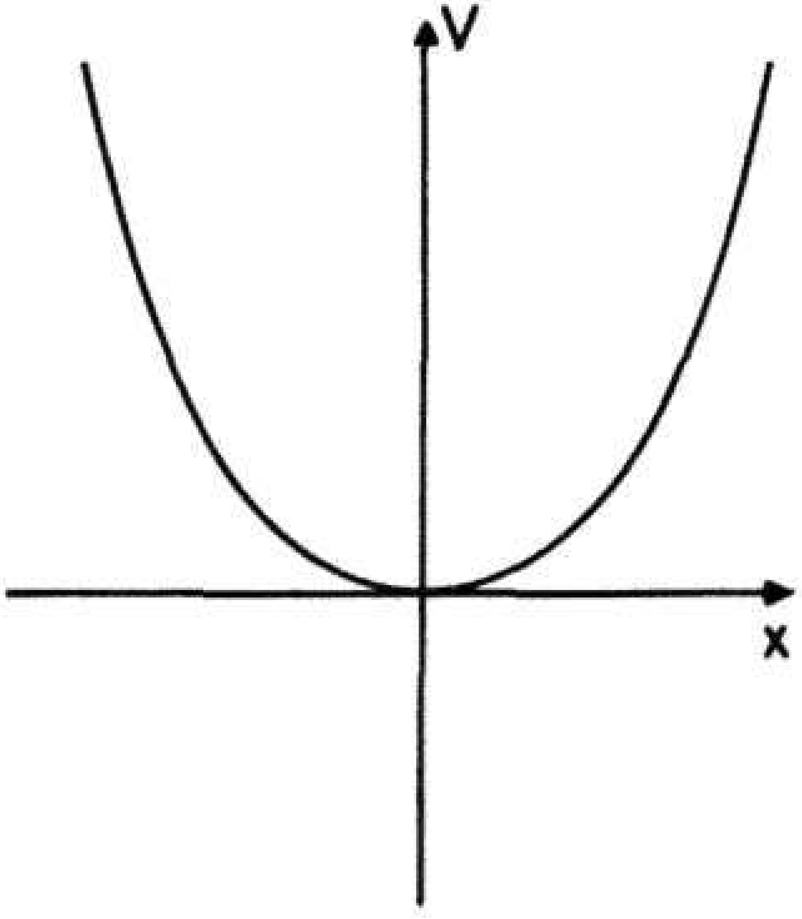
- Beyond the critical point,

  the symmetric solution becomes unstable;

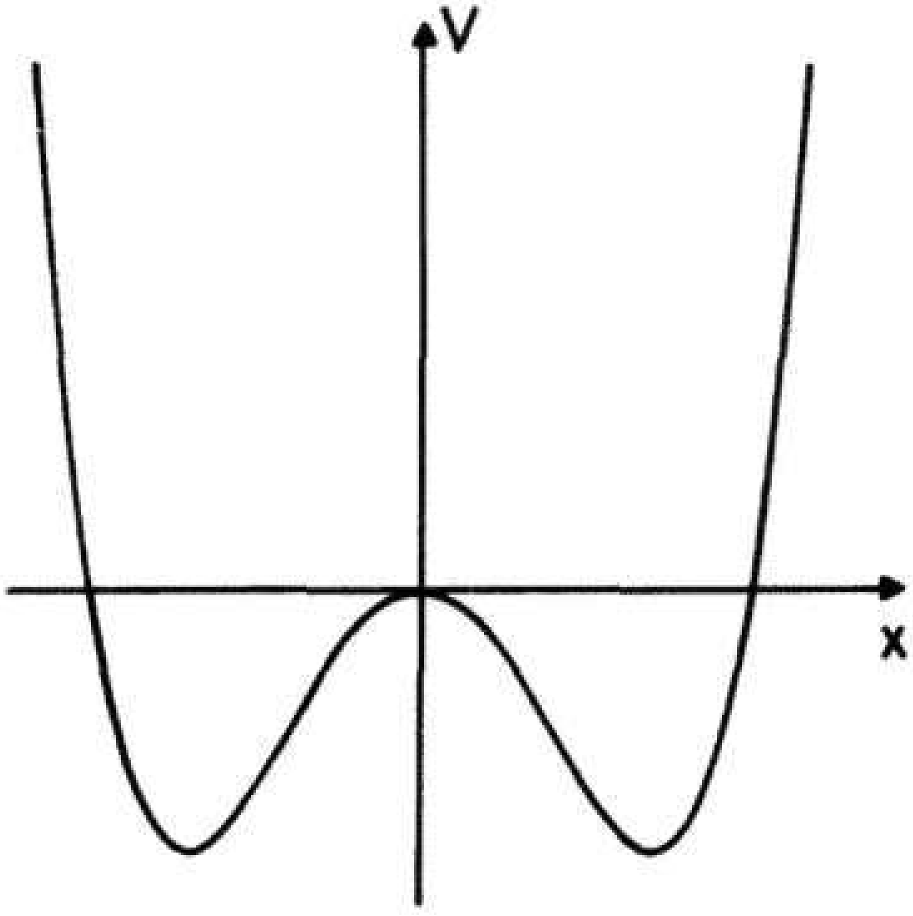
  the ground state become degenerate.

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) - V(\phi)$$

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}|\lambda|\phi^4$$



$\mu^2 > 0$



$\mu^2 < 0$

Nambu-Goldstone Boson

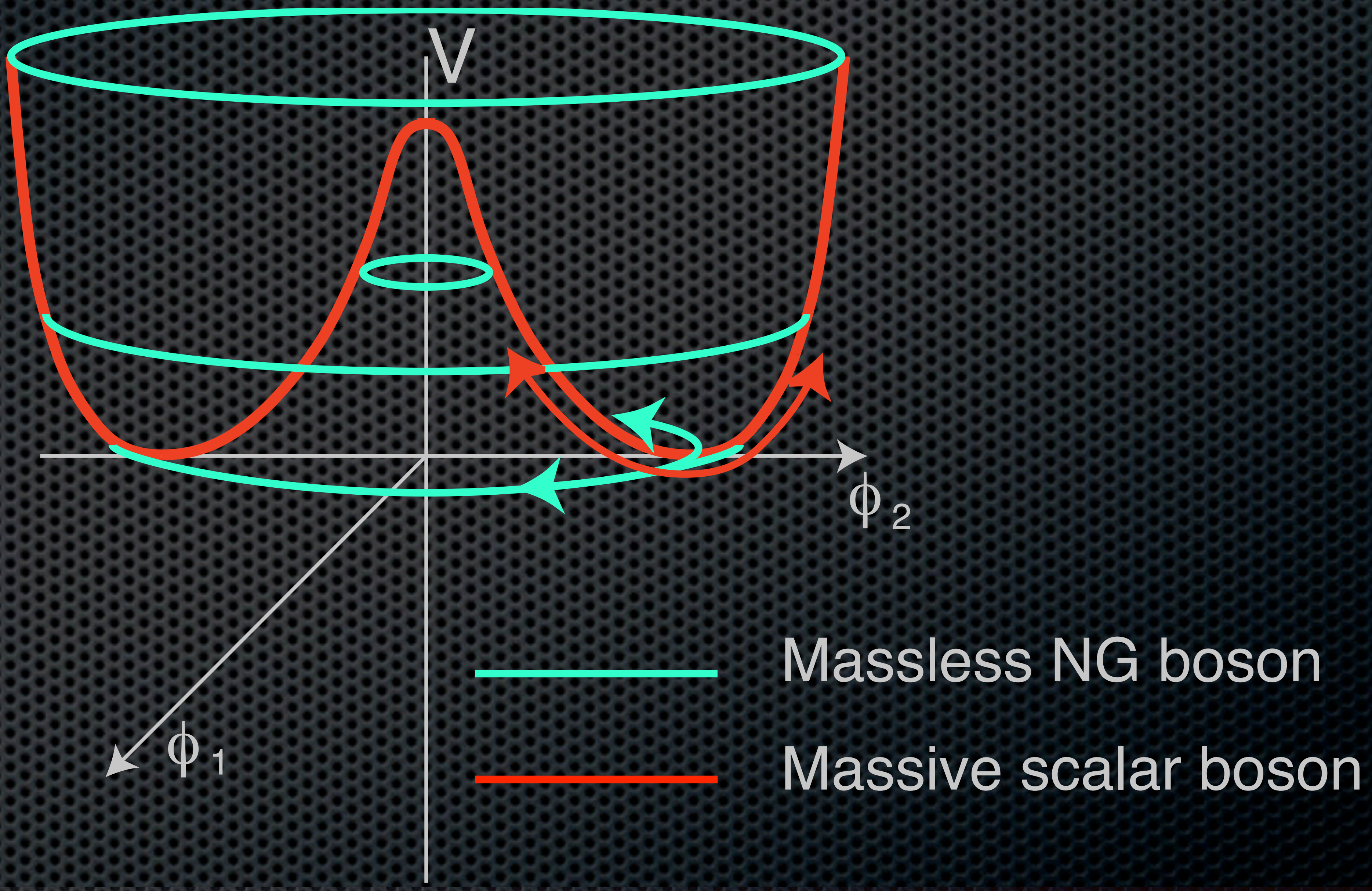
(1960)

2008
Nobel
Prize

Yoichiro
Nambu

Jeffrey
Goldstone

(1961)

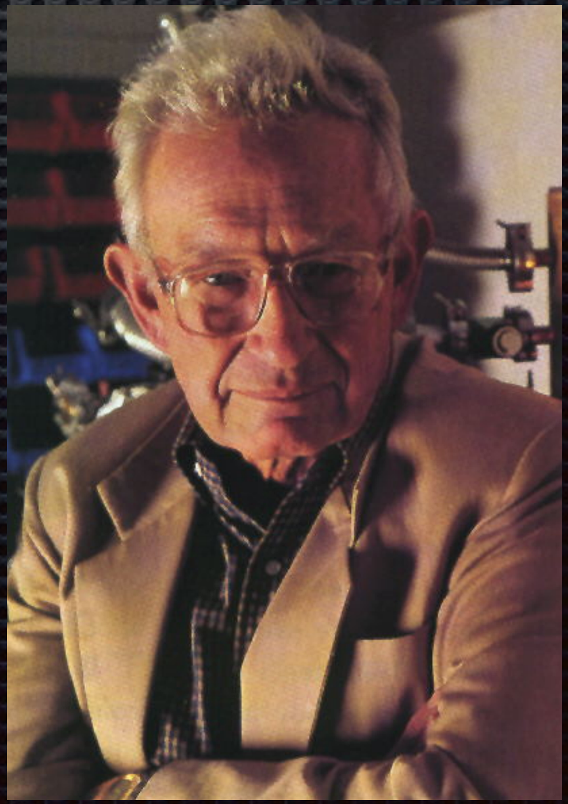


Goldstone, Salam, Weinberg (1962)

Anderson (1963)

指出超导中的Goldstone模式会因其电磁耦合获得质量，并且产生一个纵向极化模式。

- “the Goldstone zero-mass difficulty is not a serious one, because we can probably cancel it off against an equal Yang-Mills zero-mass problem”



没有指出Goldstone定理的瑕疵，
也没有探讨相对论性的理论模型

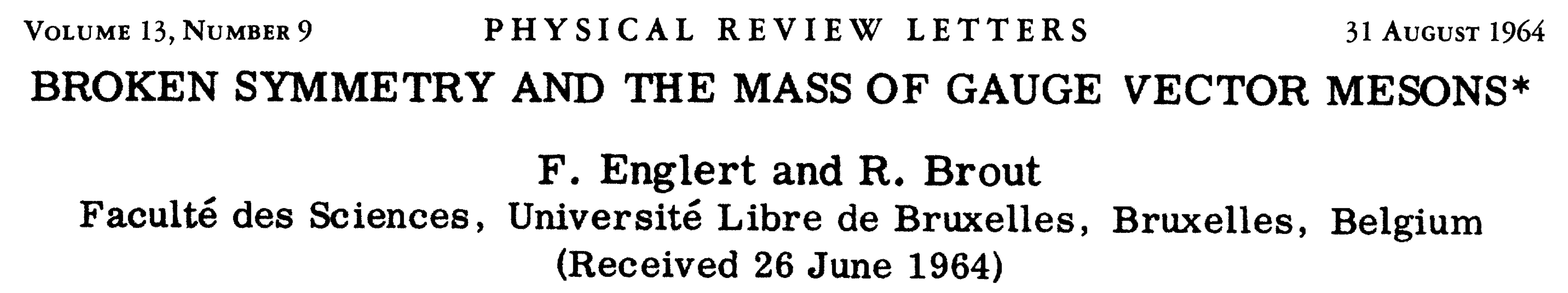
Phys. Rev. 130 (1963) 439

对称性自发破缺

1964年：Goldstone定理并不适用于规范理论

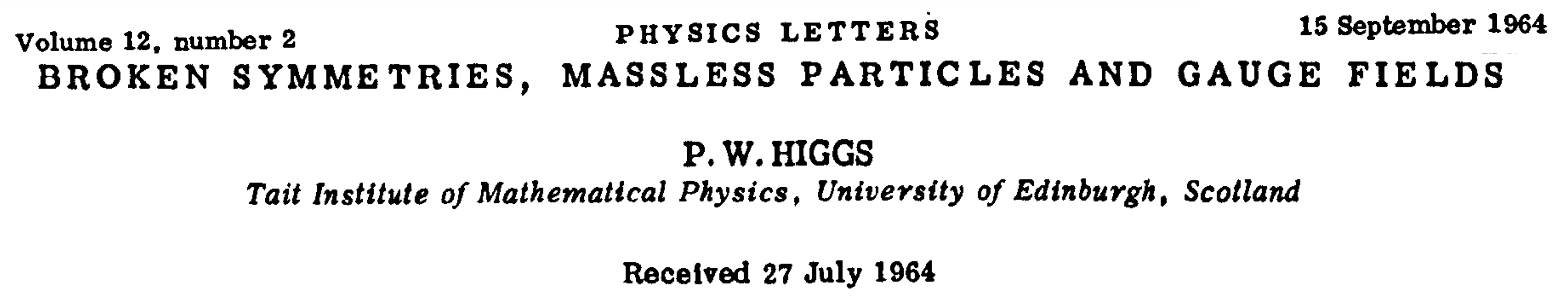
每个无质量的Goldstone玻色子和一个无质量的规范玻色子组成一个有质量的玻色子，同时还产生有质量的标量粒子

1964年3组人不约而同地…


VOLUME 13, NUMBER 9 PHYSICAL REVIEW LETTERS 31 AUGUST 1964

BROKEN SYMMETRY AND THE MASS OF GAUGE VECTOR MESONS*

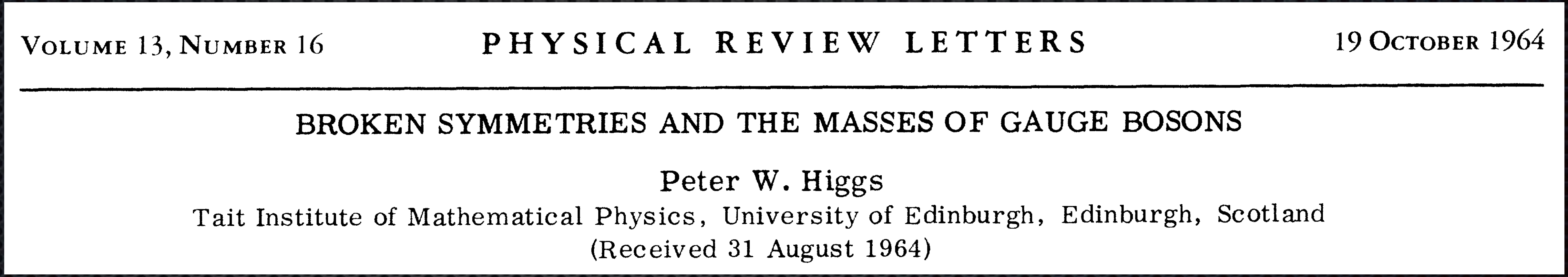
F. Englert and R. Brout
Faculté des Sciences, Université Libre de Bruxelles, Bruxelles, Belgium
(Received 26 June 1964)


Volume 12, number 2 PHYSICS LETTERS 15 September 1964

BROKEN SYMMETRIES, MASSLESS PARTICLES AND GAUGE FIELDS

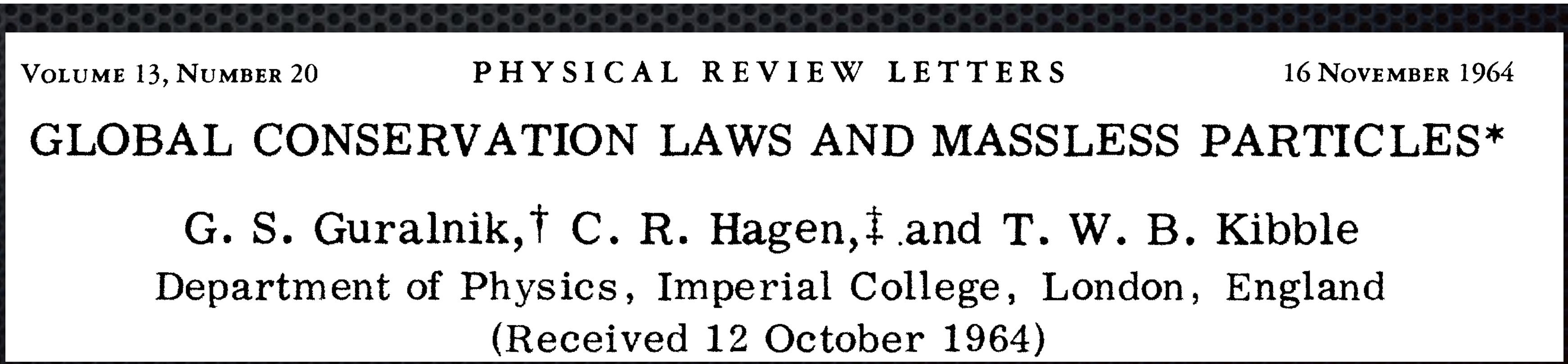
P. W. HIGGS
Tait Institute of Mathematical Physics, University of Edinburgh, Scotland

Received 27 July 1964


VOLUME 13, NUMBER 16 PHYSICAL REVIEW LETTERS 19 OCTOBER 1964

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs
Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland
(Received 31 August 1964)


VOLUME 13, NUMBER 20 PHYSICAL REVIEW LETTERS 16 NOVEMBER 1964

GLOBAL CONSERVATION LAWS AND MASSLESS PARTICLES*

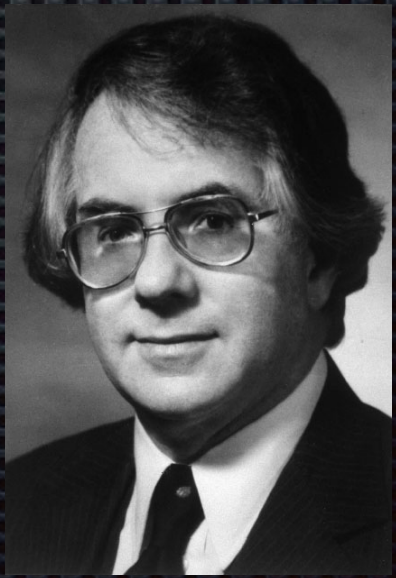
G. S. Guralnik,† C. R. Hagen,‡ and T. W. B. Kibble
Department of Physics, Imperial College, London, England
(Received 12 October 1964)

电弱理论（1967）

1979 Nobel Prize

Steven Weinberg

Abdus Salam



将希格斯机制引入到Glashow的轻子电弱理论

Shelton Glashow, Nucl. Phys. 22 (1961) 579

使用真空隐藏电弱对称性

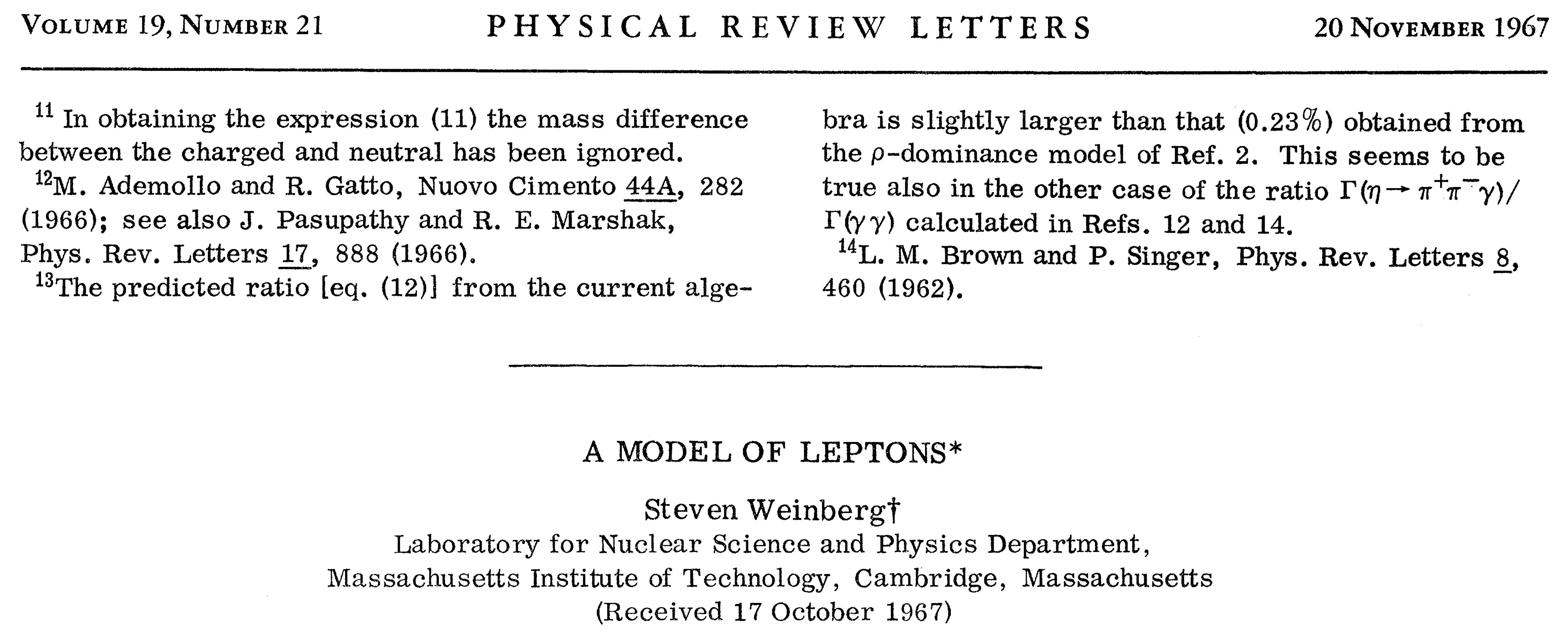
3个有质量的规范玻色子 $W^+\ W^-\ Z^0$ (1983)

1个无质量的规范玻色子 γ

1个有质量的希格斯粒子 (2012)

为何叫“希格斯机制”?

Weinberg乌龙引用



Volume 19, Number 21 PHYSICAL REVIEW LETTERS 20 November 1967

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[11] In obtaining the expression (11) the mass difference between the charged and neutral has been ignored.

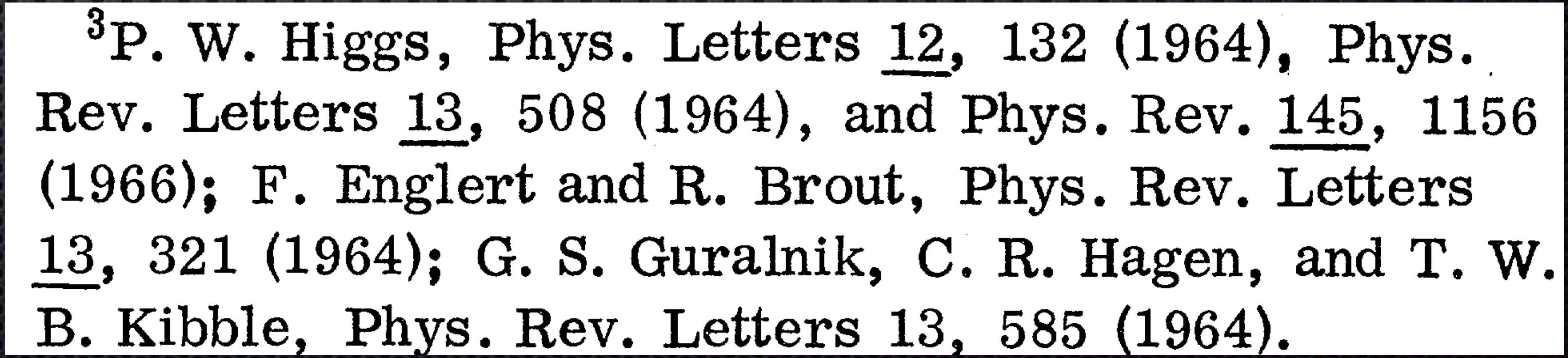
[12] M. Ademollo and R. Gatto, Nuovo Cimento 44A, 282 (1966); see also J. Pasupathy and R. E. Marshak, Phys. Rev. Letters 17, 888 (1966).

[13] The predicted ratio [eq. (12)] from the current algebra is slightly larger than that (0.23%) obtained from the ρ-dominance model of Ref. 2. This seems to be true also in the other case of the ratio $\Gamma(\eta \rightarrow \pi^+\pi^-\gamma)/\Gamma(\gamma\gamma)$ calculated in Refs. 12 and 14.

[14] L. M. Brown and P. Singer, Phys. Rev. Letters 8, 460 (1962).

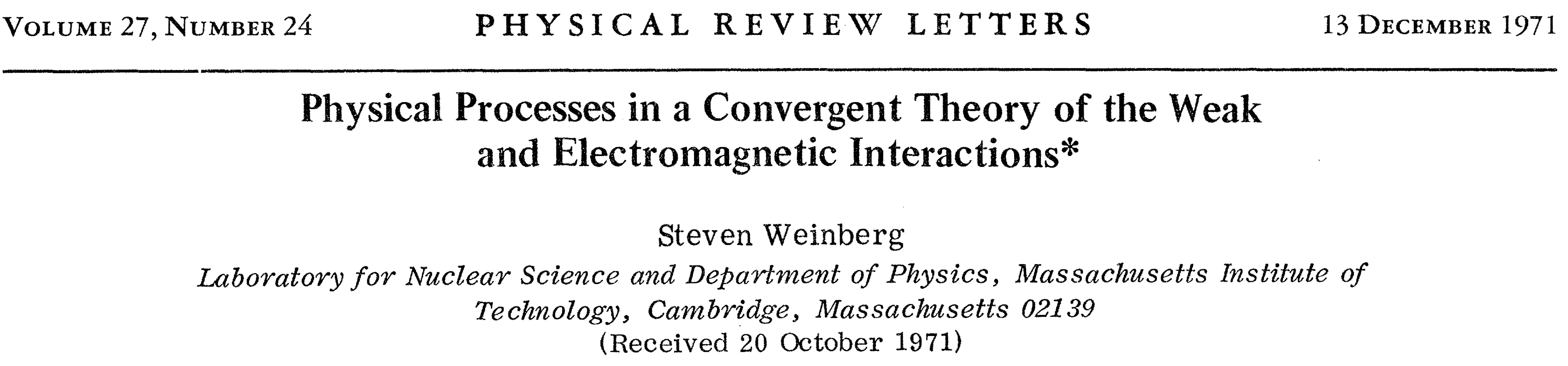
A MODEL OF LEPTONS*

Steven Weinberg†
Laboratory for Nuclear Science and Physics Department,
Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 17 October 1967)



[3] P. W. Higgs, Phys. Letters 12, 132 (1964), Phys. Rev. Letters 13, 508 (1964), and Phys. Rev. 145, 1156 (1966); F. Englert and R. Brout, Phys. Rev. Letters 13, 321 (1964); G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, Phys. Rev. Letters 13, 585 (1964).

温伯格的再次乌龙

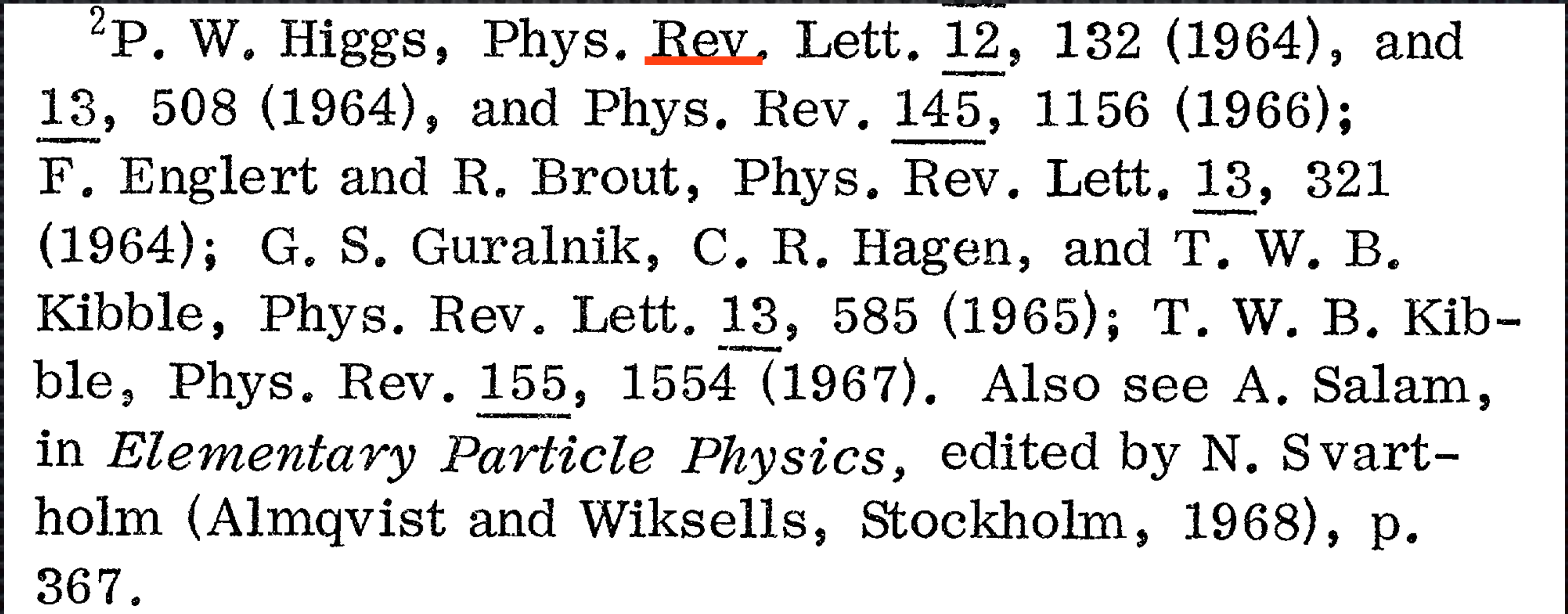


VOLUME 27, NUMBER 24 PHYSICAL REVIEW LETTERS 13 DECEMBER 1971

Physical Processes in a Convergent Theory of the Weak and Electromagnetic Interactions*

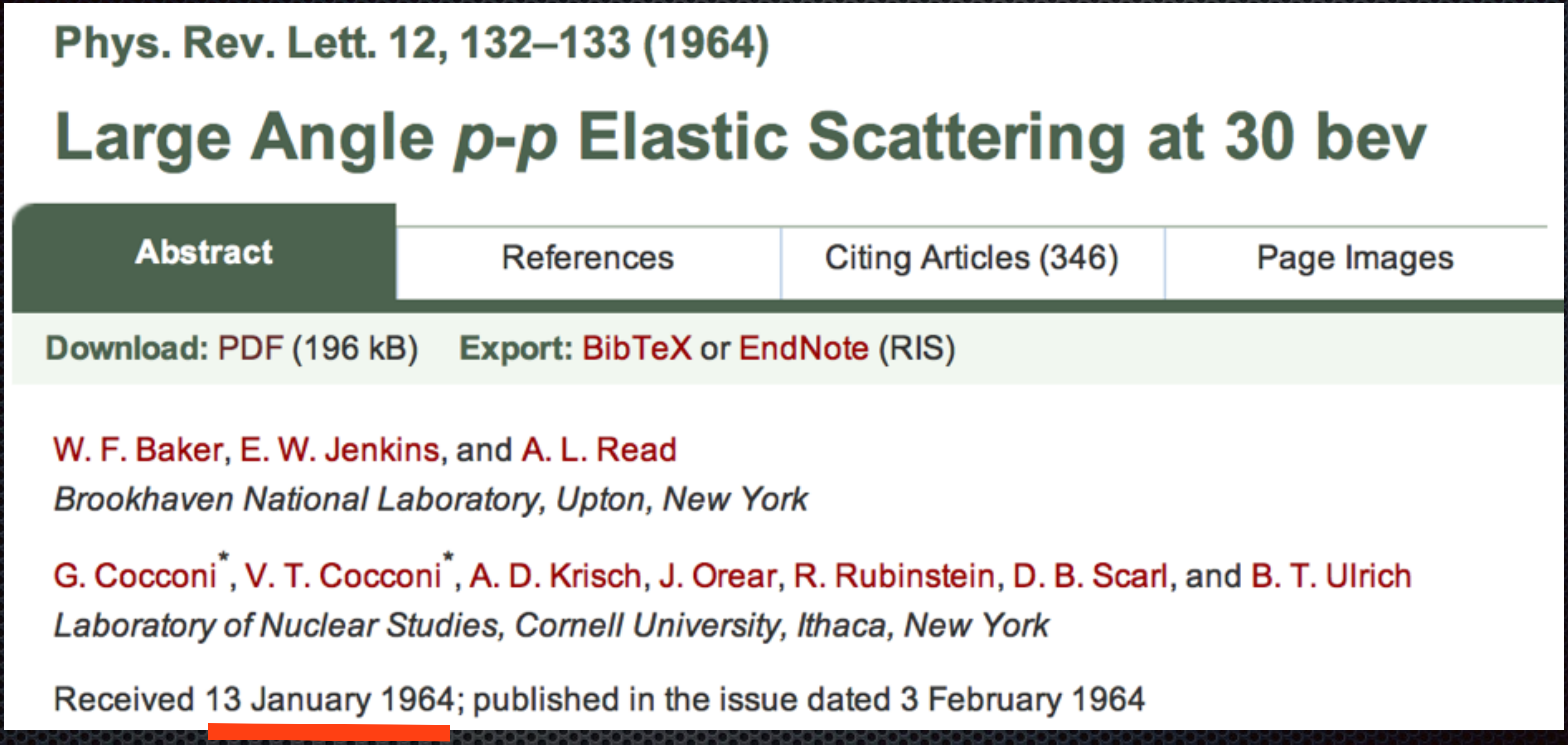
Steven Weinberg
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 20 October 1971)



[2] P. W. Higgs, Phys. Rev. Lett. 12, 132 (1964), and 13, 508 (1964), and Phys. Rev. 145, 1156 (1966); F. Englert and R. Brout, Phys. Rev. Lett. 13, 321 (1964); G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, Phys. Rev. Lett. 13, 585 (1965); T. W. B. Kibble, Phys. Rev. 155, 1554 (1967). Also see A. Salam, in *Elementary Particle Physics*, edited by N. Svartholm (Almqvist and Wiksells, Stockholm, 1968), p. 367.

匪夷所思的巧合



Phys. Rev. Lett. 12, 132–133 (1964)

Large Angle *p-p* Elastic Scattering at 30 bev

W. F. Baker, E. W. Jenkins, and A. L. Read
Brookhaven National Laboratory, Upton, New York

G. Cocconi*, V. T. Cocconi*, A. D. Krisch, J. Orear, R. Rubinstein, D. B. Scarl, and B. T. Ulrich
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York

Received 13 January 1964; published in the issue dated 3 February 1964

时间提前到1964年1月份!!!

Immediate Impact of Weinberg’s Work in 1967

ZERO

Sidney
Coleman

1968年，0次
1969年，0次
1970年，1次
1971年，4次
1972年，64次
1973年，162次

Why & What
happened?

Why so?

1) 场论在走下坡路 —> 读者少

2) Salam和Weinberg都集中在轻子部分

(有关的实验数据很少)

3) GIM(1970)机制还没有提出

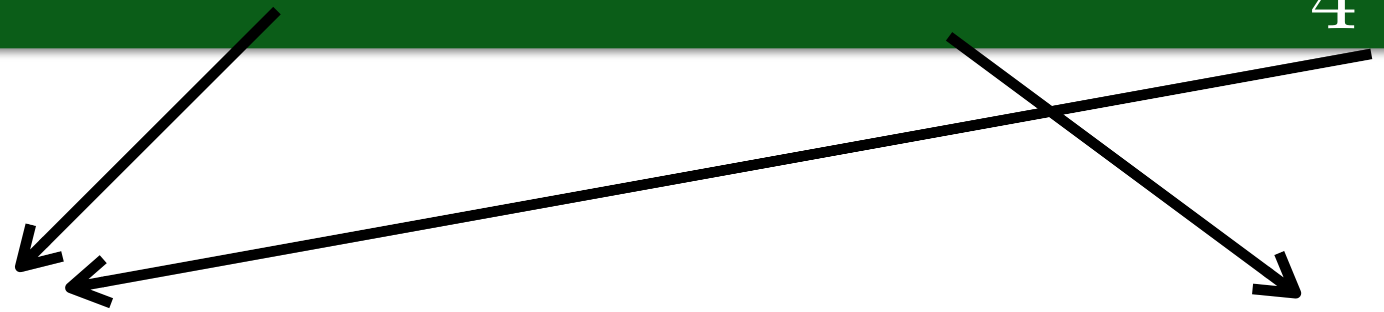
无法解释 $\Delta S = 1$ 过程

4) 量子辐射修正发散（重整化）没有解决

量子电动力学（QED）

拉格朗日量：

$$
\begin{aligned}
\mathcal{L} &= \bar{\psi}\left(i\gamma^{\mu}D_{\mu}-m\right)\psi-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\\
&= \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu}-m\right)\psi-qA_{\mu}\bar{\psi}\gamma^{\mu}\psi-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}
\end{aligned}
$$



可精确求解

不可精确求解

微扰求解

$$\alpha=\frac{e^{2}}{4\pi}=\frac{1}{137}$$ ➡ n个光子贡献α^{n}

量子力学二阶微扰项

$$E_a^{(2)} = \sum_{i \neq a} \frac{\left|\left\langle \psi_i^{(0)} | \hat{H}_I | \psi_a^{(0)} \right\rangle\right|^2}{E_a^{(0)} - E_i^{(0)}} = \sum_{i \neq a} \frac{1}{E_a^{(0)} - E_i^{(0)}} \left\langle \psi_a^{(0)} | \hat{H}_I | \psi_i^{(0)} \right\rangle \left\langle \psi_i^{(0)} | \hat{H}_I | \psi_a^{(0)} \right\rangle$$

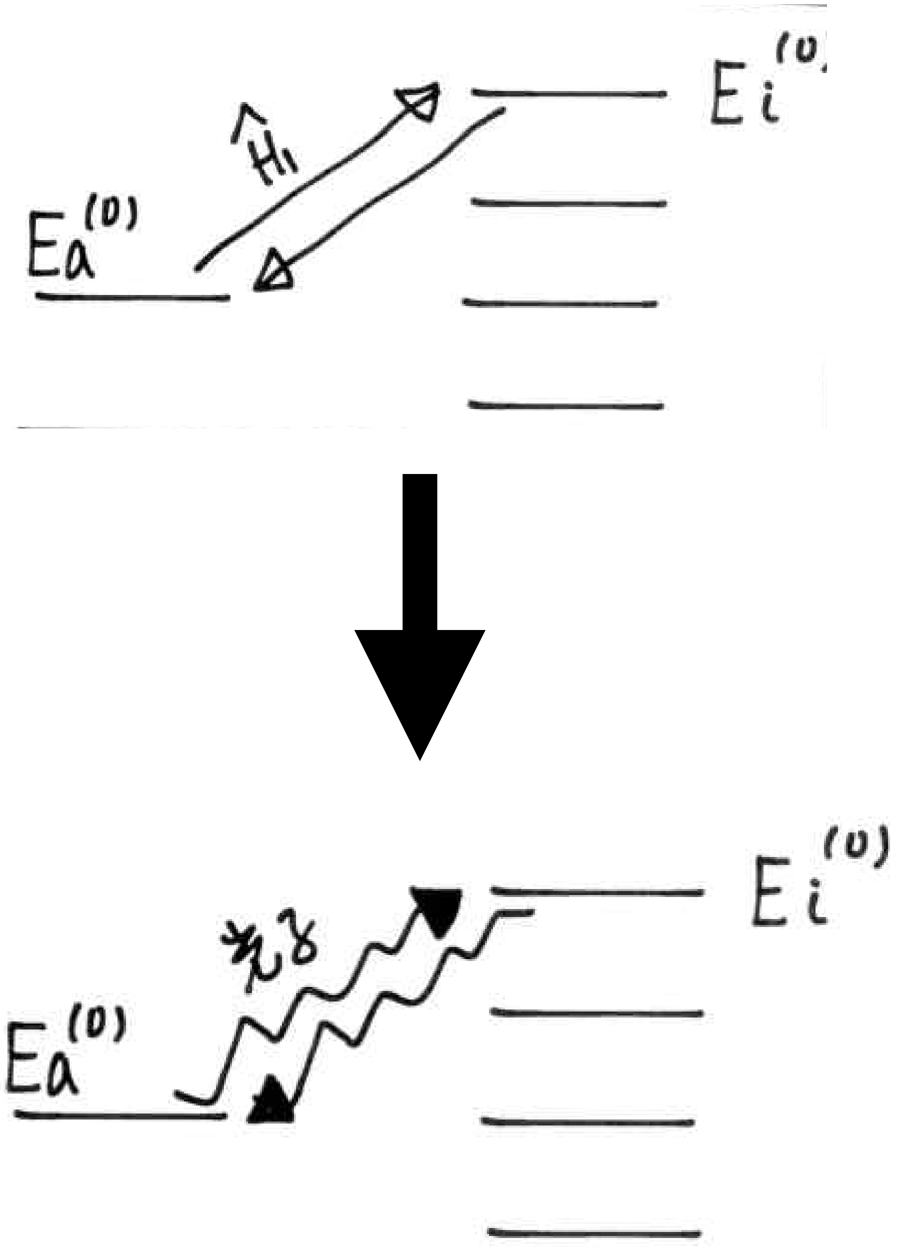
收敛性:

1) $\left\langle \psi_i^{(0)} | \hat{H}_I | \psi_a^{(0)} \right\rangle$ 很大，导致对各态求和不收敛

——> 紫外发散

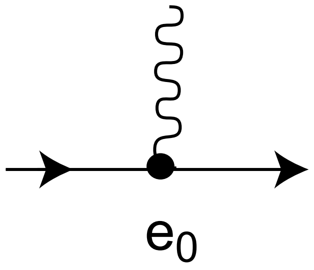
2) E_a 能级附近存在许多（或连续的）能级满足 $|E_i - E_a| \sim 0$，从而导致对各态求和不收敛

——> 红外发散

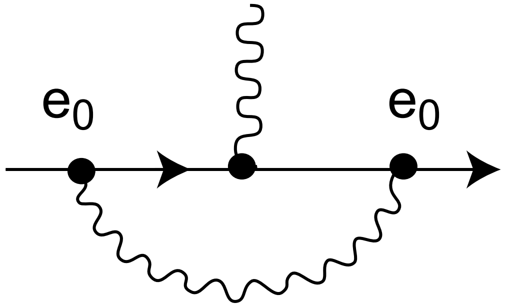


重整化

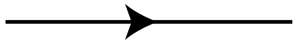
QED: 微扰展开计算中的无穷大问题



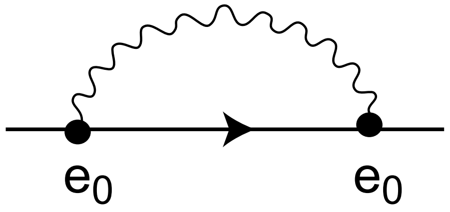
Bare vertex



Radiative correction

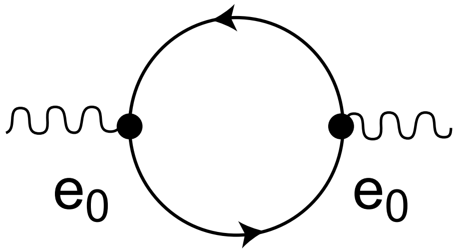


Electron propagator



Radiative correction
(Self-energy)

Photon propagator



Radiative correction
(Vacuum polarization)

“整个30年代，物理学界共识是，量子场论并不被看好。它可能有用，但只是权宜之计，需要添加全新的东西才能使它说的通。”

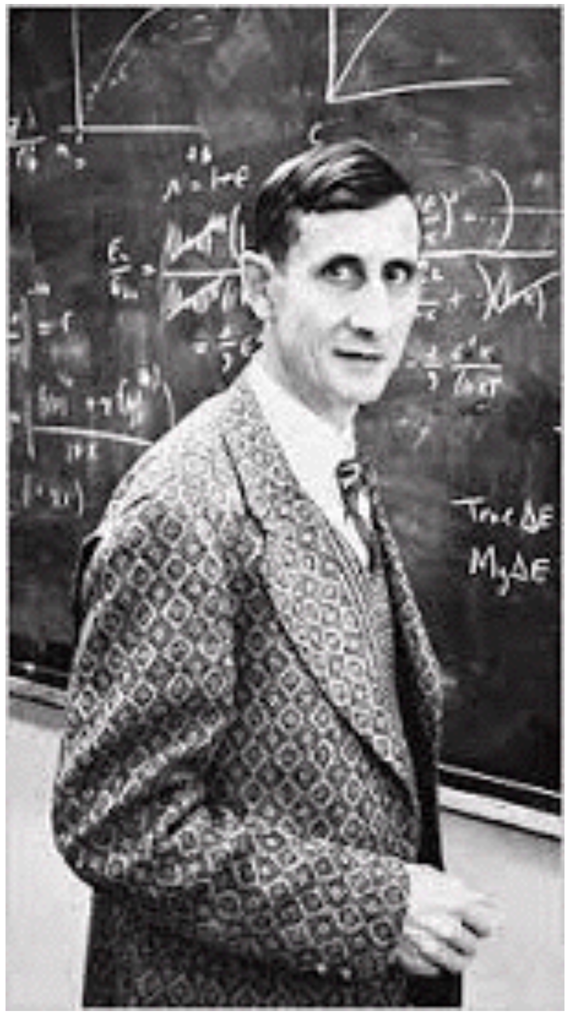
QED重整化

- 20世纪40年代后期才消除QED理论中的不健全之处

Feynman, Schwinger, Tomonaga分别提出重整化思想

1949年Dyson证明他们三种方案是等价的

1965 Nobel



Dyson

Freeman Dyson

Note: 戴森（Freeman Dyson）早年在剑桥大学追随著名的数学家G.H.哈代研究数学，1945年获得数学系的学士学位后，于1947年到美国康奈尔大学跟随汉斯·贝特和理查德·费曼学习。他证明了施温格和朝永振一郎发展的变分法方法和费曼的路径积分法的等价性，为量子电动力学的建立做出了决定性的贡献。1949年戴森提出Dyson series，这一工作启发Ward研究并提出Ward等式。

戴森没有博士学位，但由于他的杰出贡献，康奈尔大学于1951年聘请戴森为物理学教授。这在今天是难以想象的。戴森获得很多荣誉学位，其中包括Yeshiva University (1966), University of Glasgow (1974), Princeton University (1974), University of York (1980), City University of London (1981), New School of Social Research (1982), Rensselaer Polytechnic (1983), Susquehanna University (1984), Depauw University (1987), Rider College (1989), Bates College (1991), Haverford College (1991), Dartmouth College (1995), Federal Inst. of Tech. (ETH), Switzerland (1995), Scuola Normale Superiore, Pisa, Italy (1996), University of Puget Sound (1997), Oxford University (1997), Clarkson University (1998), Rockefeller University (2001), St. Peter's College (2004), Georgetown University (2005), University of Michigan (2005), University of the Sciences (2011)。

Muon g-2

$$
\begin{aligned}
\tfrac{1}{2}g_{\text{theory}} = \quad & 1 \\
& + (\alpha/2\pi) \\
& -0.32848\,(\alpha/\pi)^2 \\
& + (1.195 \pm 0.026)\,(\alpha/\pi)^3 \\
& - (1.7283\,(35))\,(\alpha/\pi)^4 + (\text{Non-QED})
\end{aligned}
$$

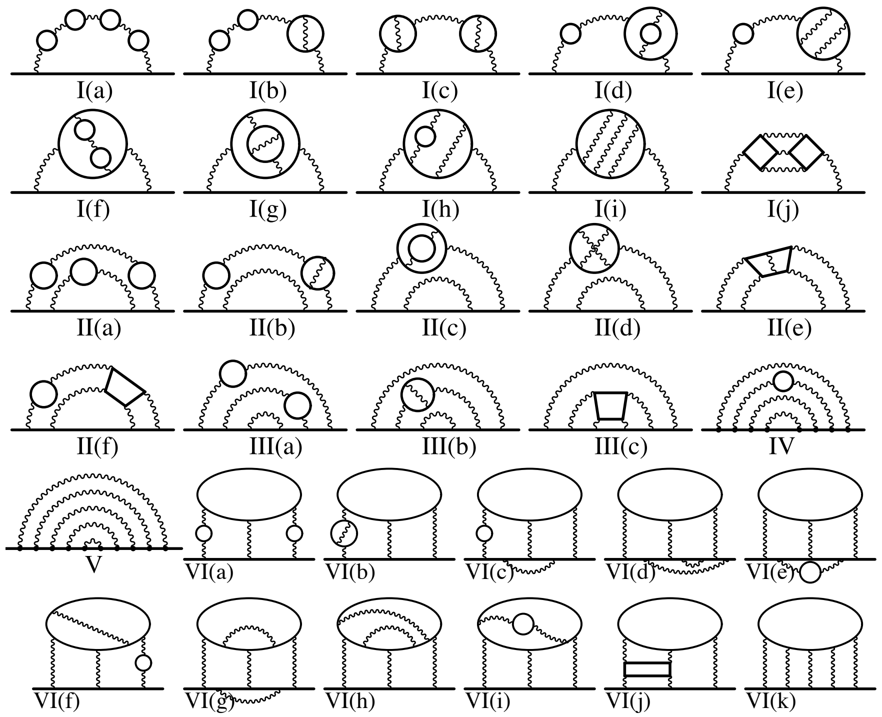
(a) 1928 (Dirac equation)
(b) 1949 (1 diagram)
(c) 1958 (18 diagrams)
(d) 1974 (72 diagrams)
(e) 2006 (891 diagrams).

Kinoshita

PRL **109,** 111808 (2012) PHYSICAL REVIEW LETTERS week ending 14 SEPTEMBER 2012

Complete Tenth-Order QED Contribution to the Muon *g* − 2

Tatsumi Aoyama,[1,2] Masashi Hayakawa,[3,2] Toichiro Kinoshita,[4,2] and Makiko Nio[2]



5圈图（总计12672个费曼图）

计算精度：10^{-12}

人类精确计算的登峰造极之作

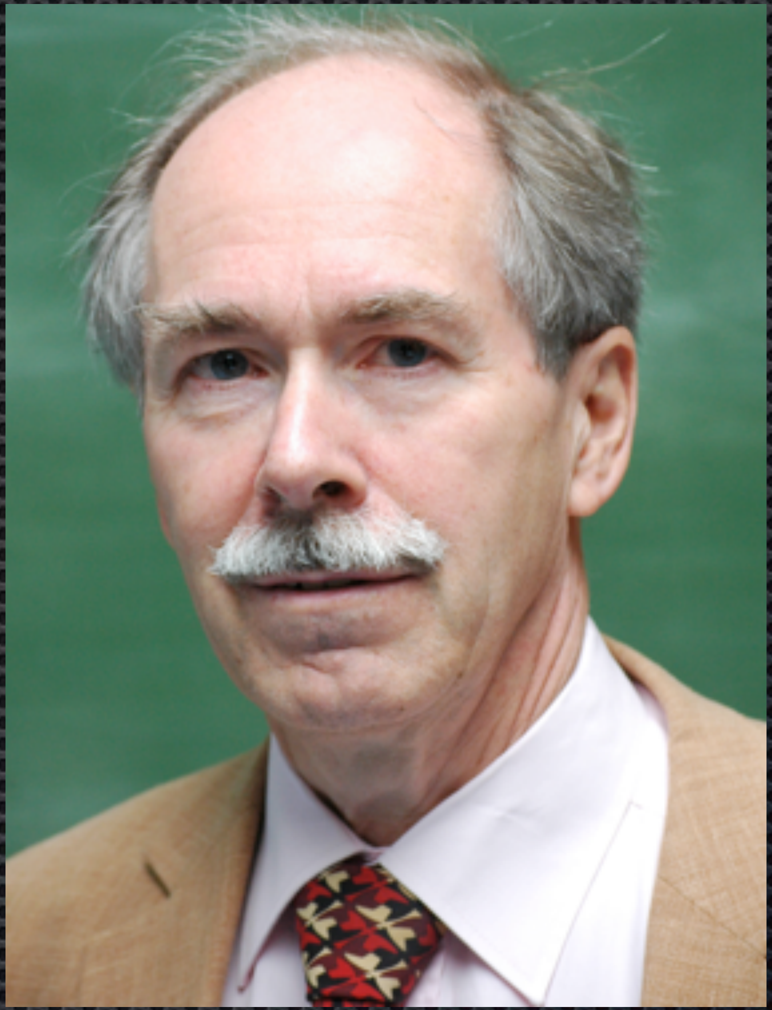
量子场论大萧条

1949年后的几年内，因为QED理论的极大成功，人们对量子场论的热情处于发烧状态。许多理论物理学家都认为很快就会完全理解所有的微观现象，不仅仅限于光子、电子和正电子而已。

然而不久，这种信心就崩溃了——量子场论的股票在物理学股市上大跌，并因此进入第二轮熊市。不幸的是，这次大萧条持续了近20年。

1971-72年

- t’Hooft 和 Veltman证明电弱理论的可重整化性



1999
Nobel
Prize

- 1972年在费米实验室举办的高能物理会议上，电弱理论部分的报告人B. W. Lee，首次提出“Higgs meson”。

Benjamin W. Lee

规范场论的传道者（1935-1977）

David Politzer (2004 Nobel Lecture)：

“ the particle physicists community at that time learned all from Lee who actually combined insights from his own work and from Russian physicists' work and encouraged 't Hooft's paper.”

1974年Rochester大会

（ICHEP会议前身）

两大议题：高能强相互作用物理学和共振态物理学

1. 高能强相互作用（280页）
2. 共振态物理（199页）
3. 弱相互作用和统一理论（115页）
4. 轻子-轻子相互作用和轻子-强子相互作用（173页）
5. 大横动量反应（80页）

Regge Theory
组分夸克模型

* 大部分都是实验文章

当时的新物理模型（标准模型）还不是主流

Standard Model shining after the revolution on November 1974 (charm discovery)

How to build up the SM?

- Step 1: Choose a gauge group G.
- Step 2: Choose the fields of the "elementary" particles and assign them to representations of G. Include scalar fields to allow for the Higgs mechanism.
- Step 3: Write the most general renormalisable Lagrangian invariant under G. At this stage gauge invariance is still exact and all gauge vector bosons are massless.
- Step 4: Choose the parameters of the Higgs potential so that spontaneous symmetry breaking occurs.
- Step 5: Translate the scalars and rewrite the Lagrangian in terms of the translated fields. Choose a suitable gauge and quantise the theory.

1305.6779

Intermediate Vector Bosons

We know that one gauge field is associated with a generator of gauge group.

W+, W-, Z, gamma → 4 generators

The simplest one is

$$SU(2)_L \times U(1)_Y$$

Gauge invariance requires the introduction of vector bosons, which act as quanta of new interactions. In gauge theories the symmetries prescribe the interactions.

The Quark and Lepton Lagrangian

$$\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi \rightarrow \bar{\psi}\gamma^{\mu}\mathcal{D}_{\mu}\psi$$

$$\boxed{\mathcal{D}_{\mu} = \partial_{\mu} - ig_1\frac{Y}{2}B_{\mu} - ig_2\frac{\tau^i}{2}W^i_{\mu} - ig_3\frac{\lambda^a}{2}G^a_{\mu}}$$

- B_{μ} is the spin-one field needed to maintain the $U(1)$ gauge invariance. g_1 is the coupling strength (to be measured experimentally. Y is the generator of $U(1)$, transformations, a constant, but in principle different for the different fermions.
- Analogous remarks describe the SU(2) and SU(3) terms. We introduce 3 and, respectively, 8 vector bosons which are needed to maintain the local gauge invariance. $\tau^i W^i_{\mu} = \tau^1 W^1_{\mu} + \tau^2 W^2_{\mu} + \tau^3 W^3_{\mu}$
- $\mathcal{D}_{\mu}$ gives a zero result when it acts on a term of different matrix form. For example $\tau^i W^i$ is a 2×2 matrix in $SU(2)$ and it gives zero acting on e_R, u_R, d_R.

$$\boxed{\mathcal{L}_{\text{ferm}} = \sum_f \bar{f}\gamma^{\mu}\mathcal{D}_{\mu}f} \qquad f = L, e_R, Q_L, u_R, d_R$$

标准模型的物质场

- 费米子 (自旋1/2)

轻子

(无强相互作用)

$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L \qquad \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L \qquad \begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_L$$

$$e^-_R \qquad \mu^-_R \qquad \tau^-_R$$

夸克

$$\underbrace{\begin{matrix}\begin{pmatrix}u\\ d\end{pmatrix}_L \begin{pmatrix}u\\ d\end{pmatrix}_L \begin{pmatrix}u\\ d\end{pmatrix}_L & \begin{pmatrix}c\\ s\end{pmatrix}_L \begin{pmatrix}c\\ s\end{pmatrix}_L \begin{pmatrix}c\\ s\end{pmatrix}_L & \begin{pmatrix}t\\ b\end{pmatrix}_L \begin{pmatrix}t\\ b\end{pmatrix}_L \begin{pmatrix}t\\ b\end{pmatrix}_L \\ u_R \quad u_R \quad u_R & c_R \quad c_R \quad c_R & t_R \quad t_R \quad t_R \\ d_R \quad d_R \quad d_R & s_R \quad s_R \quad s_R & b_R \quad b_R \quad b_R\end{matrix}}_{\text{3代}}$$

- 标量场 (自旋为0)

$$\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}$$

The Standard Model

In order to be gauge invariant, that is: unaffected by the symmetry transformation of SU(2)xU(1), all the terms in the Lagrangian must carry no net quantum numbers.

$$e^- \rightarrow \nu_e W^- \qquad \bar{\psi}_L \gamma^\mu W^a_\mu \cdot \frac{\tau^a}{2} \psi_L$$

$$\begin{pmatrix} \nu \\ e^- \end{pmatrix}_L$$

Weak isospin:

$$\begin{array}{ccccc} T_3(e^-) & \rightarrow & T_3(\nu_e) & + & T_3(W) \\ -\frac{1}{2} & & \frac{1}{2} & & -1 \end{array}$$

The U(1) Terms

$$-\mathcal{L}_{\text{ferm}}(U(1),\text{leptoni}) = \bar{L} i\gamma^{\mu}\left(ig_1 \frac{Y_L}{2} B_{\mu} \right) L + \bar{e}_R i\gamma^{\mu}\left(ig_1 \frac{Y_R}{2} B_{\mu} \right) e_R$$

$$\bar{L}\gamma^{\mu} L = \bar{\nu}_L \gamma^{\mu} \nu_L + \bar{e}_L \gamma^{\mu} e_L$$

$$\mathcal{L}_{\text{ferm}}(U(1),\text{leptoni}) = \frac{g_1}{2}\left[Y_L \left(\bar{\nu}_L \gamma^{\mu} \nu_L + \bar{e}_L \gamma^{\mu} e_L \right) + Y_R \bar{e}_R \gamma^{\mu} e_R \right] B_{\mu}$$

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \qquad e_R^-$$

The SU(2) Terms

$$-\mathcal{L}_{\text{ferm}}\left(SU(2),\text{leptoni}\right)=\overline{L}\,i\gamma^{\mu}\left(ig_2\frac{\tau^i}{2}W^i_{\mu}\right)L$$

$$=-\frac{g_2}{2}\left(\overline{\nu}_L \quad \overline{e}_L\right)\gamma^{\mu}\begin{pmatrix} W^3_{\mu} & W^1_{\mu}-iW^2_{\mu} \\ W^1_{\mu}+iW^2_{\mu} & -W^3_{\mu}\end{pmatrix}\begin{pmatrix}\nu_L \\ e_L\end{pmatrix}$$

$$=-\frac{g_2}{2}\left(\overline{\nu}_L \quad \overline{e}_L\right)\gamma^{\mu}\begin{pmatrix} W^0_{\mu} & -\sqrt{2}W^+_{\mu} \\ -\sqrt{2}W^-_{\mu} & -W^0_{\mu}\end{pmatrix}\begin{pmatrix}\nu_L \\ e_L\end{pmatrix}$$

$$=-\frac{g_2}{2}\left(\overline{\nu}_L \quad \overline{e}_L\right)\gamma^{\mu}\begin{pmatrix} W^0_{\mu}\nu_L-\sqrt{2}W^+_{\mu}e_L \\ -\sqrt{2}W^-_{\mu}\nu_L-W^0_{\mu}e_L\end{pmatrix}$$

$$=-\frac{g_2}{2}\left[\overline{\nu}_L\gamma^{\mu}\nu_L W^0_{\mu}-\sqrt{2}\,\overline{\nu}_L\gamma^{\mu}e_L W^+_{\mu}-\sqrt{2}\,\overline{e}_L\gamma^{\mu}\nu_L W^-_{\mu}-\overline{e}_L\gamma^{\mu}e_L W^0_{\mu}\right]$$

The Neutral Current

Electromagnetic interaction of particles of charge Q:

$$\mathcal{L}_{\mathrm{EM}} = QA_\mu \left[\bar{e}_L \gamma^\mu e_L + \bar{e}_R \gamma^\mu e_R\right]$$

There are terms involving neutrinos $\left(-\frac{g_1}{2} Y_L B_\mu - \frac{g_2}{2} W^0_\mu\right) \overline{\nu_L} \gamma^\mu \nu_L$

We assume the the electromagnetic field A_μ is the orthogonal combination:

$$A_\mu \propto g_2 B_\mu - g_1 Y_L W^0_\mu \qquad Z_\mu \propto g_1 Y_L B_\mu + g_2 W^0_\mu$$

$$A_\mu = \frac{g_2 B_\mu - g_1 Y_L W^0_\mu}{\sqrt{g_2^2 + g_1^2 Y_L^2}} \qquad Z_\mu = \frac{g_1 Y_L B_\mu + g_2 W^0_\mu}{\sqrt{g_2^2 + g_1^2 Y_L^2}}$$

The Neutral Current

Terms involving electrons: $\bar{e}_L\gamma^\mu e_L\left(-\frac{g_1}{2}Y_L B_\mu+\frac{g_2}{2}W^0_\mu\right)+\bar{e}_R\gamma^\mu e_R\left(-\frac{g_1}{2}Y_R B_\mu\right)$

$$B_\mu=\frac{g_2A_\mu+g_1Y_LZ_\mu}{\sqrt{g_2^2+g_1^2Y_L^2}}\qquad W^0_\mu=\frac{-g_1Y_LA_\mu+g_2Z_\mu}{\sqrt{g_2^2+g_1^2Y_L^2}}$$

$$-A_\mu\left\{\bar{e}_L\gamma^\mu e_L\left[\frac{g_1g_2Y_L}{\sqrt{g_2^2+g_1^2Y_L^2}}\right]+\bar{e}_R\gamma^\mu e_R\left[\frac{g_1g_2Y_R}{2\sqrt{g_2^2+g_1^2Y_L^2}}\right]\right\}$$

$$-Z_\mu\left\{\bar{e}_L\gamma^\mu e_L\left[\frac{g_1^2Y_L^2-g_2^2}{2\sqrt{g_2^2+g_1^2Y_L^2}}\right]+\bar{e}_R\gamma^\mu e_R\left[\frac{g_1^2Y_RY_L}{2\sqrt{g_2^2+g_1^2Y_L^2}}\right]\right\}$$

The term in A_μ must be the usual electromagnetic current.
The term in Z_μ can be an additional interaction, to be checked experimentally.

The Neutral Current

$$-e=\frac{g_1g_2Y_L}{\sqrt{g_2^2+g_1^2Y_L^2}} \qquad -e=\frac{g_1g_2Y_R}{2\sqrt{g_2^2+g_1^2Y_L^2}}$$

$$Y_R=2Y_L$$

$$Y_L=-e\frac{\sqrt{g_2^2+g_1^2Y_L^2}}{g_1g_2}$$

We can choose Y_L=-*1*, since any change in Y_L can be absorbed by a redefinition of g_1.

$$Y_L=-1 \quad \Rightarrow \quad e=\frac{g_1g_2}{\sqrt{g_2^2+g_1^2}}$$

The theory we have been writing can be interpreted to contain the usual electromagnetic interaction, plus an additional neutral current interaction with Z_μ for both electrons and neutrinos.

The Neutral Current

Define:

$$\sin\theta_W = \frac{g_1}{\sqrt{g_2^2 + g_1^2}}$$

$$\cos\theta_W = \frac{g_2}{\sqrt{g_2^2 + g_1^2}}$$

θ_W weak mixing angle (Weinberg angle)

$$g_1 = \frac{e}{\cos\theta_W}$$

$$g_2 = \frac{e}{\sin\theta_W}$$

g_1 and g_2 are written in terms of the known e ($e^2/4\pi \approx 1/137$) and the electroweak mixing angle, which needs to be measured or calculated some other way.

$$\sin^2\theta_W \approx 0.23$$

Neutrino and Z-boson coupling

$$-\frac{\sqrt{g_2^2+g_1^2}}{2} Z_\mu \bar{\nu}_L \gamma^\mu \nu_L = -\frac{g_2}{2\cos\theta_W} Z_\mu \bar{\nu}_L \gamma^\mu \nu_L$$

$\frac{g_2}{2\cos\theta_W}$ quantity to be associated to each ν_L-Z vertex.
“electroweak charge” of the left-handed neutrino.

$$\sqrt{g_2^2+g_1^2} = \left[\frac{e^2}{\cos^2\theta_W} + \frac{e^2}{\sin^2\theta_W}\right]^{1/2}$$

$$= \left[\frac{e^2}{\cos^2\theta_W \sin^2\theta_W}\right]^{1/2}$$

$$= \frac{e}{\cos\theta_W \sin\theta_W}$$

Electron and Z-boson coupling

$$-Z_\mu\left\{\bar{e}_L\gamma^\mu e_L\left[\frac{g_1^2-g_2^2}{2\sqrt{g_2^2+g_1^2}}\right]+\bar{e}_R\gamma^\mu e_R\left[\frac{g_1^2}{\sqrt{g_2^2+g_1^2}}\right]\right\}$$

$$\frac{g_1^2-g_2^2}{2\sqrt{g_2^2+g_1^2}}=\frac{e^2}{2\sqrt{g_2^2+g_1^2}}\left(\frac{1}{\cos^2\theta_W}-\frac{1}{\sin^2\theta_W}\right)$$

$$=\frac{e}{\cos\theta_W\sin\theta_W}\left(-\frac{1}{2}+\sin^2\theta_W\right)$$

e_L Coupling

$$\frac{g_1^2}{\sqrt{g_2^2+g_1^2}}=-\frac{e^2}{\cos^2\theta_W}\frac{\cos\theta_W\sin\theta_W}{e}$$

$$=\frac{e}{\cos\theta_W\sin\theta_W}\left(\sin^2\theta_W\right)$$

e_R Coupling

The Charged Current

The $U(1)$ part of the Lagrangian contains only terms diagonal in the fermions, whereas the $SU(2)$ part has also non diagonal terms.

$$\mathcal{L}_{\text{ferm}} = \frac{g_2}{\sqrt{2}}\left[\bar{\nu}_L \gamma^\mu e_L W_\mu^+ + \bar{e}_L \gamma^\mu \nu_L W_\mu^-\right]$$

charged current

$$\bar{\nu}_L \gamma^\mu e_L = \frac{1}{2}\bar{\nu}\gamma^\mu \left(1-\gamma^5\right)e$$

V-A interaction

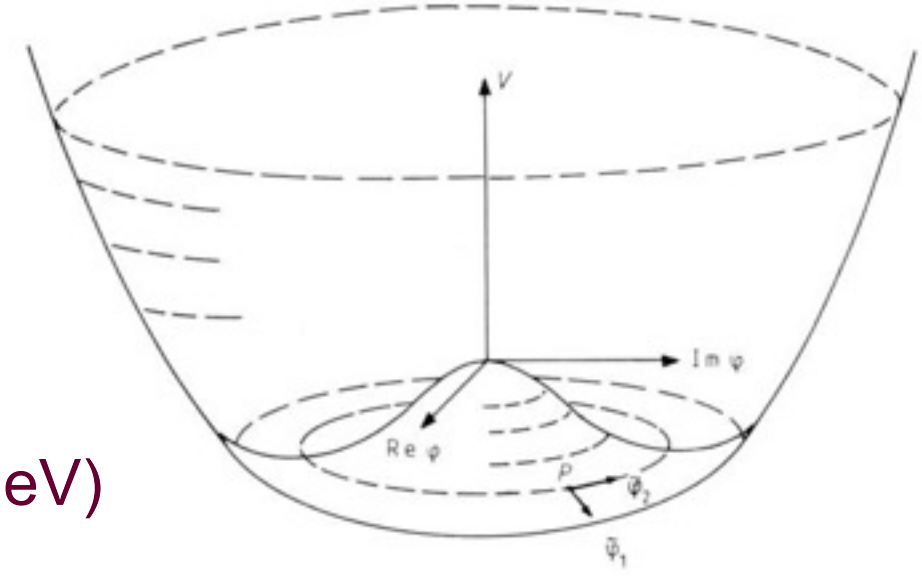
We thus expect $W^\pm$ bosons and the associated charged current transitions. The observed charged currents occur with a strength much smaller than one would expect:

$$\frac{\left(g_2/\sqrt{2}\right)^2}{4\pi} = \frac{\left(e^2/4\pi\right)}{2\sin^2\theta_W} \approx \frac{2}{137}$$

Higgs Mechanism in the SM

A fundamental (complex) scalar doublet:

$$\phi = \begin{pmatrix}\phi^{\dagger} \\ \phi^{0}\end{pmatrix}$$



- The cause of Electroweak Symmetry Breaking

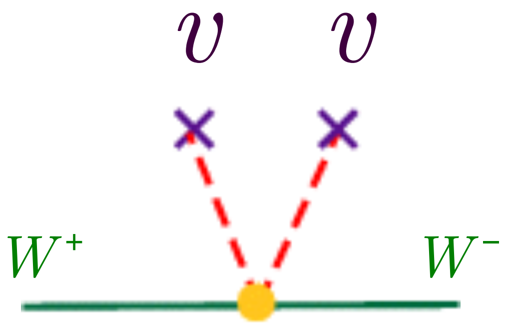
(M$_W$ = 80 GeV, M$_Z$ = 91 GeV)

- The origin of Flavor Symmetry Breaking

(Quarks and Leptons have diverse masses.)

- To generate M$_W$ and M$_Z$

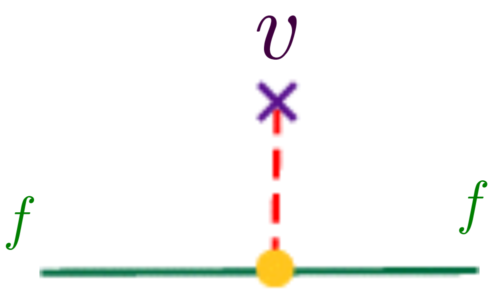
$$L_\phi = (D_\mu\phi)^{+} (D^\mu\phi) - \lambda\left(\phi^{+}\phi - \frac{v^2}{2}\right)^2$$



⇨ $M_W = \frac{1}{2} g v$

- To generate m_f

$$y_f \bar{F}_L \phi f_R + \cdots$$

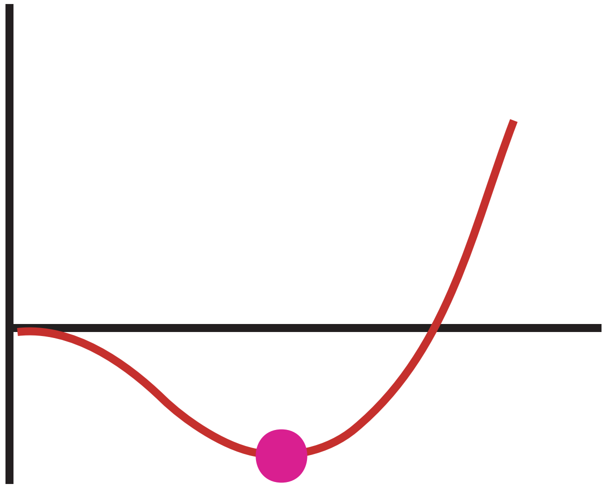


⇨ $m_f = y_f \frac{v}{\sqrt{2}}$

The Lagrangian for φ is

$$\mathcal{L} = |D_\mu \varphi|^2 - V(|\varphi|) - \frac{1}{4}(F^a_{\mu\nu})^2 - \frac{1}{4}(G_{\mu\nu})^2$$

+ (coupling to quarks and leptons)



$$v \to v + h(x)$$

$$\varphi = \begin{pmatrix} \pi^+ \\ (v + h + i\pi^0)/\sqrt{2} \end{pmatrix}$$

$$|D_\mu \varphi|^2$$

$$= \begin{pmatrix} 0 & v/\sqrt{2} \end{pmatrix} \left| \frac{g}{\sqrt{2}} W^+ \sigma^+ + \frac{g}{\sqrt{2}} W^- \sigma^- + \frac{g}{2} W^0 \sigma^3 + \frac{g'}{2} B \right|^2 \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

$$= \frac{v^2}{4}[g^2 W^+ W^- + \frac{1}{2}(-gW^0 + g'B)^2]$$

$$\boxed{\begin{array}{l} g = g_2 \\ g' = g_1 \end{array}}$$

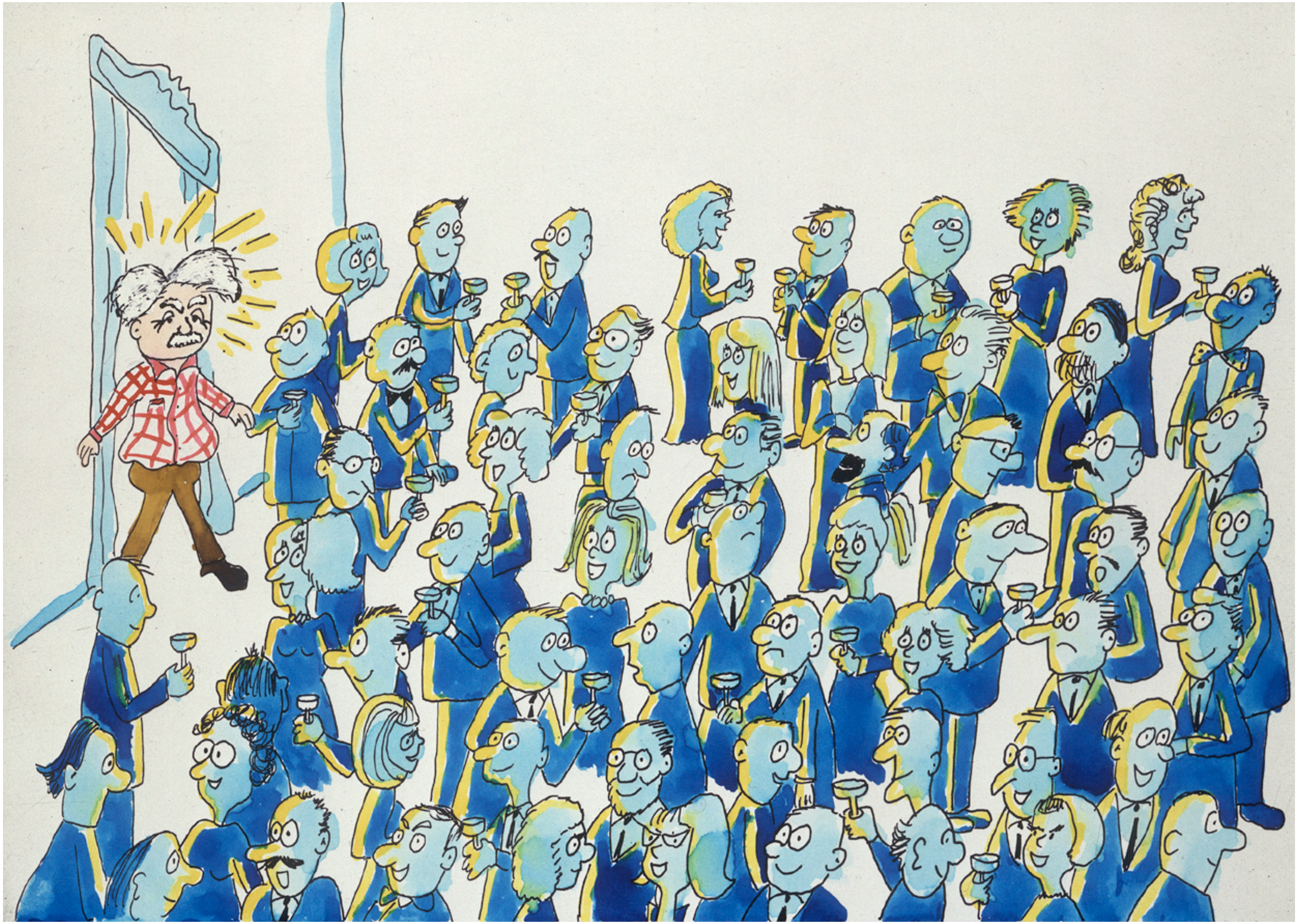
$$Z = \cos\theta_w W^3 - \sin\theta_w B$$
$$A = \sin\theta_w W^3 + \cos\theta_w B$$

$$\cos\theta_w = \frac{g}{\sqrt{g^2 + g'^2}} \qquad \sin\theta_w = \frac{g'}{\sqrt{g^2 + g'^2}}$$

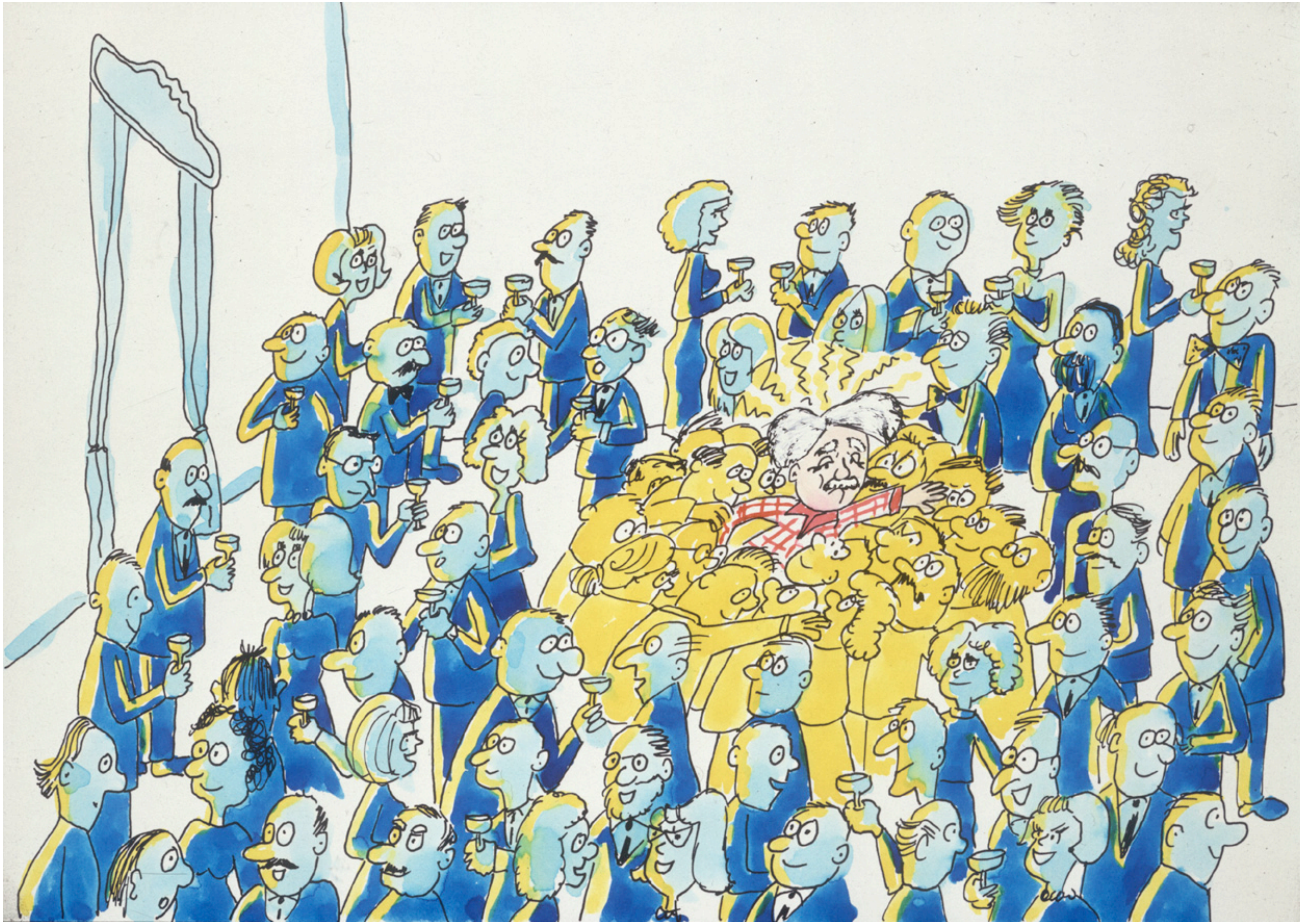
$$\boxed{m_W^2 = \frac{g^2}{4} v^2 \qquad m_Z^2 = \frac{g^2 + g'^2}{4} v^2 \qquad m_W / m_Z = \cos\theta_w}$$

标准模型的希格斯机制

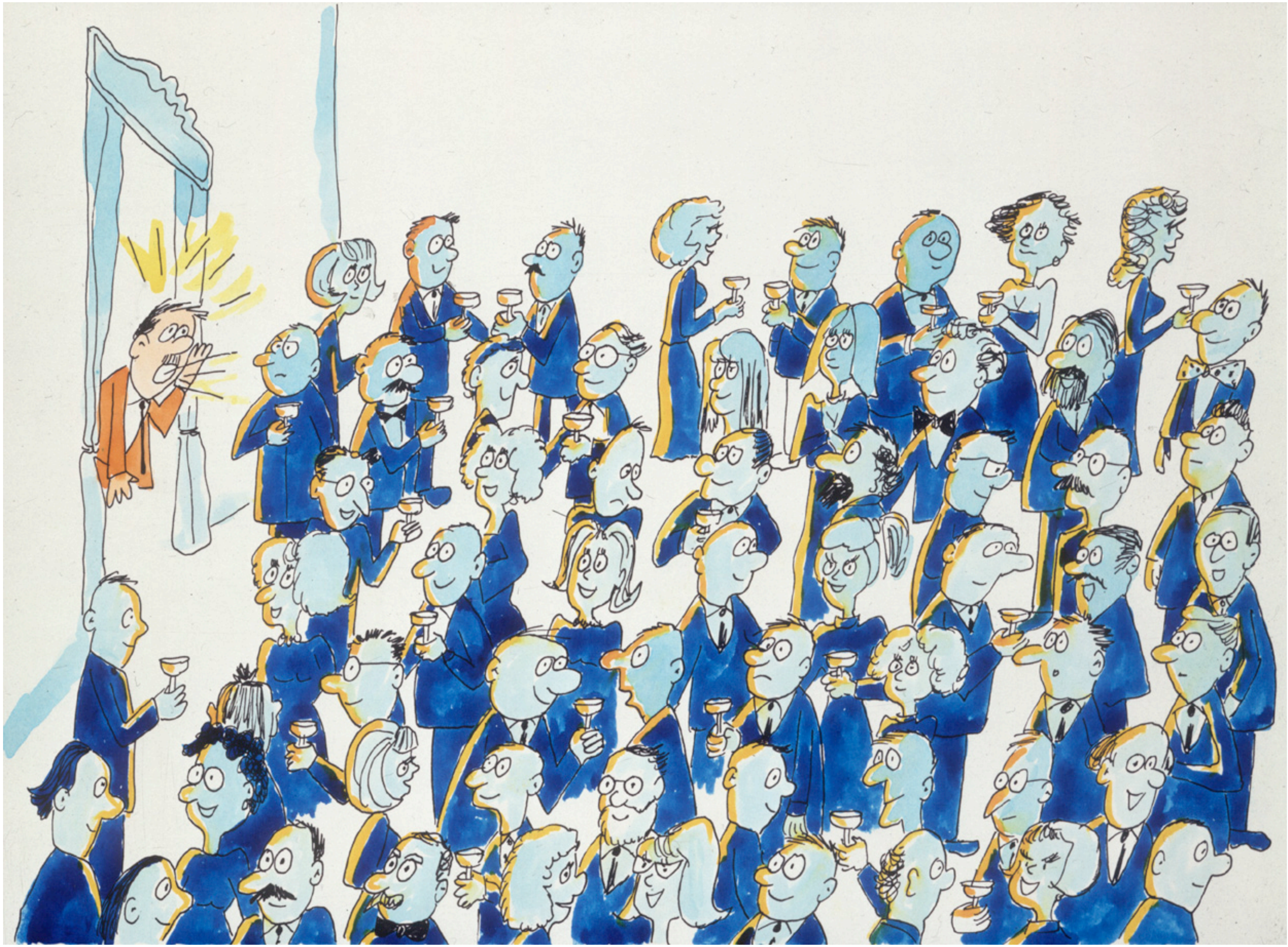
标准模型的希格斯机制



标准模型的希格斯机制



标准模型的希格斯机制



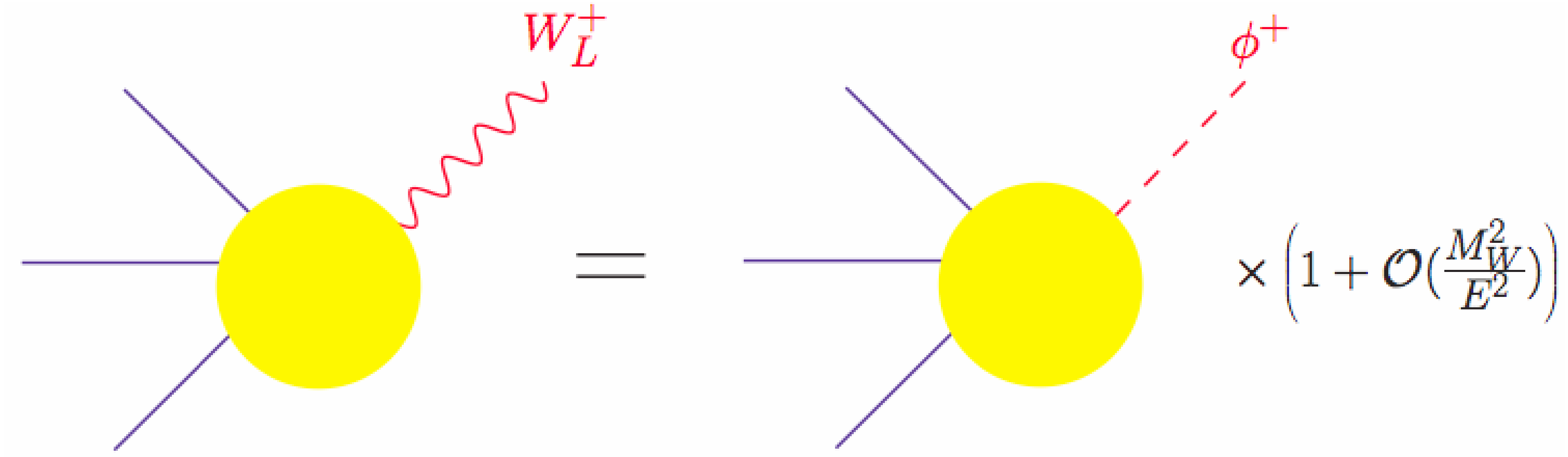
标准模型的希格斯机制

1) Why are we not anxious when LEP & Tevatron missed the Higgs boson?

2) Why does it take such a long time to observe the spin-0 particle?

Footprint of Higgs Boson

- Equivalence theorem:



Longitudinal polarized massive gauge boson → unphysical Goldstone boson

At the rest frame of gauge boson:
three polarization states
are equivalent.

$$k = \begin{pmatrix} m & 0 & 0 & 0 \end{pmatrix}$$
$$\varepsilon_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}$$
$$\varepsilon_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix}$$
$$\varepsilon_3 = \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}$$

In the high energy limit:
Longitudinal polarization state
is distinctive.

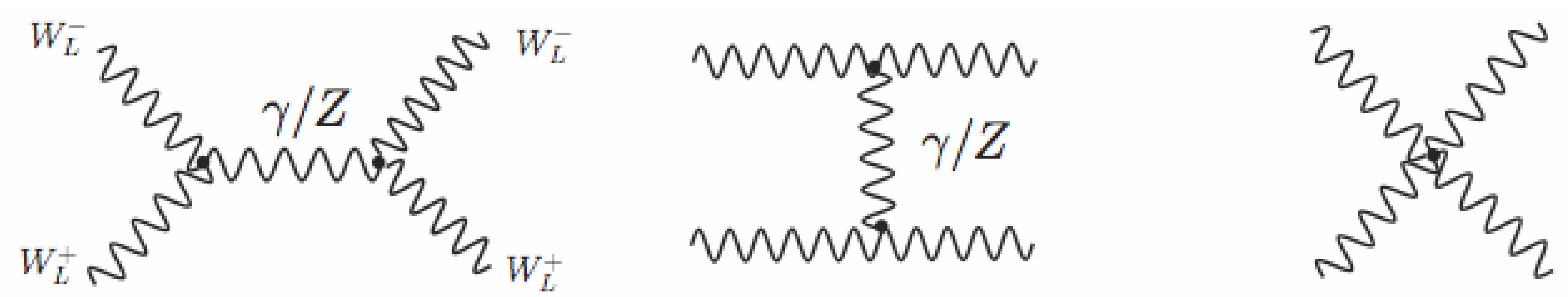
$$k = \begin{pmatrix} E & 0 & 0 & p \end{pmatrix}$$
$$\varepsilon_T^{1,2} = \begin{pmatrix} 0 & 1 & \pm i & 0 \end{pmatrix}$$
$$\varepsilon_L = \begin{pmatrix} \frac{p}{m_W} & 0 & 0 & \frac{E}{m_W} \end{pmatrix} \xrightarrow{p \to \infty} \varepsilon_L = \frac{k}{m} + O\left(\frac{m}{E}\right)$$

Theoretical Bound of Higgs Boson Mass

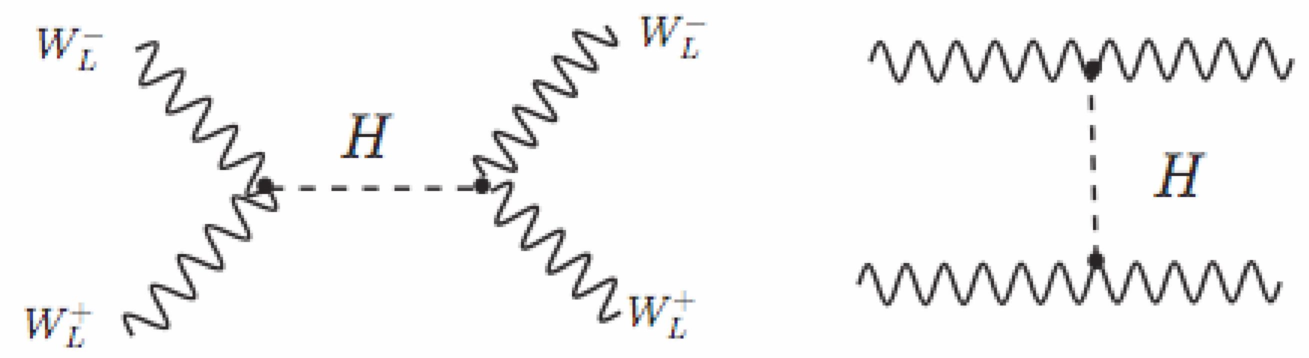
- Imaginary experiment:

$$W_L^+ W_L^- \to W_L^+ W_L^-$$

Longitudinal gauge boson scattering cross section at high energy grows with M_H



$$\propto a \frac{E^2}{M_W^2} + \text{finite term}$$



Theorem:
The only way to unitarize amplitude is by a scalar Higgs boson

$$\propto -a \frac{E^2}{M_W^2} + \text{finite term}$$

Theoretical Bound of Higgs Boson Mass

- Unitarity

For any $2 \rightarrow 2$ elastic scattering,

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|M|^2$$

one can make the partial wave decomposition:

$$M = 16\pi \sum_{l=0}^{\infty} (2l+1)\ P_l(\cos\theta)\ a_l$$

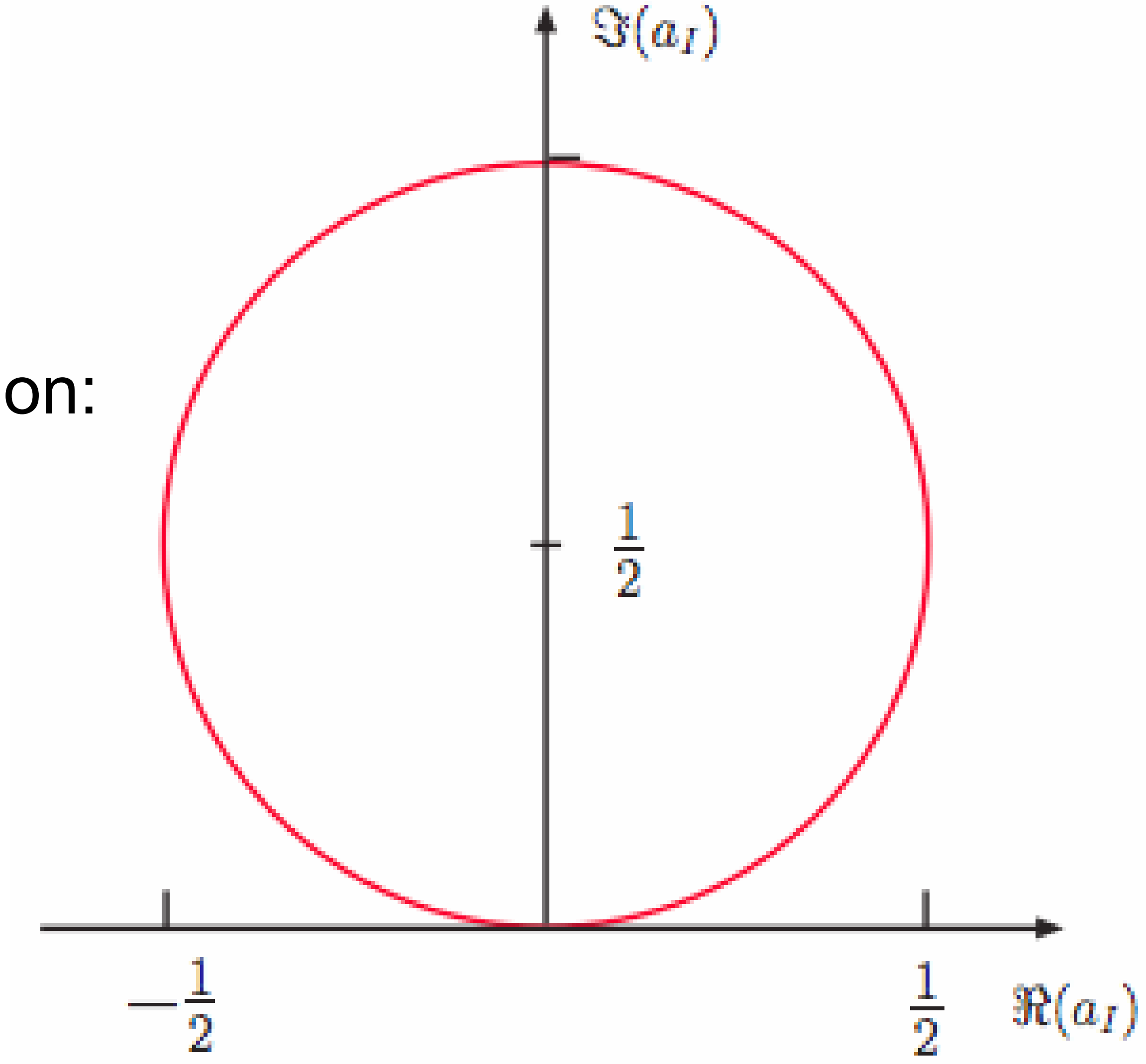
Therefore,

$$\sigma = \frac{16\pi}{s}\sum_{l=0}^{\infty}(2l+1)\ |a_l|^2$$

Optical theorem:

$$\sigma = \frac{1}{s}\mathrm{Im}\left[M(\theta=0)\right] = \frac{16\pi}{s}\sum_{l=0}^{\infty}(2l+1)|a_l|^2$$

$$\longrightarrow \quad |\mathrm{Re}(a_l)| < \frac{1}{2}$$



Theoretical Bound of Higgs Boson Mass

For $W_L^+ W_L^- \to W_L^+ W_L^-$,

$$a_0 = -\frac{M_H^2}{16\pi v^2}\left[2 + \frac{M_H^2}{s - M_H^2} - \frac{M_H^2}{s}\log\left(1 + \frac{s}{M_H^2}\right)\right]$$

- When $s \gg M_H^2,\ a_0 \to -\dfrac{M_H^2}{8\pi v^2}$

$$\left|\mathrm{Re}(a_0)\right| < \frac{1}{2} \longrightarrow M_H < 870\ \mathrm{GeV}$$

- If there is no Higgs boson observed,

  i.e. $M_H \gg s,\ a_0 \to -\dfrac{s}{32\pi v^2}$

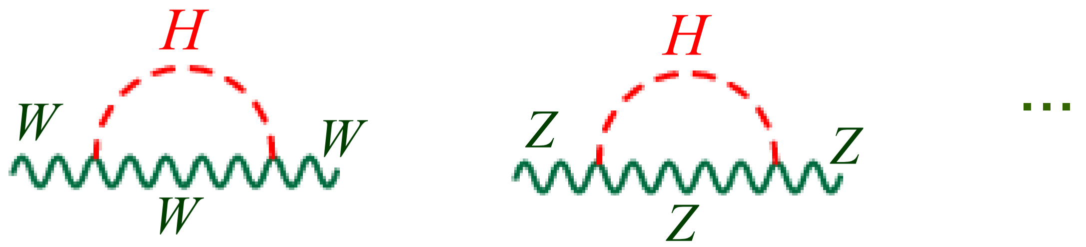
$$\left|\mathrm{Re}(a_0)\right| < \frac{1}{2} \longrightarrow \sqrt{s_c} < 1.8\ \mathrm{TeV}$$

Best constraints are derived from coupled channels: *WW*, *ZZ*, *HH*, *HZ*.

New physics expected at TeV scale (LHC)

Why so difficult to see Higgs boson?

- The direct search of Higgs boson is negative due to many reasons (theoretical uncertainties, copious backgrounds, etc.), but why can we not see it from the quantum corrections?



Strong interaction for large m_H

- Screening theorem (Veltman 1977):

Radiative corrections which are dependent on the Higgs mass are of form,

$$g^2\left[\ln\left(\frac{m_H}{m_W}\right)+g^2\left(\frac{m_H^2}{m_W^2}\right)+\cdots\right]$$

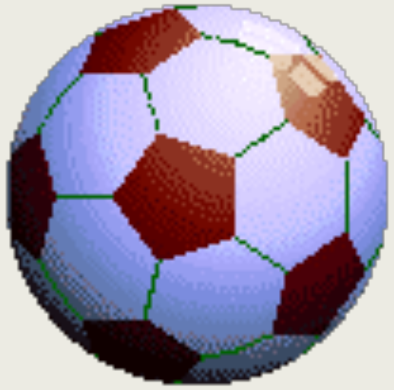
⇒ Low energy observables are relatively insensitive to M_H.

Deeper understanding of the source of this screening cf. M.B. Einhorn and Jose Wudka, PRD39 (1989) 2758.

Summary

Theory (1970-1975): All fundamental subatomic particles can be understood as the energy quanta of fields:

- vector fields: spin 1 particles
- spinor fields: Dirac particles with spin ½
- scalar fields: spin 0 particles



These fields interact. Particles interact.

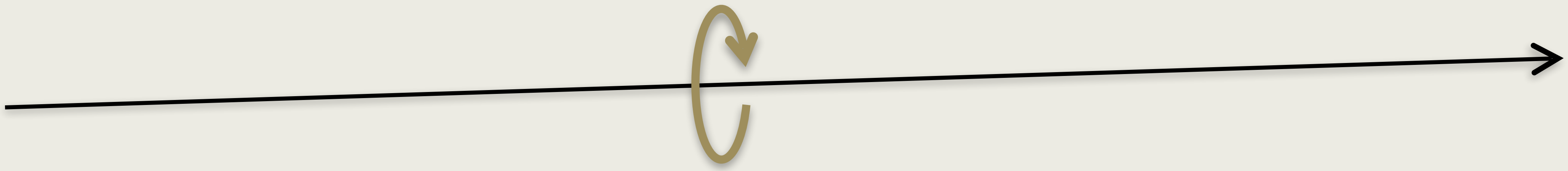
Due to these interactions, they tend to accumulate **infinite** amounts of energy: hence infinite mass !

Unless there is some protection mechanism,

keeping the ***mass = 0*** *,*

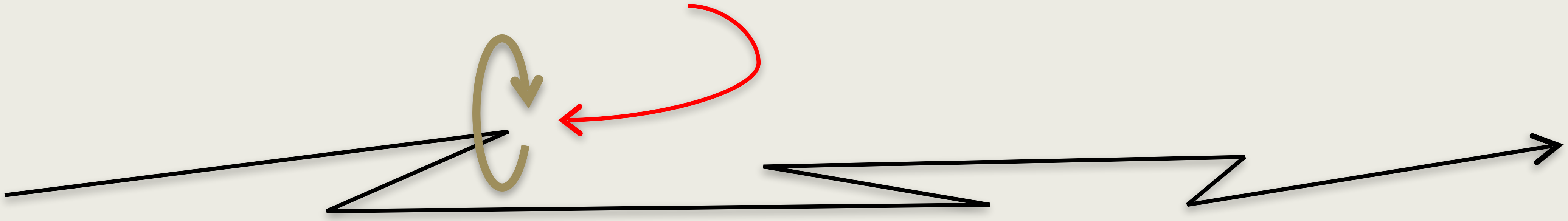
spinning *particles, interacting with* ***vector fields****, enjoy such a protection mechanism !*

Spin = angular momentum

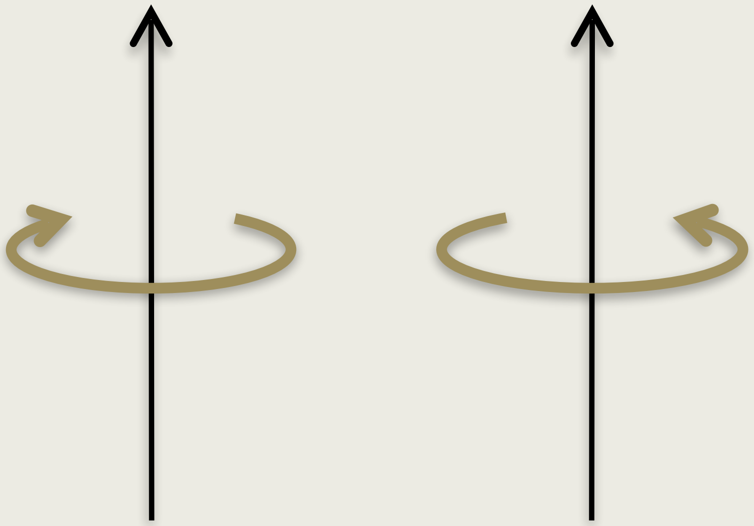


Massless particle: *always goes with speed of light*

Helicity has to ***flip***



Particle with mass: *always goes slower*



Helicity =

spin along
axis parallel
to motion

The electromagnetic force and the weak force cannot flip the helicity of a particle, so

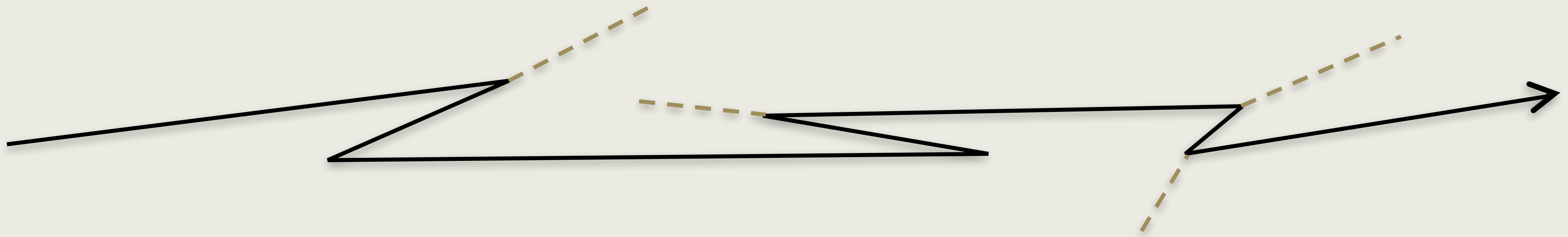
These forces cannot generate mass !!

When a force acts differently on particles with *Left-helicity* and *right helicity*, then

Mass is forbidden !!

This is why electrons, neutrinos, quarks should all be strictly massless …

But they do have mass !!



Particle with mass

A particle that can vanish
into empty space:
the Higgs particle.

It carries away the weak charges that determine the effects of the weak forces.

Now the helicity can flip !!

THE HIGHWAY ACROSS THE DESERT

Planck length : 10^{-35} m

GUTs

TODAY's limit