Electroweak Theory of the Standard Model

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20世纪基础物理学

- 源于1895年伦琴射线，止于2012年希格斯粒子发现

量子力学

相对论

量子场论

规范场论

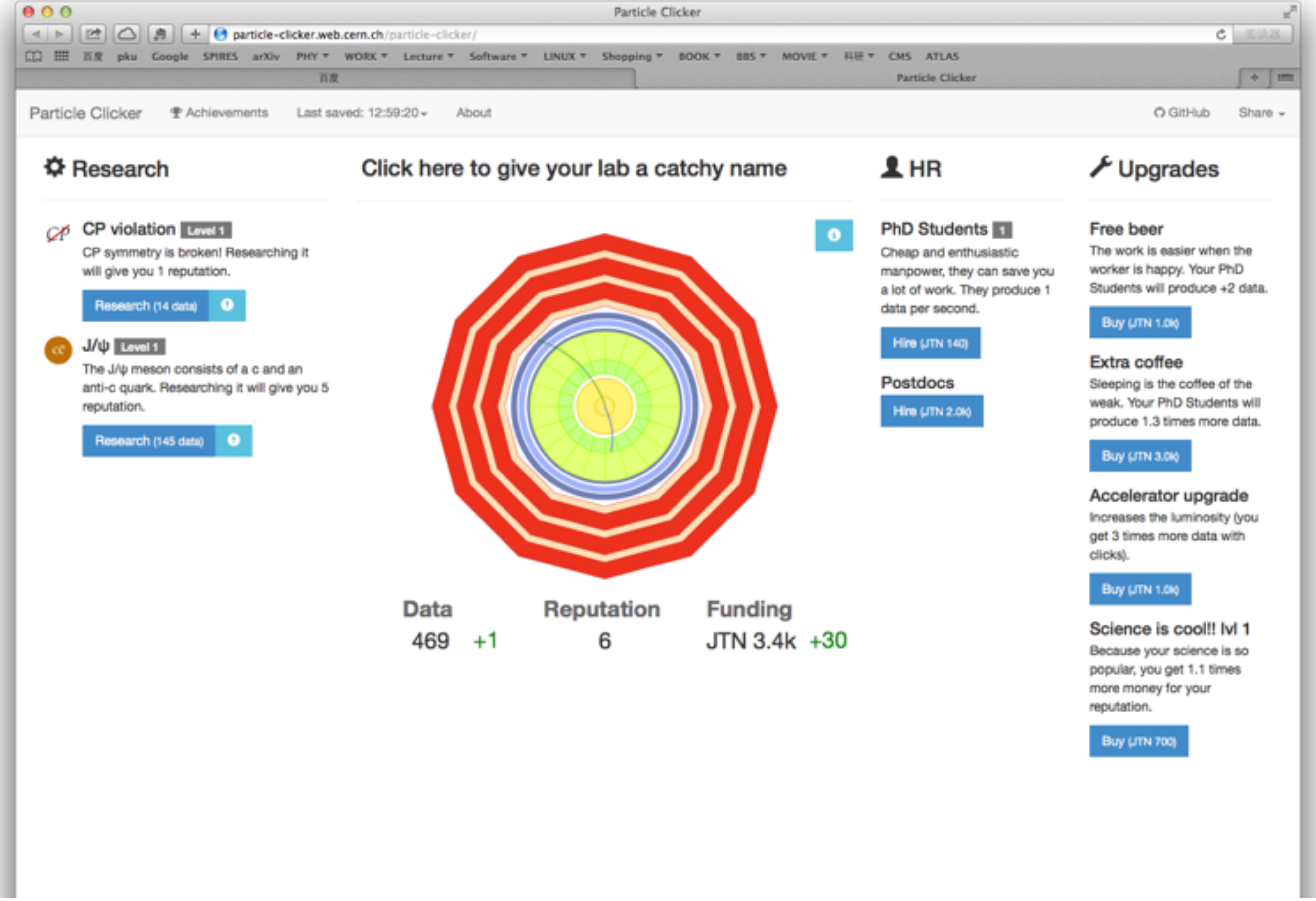
标准模型

量子色动力学

电弱理论

Want to be a great physicist? Piece of Cake!

http://particle-clicker.web.cern.ch/particle-clicker/



Research

Click here to give your lab a catchy name

HR

Upgrades

CP violation Level 1

CP symmetry is broken! Researching it will give you 1 reputation.

Research (14 data)

J/ψ Level 1

The J/ψ meson consists of a c and an anti-c quark. Researching it will give you 5 reputation.

Research (145 data)

Data	Reputation	Funding
469 +1	6	JTN 3.4k +30

PhD Students 1

Cheap and enthusiastic manpower, they can save you a lot of work. They produce 1 data per second.

Hire (JTN 140)

Postdocs

Hire (JTN 2.0k)

Free beer

The work is easier when the worker is happy. Your PhD Students will produce +2 data.

Buy (JTN 1.0k)

Extra coffee

Sleeping is the coffee of the weak. Your PhD Students will produce 1.3 times more data.

Buy (JTN 3.0k)

Accelerator upgrade

Increases the luminosity (you get 3 times more data with clicks).

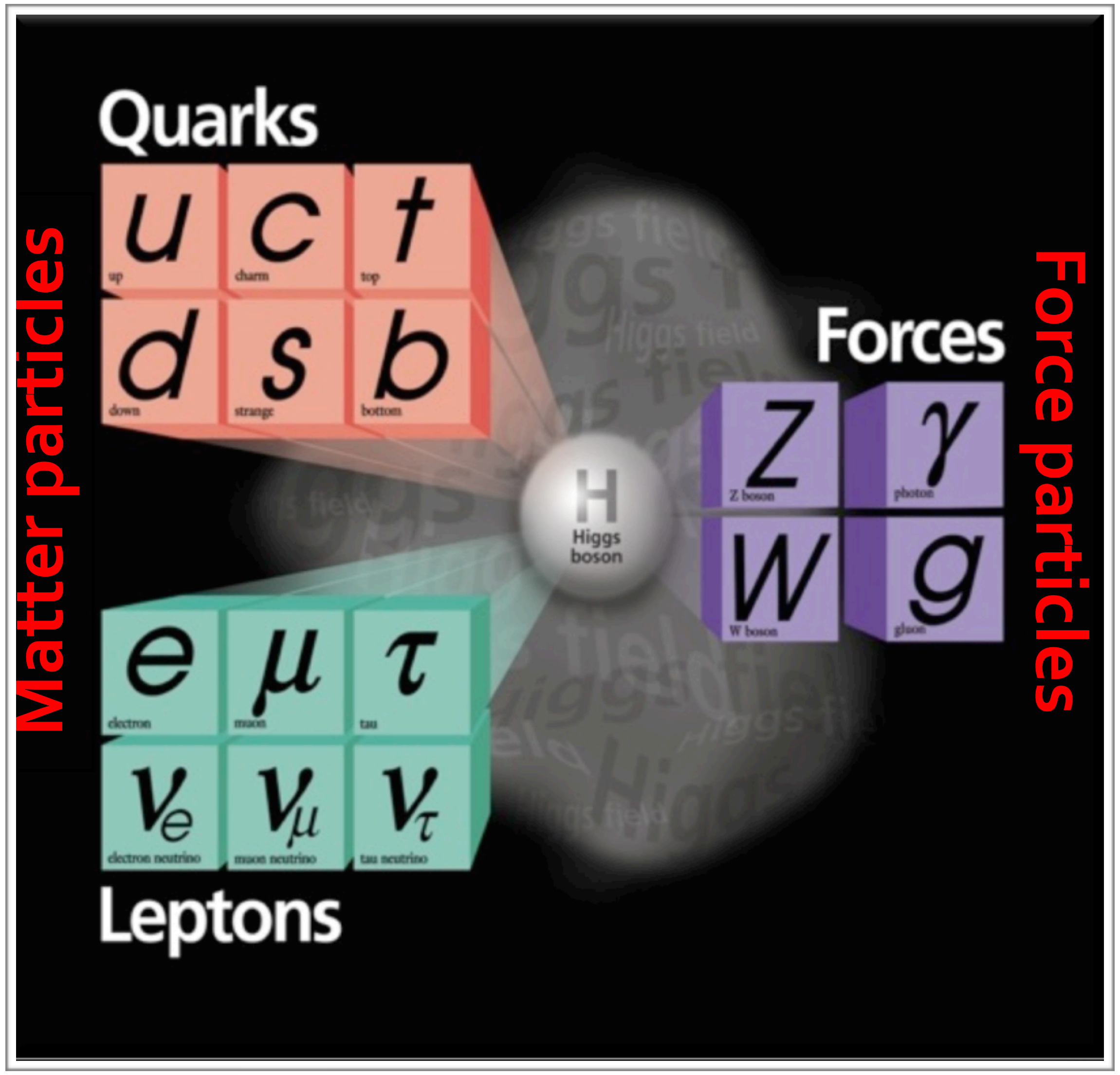
Buy (JTN 1.0k)

Science is cool!! lvl 1

Because your science is so popular, you get 1.1 times more money for your reputation.

Buy (JTN 700)

粒子物理的标准模型



集百年物理之大成

新“元素周期表”

20世纪自然科学
的卓越成就之一

宇宙万物可以用一个简单公式描述

$\mathcal{L}_{\rm SM} =$

$$-\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \frac{1}{2}ig_s^2(\bar{q}^\sigma_i \gamma^\mu q^\sigma_j)g^a_\mu + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu$$

$$-\partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H - \frac{1}{2}m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^-$$

$$-M^2\phi^+\phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \frac{1}{2c_w^2}M\phi^0\phi^0 - \beta_h\left[\frac{2M^2}{g^2} + \frac{2M}{g}H + \frac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)\right] + \frac{2M^4}{g^2}\alpha_h$$

$$-igc_w\left[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)\right]$$

$$-igs_w\left[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)\right]$$

$$-\frac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2\left(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu\right) + g^2 s_w^2\left(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu\right)$$

$$+g^2 s_w c_w\left[A_\mu Z^0_\nu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu\right] - g\alpha\left[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-\right]$$

$$-\frac{1}{8}g^2\alpha_h\left[H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2\right] - gMW^+_\mu W^-_\mu H - \frac{1}{2}g\frac{M}{c_w^2}Z^0_\mu Z^0_\mu H$$

$$-\frac{1}{2}ig\left[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)\right] + \frac{1}{2}g\left[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right]$$

$$+\frac{1}{2}g\frac{1}{c_w}Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_wMA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^-$$

$$-\phi^-\partial_\mu\phi^+) + igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac{1}{4}g^2W^+_\mu W^-_\mu\left[H^2 + (\phi^0)^2 + 2\phi^+\phi^-\right] - \frac{1}{4}g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2 + (\phi^0)^2$$

$$+2(2s_w^2 - 1)^2\phi^+\phi^-] - \frac{1}{2}g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \frac{1}{2}ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac{1}{2}g^2s_wA_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+)$$

$$+\frac{1}{2}ig^2s_wA_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\frac{s_w}{c_w}(2c_w^2 - 1)Z^0_\mu A_\mu\phi^+\phi^- - g^1s_w^2A_\mu A_\mu\phi^+\phi^- - \bar{e}^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar{\nu}^\lambda\gamma\partial\nu^\lambda$$

$$-\bar{u}_j^\lambda(\gamma\partial + m_u^\lambda)u_j^\lambda - \bar{d}_j^\lambda(\gamma\partial + m_d^\lambda)d_j^\lambda + igs_wA_\mu[-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}_j^\lambda\gamma^\mu u_j^\lambda) - \frac{1}{3}(\bar{d}_j^\lambda\gamma^\mu d_j^\lambda)]$$

$$+\frac{ig}{4c_w}Z^0_\mu\left[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{u}_j^\lambda\gamma^\mu(\frac{4}{3}s_w^2 - 1 - \gamma^5)u_j^\lambda) + (\bar{d}_j^\lambda\gamma^\mu(1 - \frac{8}{3}s_w^2 - \gamma^5)d_j^\lambda)\right]$$

$$+\frac{ig}{2\sqrt{2}}W^+_\mu\left[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) + (\bar{u}_j^\lambda\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d_j^\kappa)\right] + \frac{ig}{2\sqrt{2}}W^-_\mu\left[(\bar{e}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}_j^\kappa C^\dagger_{\lambda\kappa}\gamma^\mu(1+\gamma^5)u_j^\lambda)\right]$$

$$+\frac{ig}{2\sqrt{2}}\frac{m_e^\lambda}{M}\left[-\phi^+(\bar{\nu}^\lambda(1-\gamma^5)e^\lambda) + \phi^-(\bar{e}^\lambda(1+\gamma^5)\nu^\lambda)\right] - \frac{g}{2}\frac{m_e^\lambda}{M}\left[H(\bar{e}^\lambda e^\lambda) + i\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda)\right]$$

$$+\frac{ig}{2M\sqrt{2}}\phi^+\left[-m_d^\kappa(\bar{u}_j^\lambda C_{\lambda\kappa}(1-\gamma^5)d_j^\kappa) + m_u^\lambda(\bar{u}_j^\lambda C_{\lambda\kappa}(1+\gamma^5)d_j^\kappa)\right] + \frac{ig}{2M\sqrt{2}}\phi^-\left[m_d^\lambda(\bar{d}_j^\lambda C^\dagger_{\lambda\kappa}(1+\gamma^5)u_j^\kappa) - m_u^\kappa(\bar{d}_j^\lambda C^\dagger_{\lambda\kappa}(1-\gamma^5)u_j^\kappa\right.$$

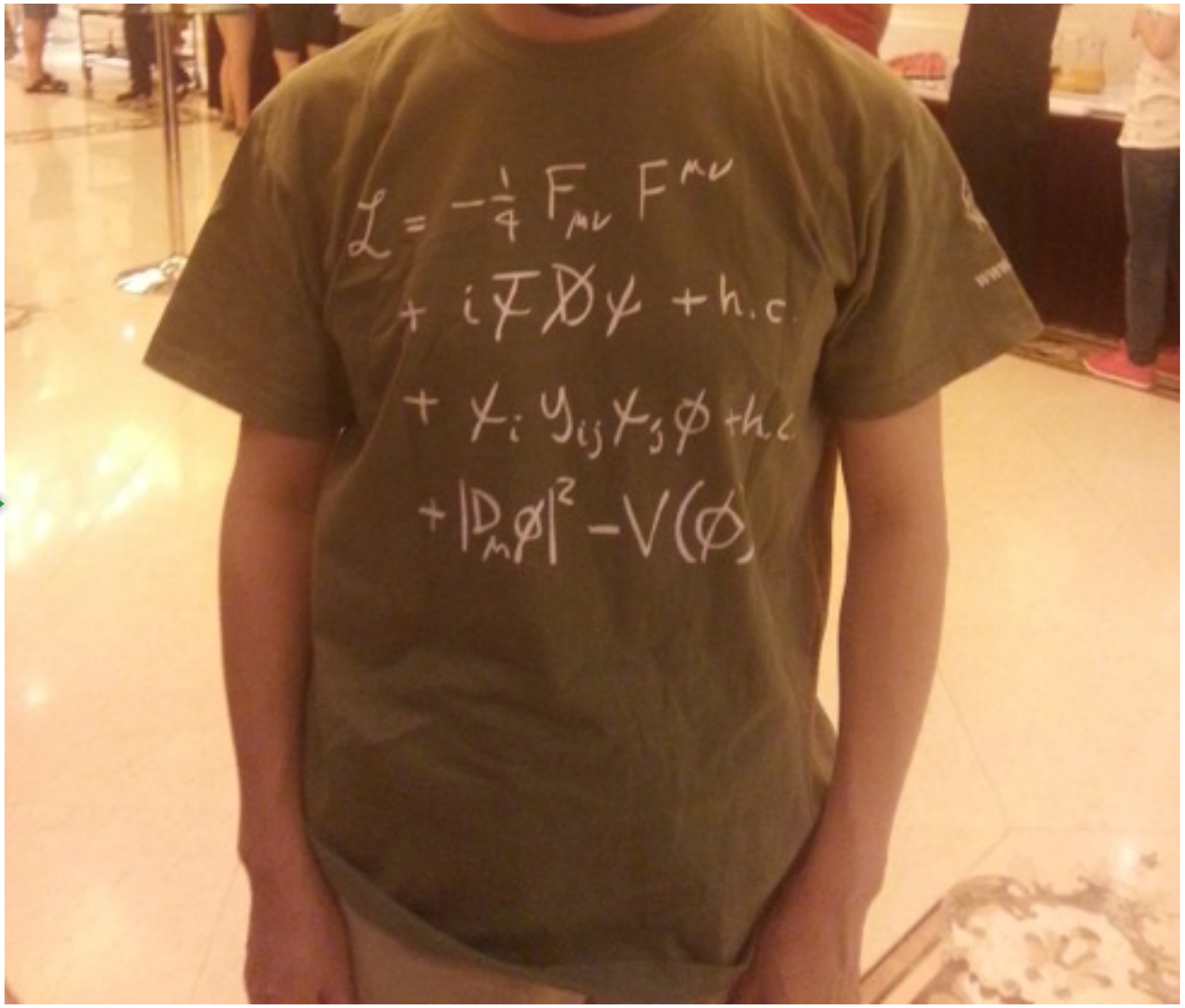
$$-\frac{g}{2}\frac{m_u^\lambda}{M}H(\bar{u}_j^\lambda u_j^\lambda) - \frac{g}{2}\frac{m_d^\lambda}{M}H(\bar{d}_j^\lambda d_j^\lambda) + \frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar{u}_j^\lambda\gamma^5u_j^\lambda) - \frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar{d}_j^\lambda\gamma^5d_j^\lambda) + \bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^-$$

$$+\bar{X}^0\left(\partial^2 - \frac{M^2}{c_w^2}\right)X^0 + \bar{Y}\partial^2Y + igc_wW^+_\mu(\partial_\mu\bar{X}^0X^- - \partial_\mu\bar{X}^+X^0) + igs_wW^+_\mu(\partial_\mu\bar{Y}X^- - \partial_\mu\bar{X}^+Y) + igc_wW^-_\mu(\partial_\mu\bar{X}^-X^0 - \partial_\mu\bar{X}^0X^+)$$

$$+igs_wW^-_\mu(\partial_\mu\bar{X}^-Y - \partial_\mu\bar{Y}X^+) + igc_wZ^0_\mu(\partial_\mu\bar{X}^+X^+ - \partial_\mu\bar{X}^-X^-) + igs_wA_\mu(\partial_\mu\bar{X}^+X^+ - \partial_\mu\bar{X}^-X^-) - \frac{1}{2}gM[\bar{X}^+X^+H + \bar{X}^-X^-H$$

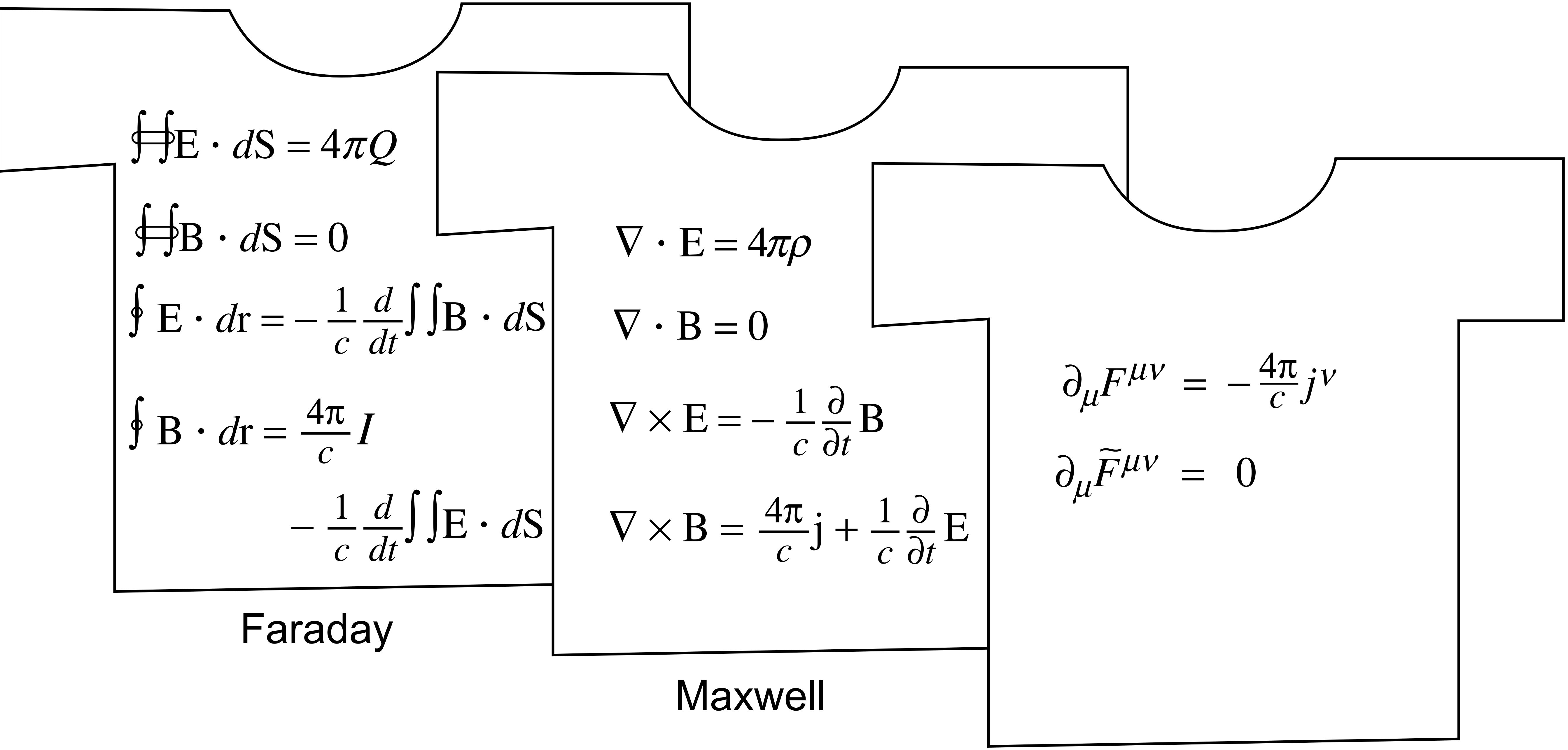
$$+\frac{1}{c_w^2}\bar{X}^0X^0H] + \frac{1-2c_w^2}{2c_w}igM[\bar{X}^+X^0\phi^+ - \bar{X}^-X^0\phi^-] + \frac{1}{2c_w}igM[\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-] + igMs_w[\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-]$$

Standard Model of Particle Physics



Maxwell Equations

1864年10月27日，麦克斯韦写下方程组：
283种符号，20个变量，20个方程



$$\oiint \mathrm{E} \cdot d\mathrm{S} = 4\pi Q$$

$$\oiint \mathrm{B} \cdot d\mathrm{S} = 0$$

$$\oint \mathrm{E} \cdot d\mathrm{r} = -\frac{1}{c}\frac{d}{dt}\iint \mathrm{B} \cdot d\mathrm{S}$$

$$\oint \mathrm{B} \cdot d\mathrm{r} = \frac{4\pi}{c} I - \frac{1}{c}\frac{d}{dt}\iint \mathrm{E} \cdot d\mathrm{S}$$

Faraday

$$\nabla \cdot \mathrm{E} = 4\pi\rho$$

$$\nabla \cdot \mathrm{B} = 0$$

$$\nabla \times \mathrm{E} = -\frac{1}{c}\frac{\partial}{\partial t}\mathrm{B}$$

$$\nabla \times \mathrm{B} = \frac{4\pi}{c}\mathrm{j} + \frac{1}{c}\frac{\partial}{\partial t}\mathrm{E}$$

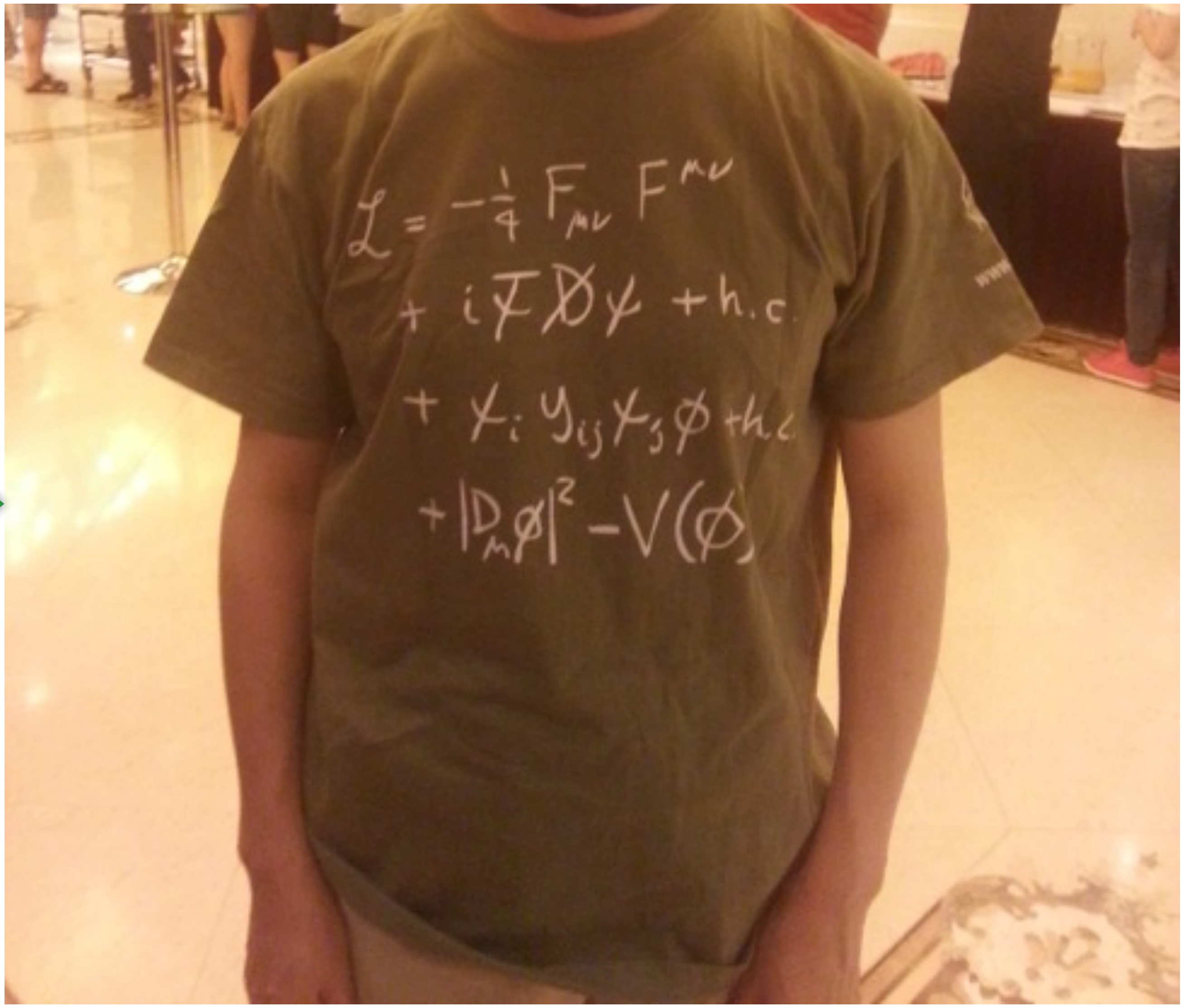
Maxwell

$$\partial_\mu F^{\mu\nu} = -\frac{4\pi}{c} j^\nu$$

$$\partial_\mu \tilde{F}^{\mu\nu} = 0$$

Einstein

Standard Model of Particle Physics



Four Forces in Nature

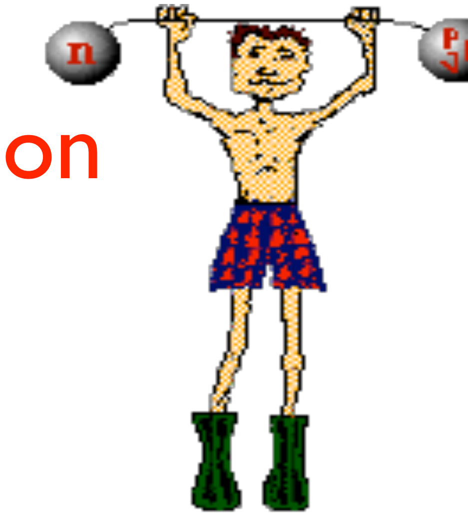
1 Gravity

Newton ⟺

2 Electromagnetism

Faraday ⟺

3 Weak Interaction



Beta-decay
Muon-decay

Time scale: $10^{-12} \sim 10^{3}$ s

4 Strong Interaction

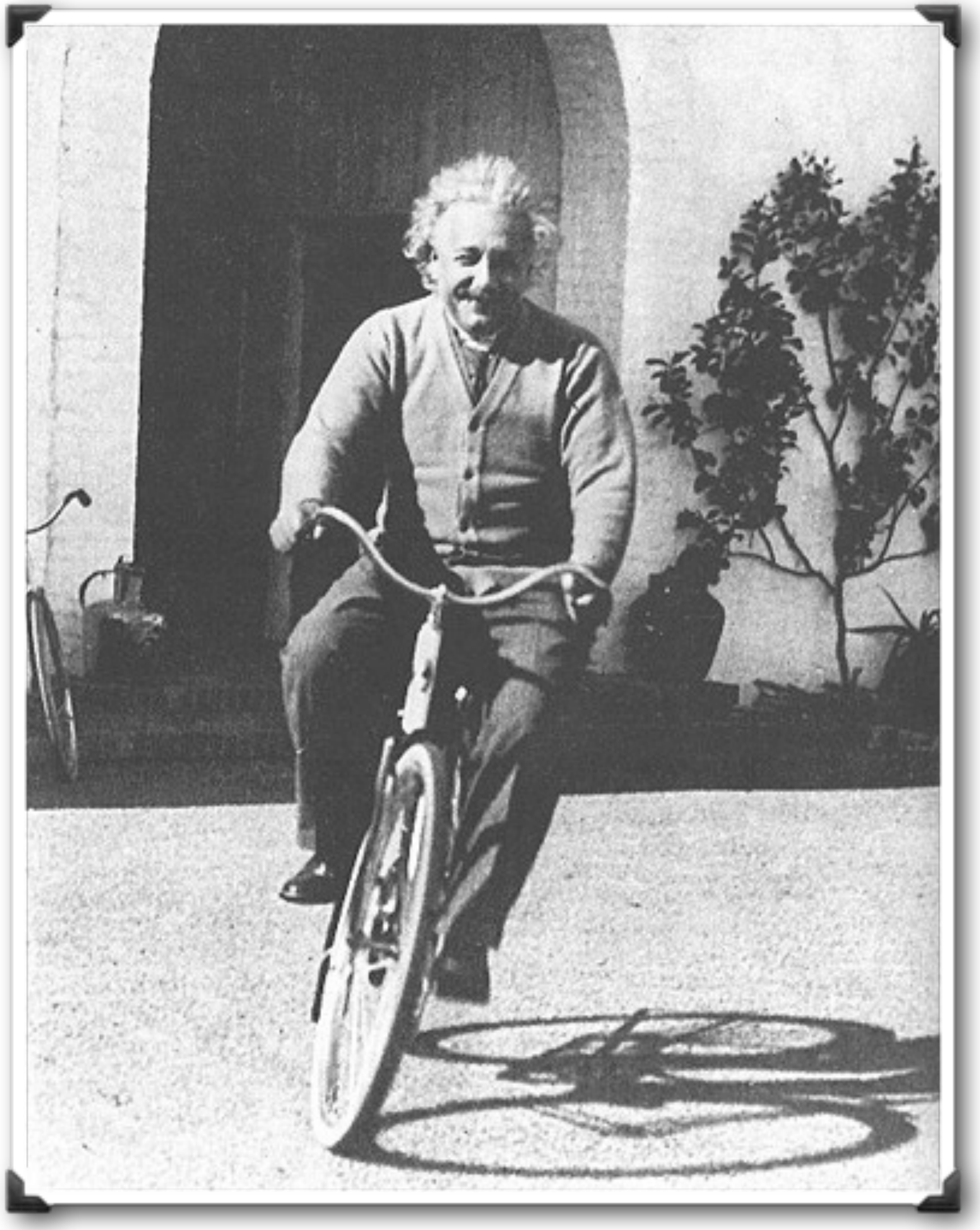
将核子紧紧
结合起来

Time scale: 10^{-23} s

爱因斯坦的统一之梦

- *Einstein dreamed to come up with a unified description*

- *But he failed to unify electromagnetism and gravity (GR)*

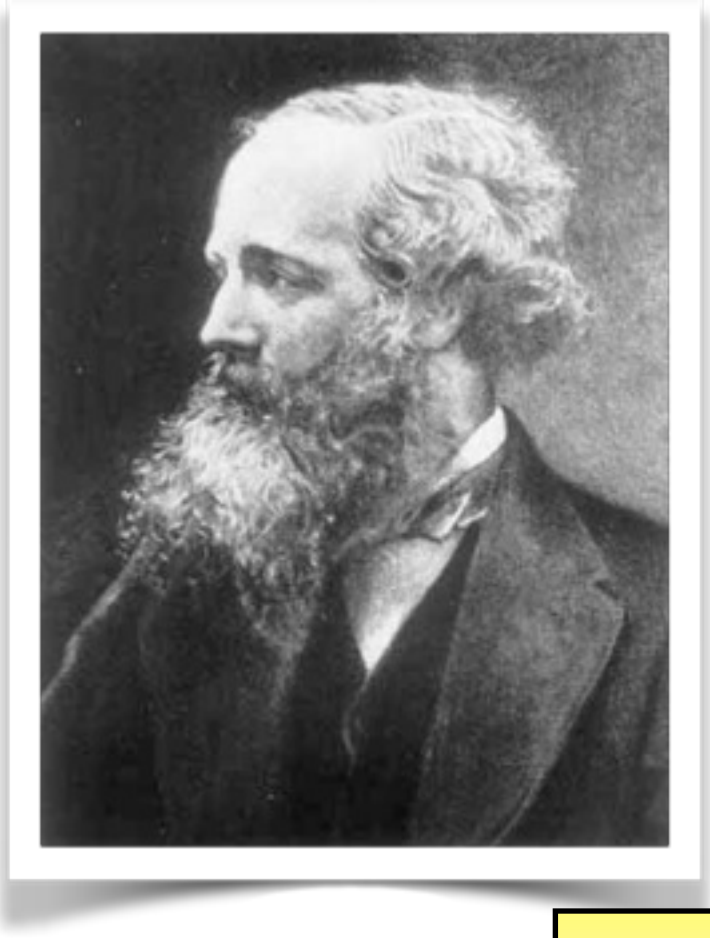


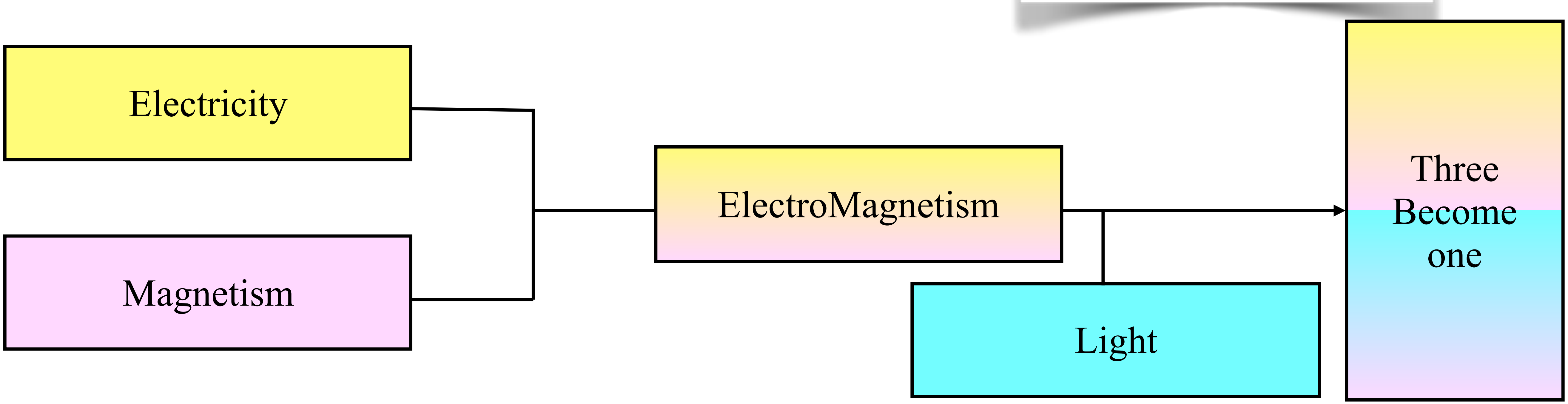
太阳、地球和苹果

Maxwell: Electromagnetism

$$\vec{\nabla} \times \vec{D} = \rho \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = 0 \qquad \vec{\nabla} \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$$





Electricity

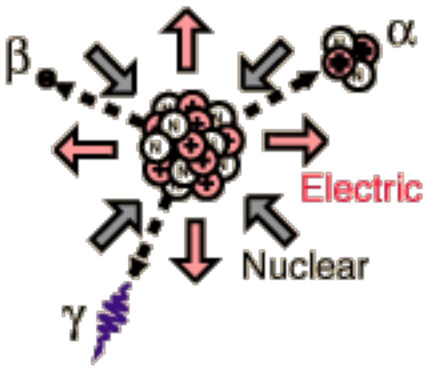
Magnetism

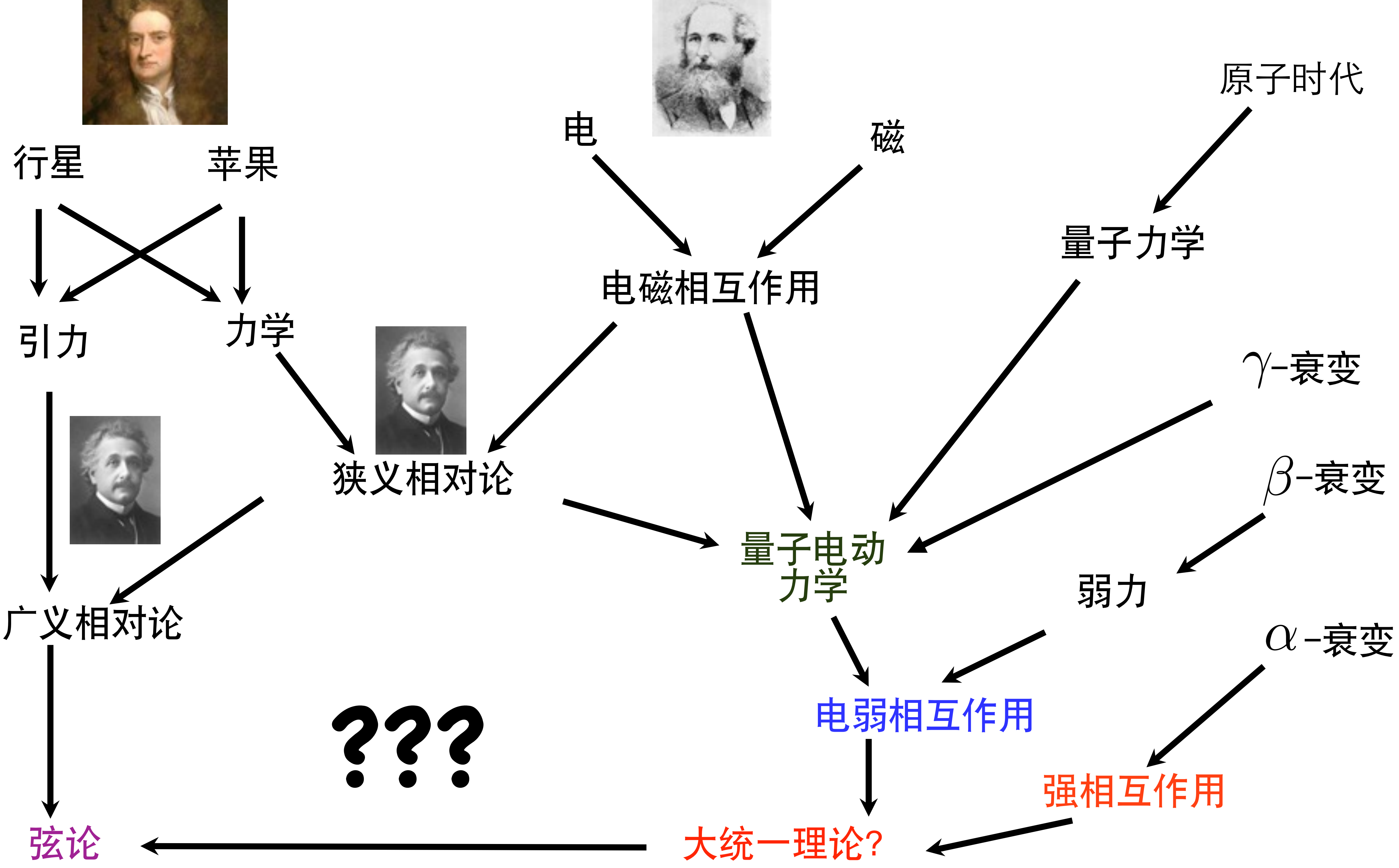
ElectroMagnetism

Light

Three
Become
one

Unification





行星

苹果

引力

力学

电

磁

电磁相互作用

原子时代

量子力学

狭义相对论

广义相对论

γ-衰变

β-衰变

量子电动力学

弱力

α-衰变

电弱相互作用

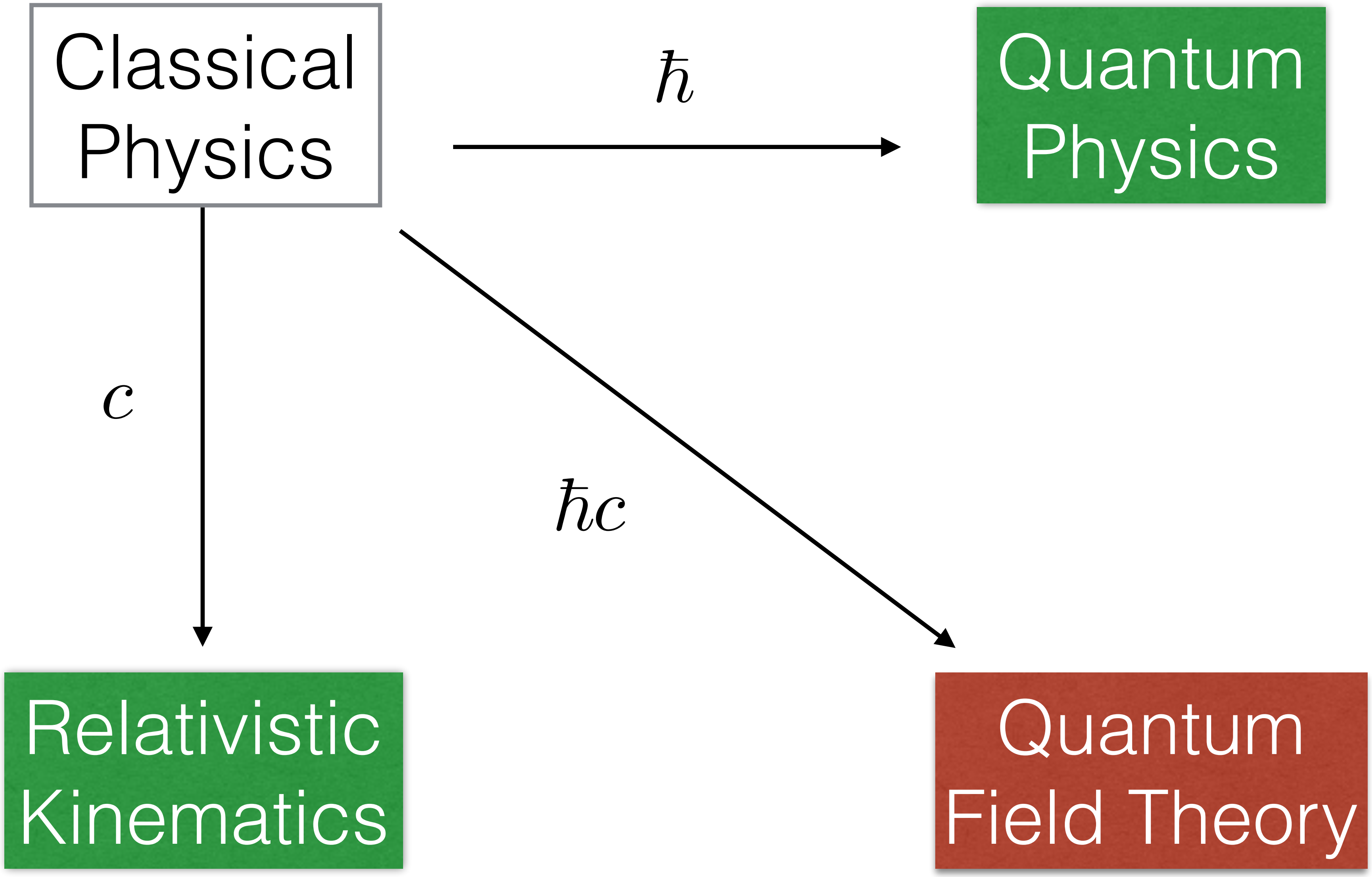
强相互作用

???

弦论

大统一理论?

The Magic of Constants



Classical Physics

$\hbar$

Quantum Physics

c

$\hbar c$

Relativistic Kinematics

Quantum Field Theory

自然单位制：微观世界语言

高能物理中大部分情形下，基本粒子间的
相互作用仅仅发生在极高能量和极短距离

$$\hbar = c = k_B = 1$$

[长度]=[时间]=[质量]$^{-1}$=[温度]$^{-1}$=[能量]$^{-1}$

$\hbar$	量子性质
c	相对论性质
k_B	热力学性质

需要仔细处理
微观世界的理论结论
推广到
宏观世界的观测量

The Magic of Units

$$[c] = m/s$$

$$[\hbar] = J \cdot s = \text{MeV} \cdot s = \frac{[\text{mass}]\,[\text{length}]}{[\text{time}]}$$

$$\hbar c = 197.3\text{MeV} \cdot \text{fb}$$

$$[e] = \text{Coulomb} = \sqrt{\frac{[\text{mass}][\text{length}]^3}{[time]^2}} \qquad \frac{e^2}{r} = ma$$

$$\alpha = \frac{e^2}{\hbar c} = \frac{1}{137.036}$$

$$\hbar = c = 1$$

$$c = 3 \times 10^8 m/s \Longrightarrow 1\mathrm{sec} = 3 \times 10^8 m$$

$$\hbar c = 197.3\mathrm{MeV} \cdot \mathrm{fm} \Longrightarrow 1\mathrm{fm} \sim \frac{1}{200\mathrm{MeV}}$$

$$[\mathrm{length}] = [\mathrm{time}] \sim \frac{1}{\mathrm{MeV}}$$

The History of Electroweak Theory

The Birth: Beta Decay

$$A \rightarrow B + e^-$$

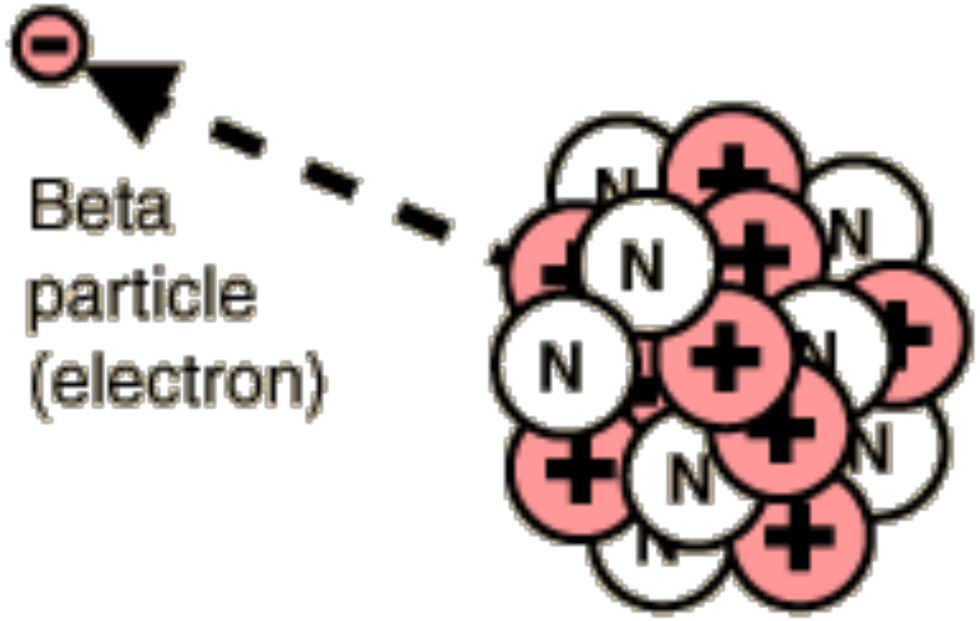
$$(Z, N) \rightarrow (Z+1, N-1) + e^-$$

Ⓝ $m_n = 939.5656 \;\; MeV$

⊕ $m_p = 938.2723 \;\; MeV$

⊖ $m_e = 0.510999 \;\; MeV$

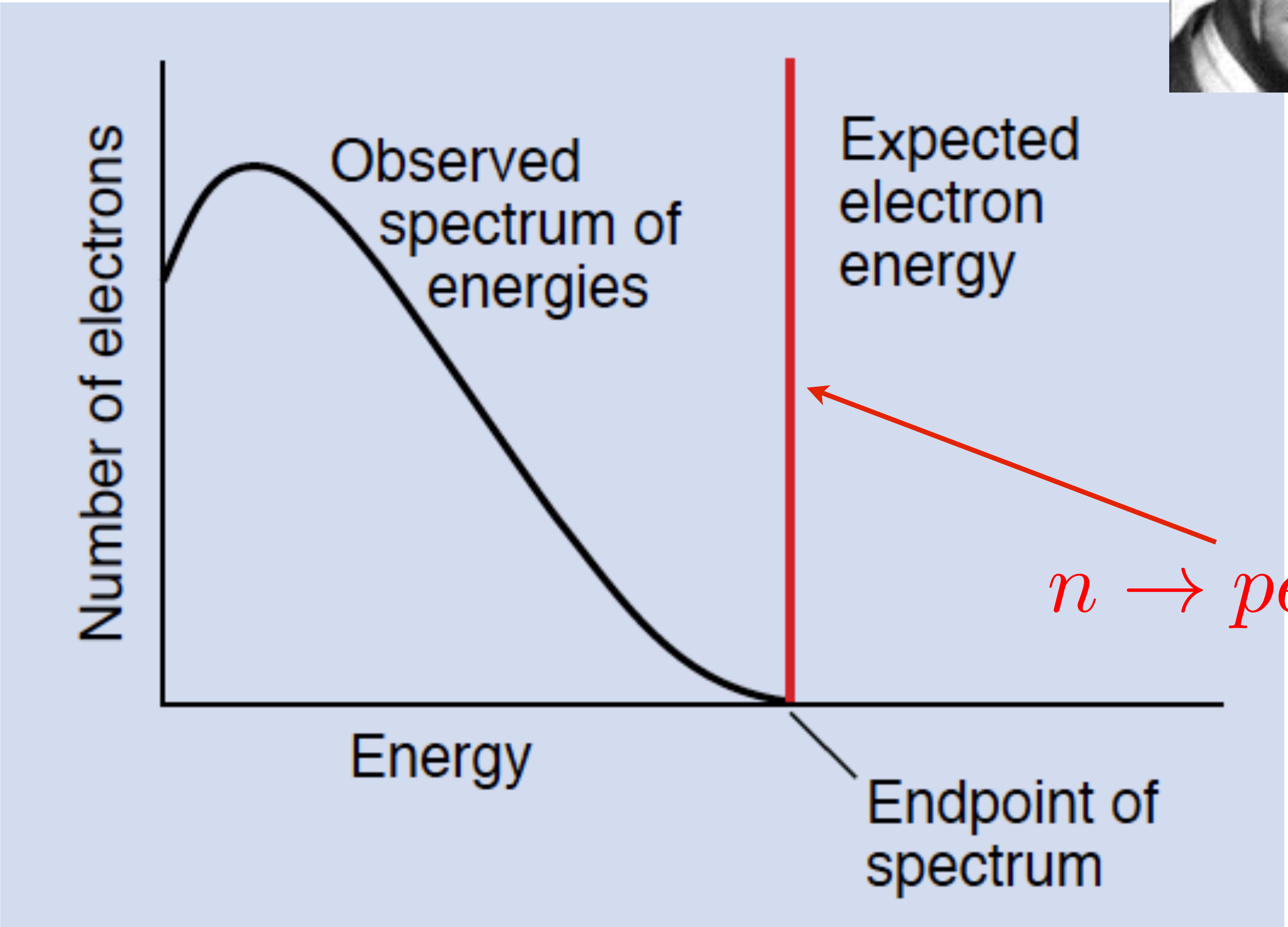
$0.7823 \;\; MeV = Q \;\; for \;\; n \rightarrow p + e^-$



The conservation of Energy and momentum requires the electron have a single value of energy.

Beta Decay

1914, Chadwick



$n \rightarrow pe^-$

What is Wrong?

Something to loose

or

Something to add

Neil Bohr

1929

- ready to abandon the law of conservation of energy
- propose a statistical version of the conservation laws of energy, momentum, angular momentum

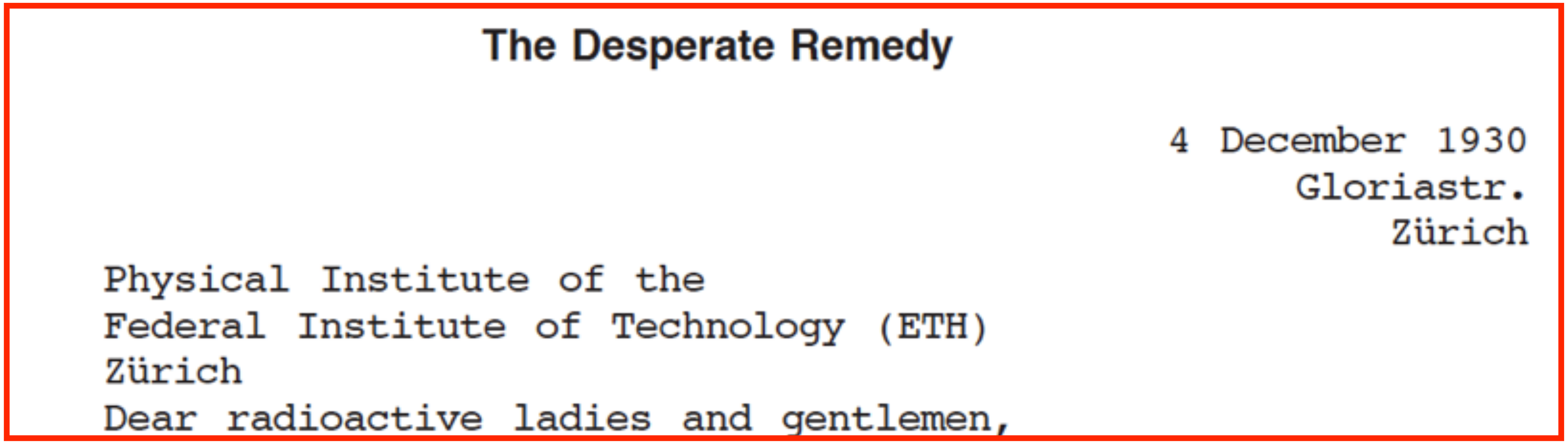
1924, Borh, Kramers, Slater, “辐射的量子理论”：能量和动量在单个微观相互作用过程中不必守恒，而只需要在统计意义上守恒。

1925年，康普顿电子-光子散射验证了微观散射过程中能动量守恒。

Neutrino

Wolfgang Pauli 1930

Letter to the physical Institute of the Federal Institute of Technology, Zurich

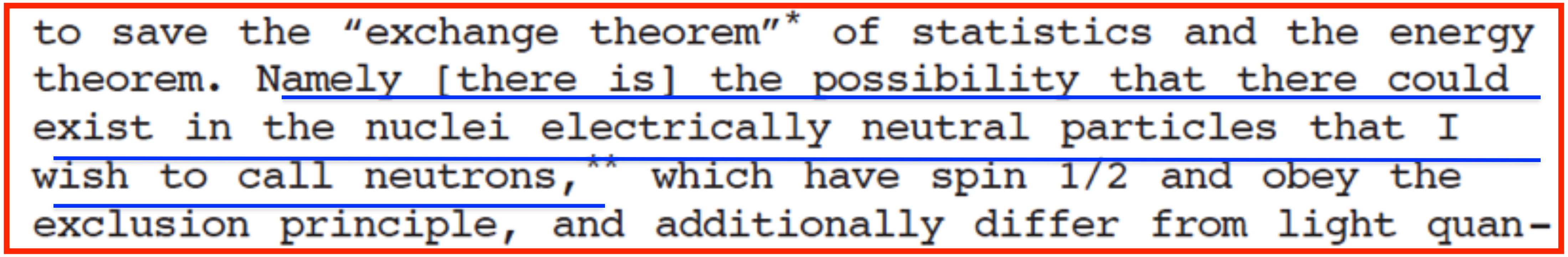


The Desperate Remedy

4 December 1930
Gloriastr.
Zürich

Physical Institute of the
Federal Institute of Technology (ETH)
Zürich

Dear radioactive ladies and gentlemen,



to save the “exchange theorem”[*] of statistics and the energy theorem. Namely [there is] the possibility that there could exist in the nuclei electrically neutral particles that I wish to call neutrons,[**] which have spin 1/2 and obey the exclusion principle, and additionally differ from light quan-

Neutrino

In 1932 Chadwick discovered a neutral nuclear constituent. By studying the properties of the neutral radiation n emitted in the process

$$^{9}\mathrm{Be} + \alpha \rightarrow {}^{12}\mathrm{C} + n$$

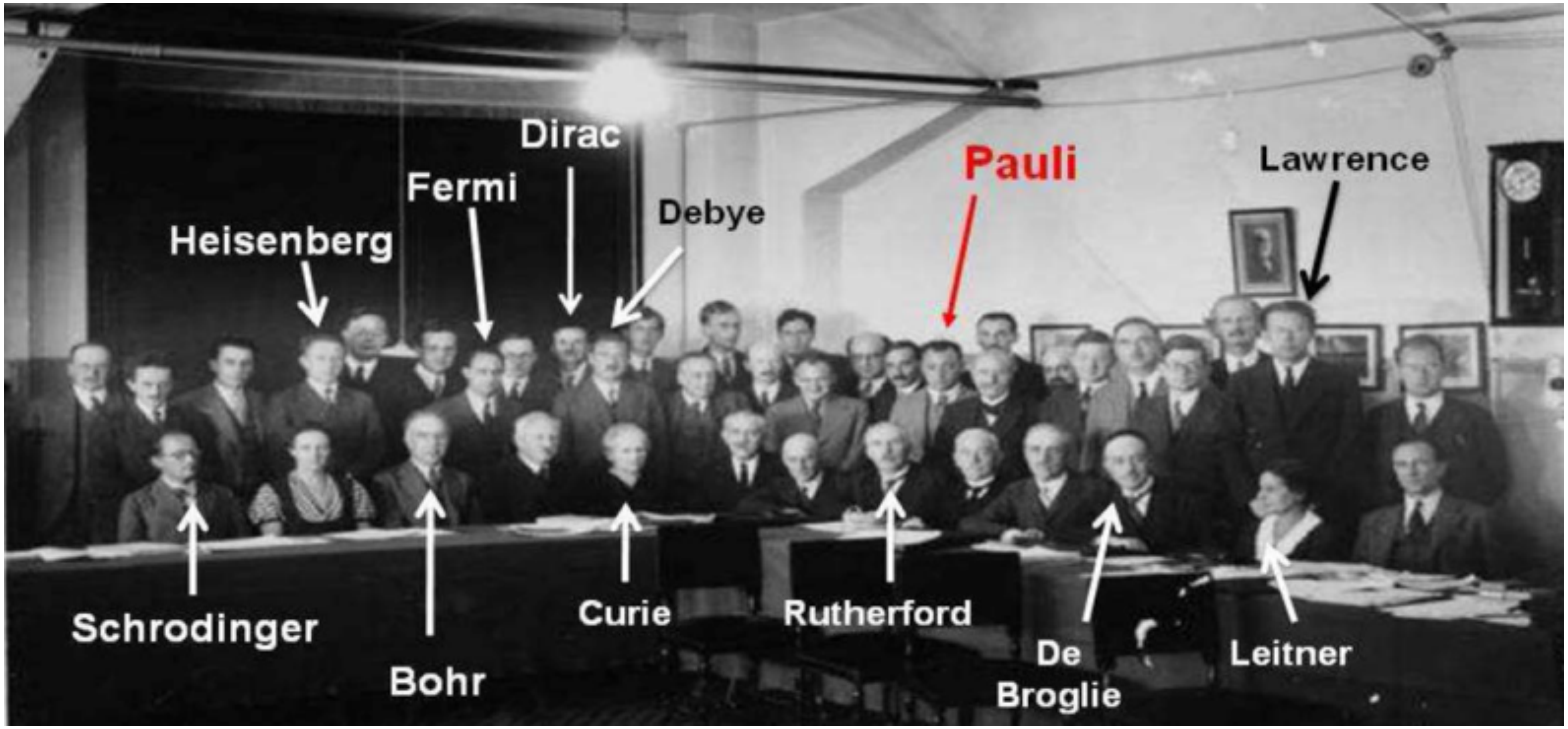
He found out that n was a deeply penetrating neutral particle slightly heavier than the proton, quite distinct from gamma-rays.

Pauli 说的“neutron”被Fermi改成“little neutral one”，成为今天常说的 “Neutrino”

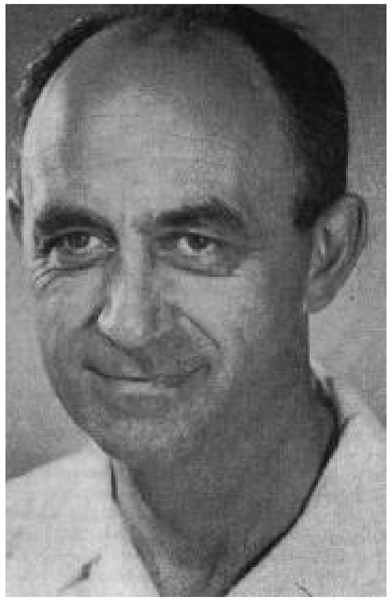
Neutrino

Solvay 1933 Physics Conference (Brussels, Belgium)

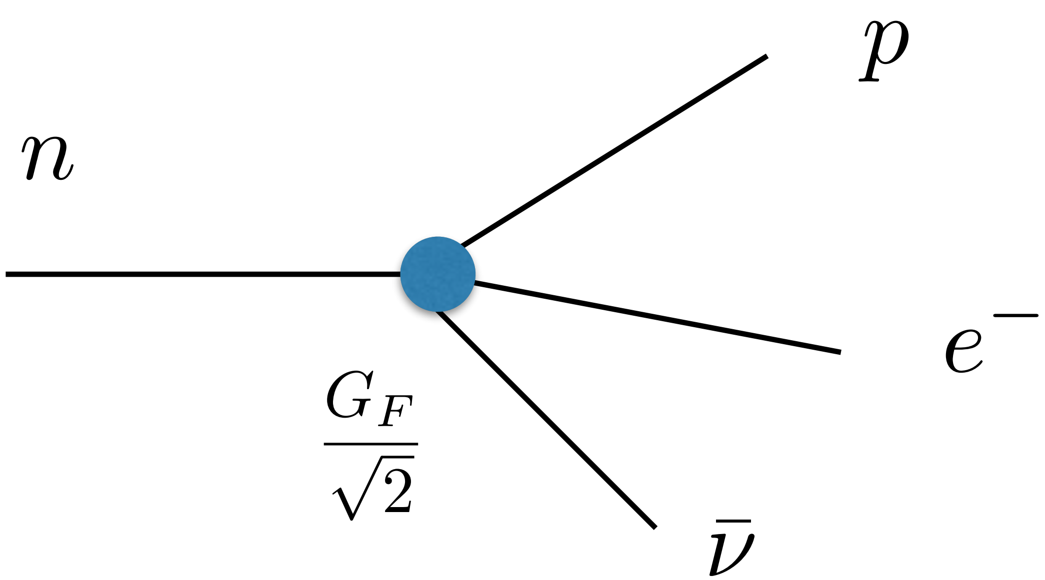
Pauli 报告了他的中微子设想



Fermi Theory



1934



$$M_{fi} = G_F\,\left[\overline{\psi_n}\gamma^\mu\psi_p\right]\,\left[\overline{\psi_e}\gamma^\mu\psi_\nu\right]$$

$$G_F \sim 10^{-5}\,(\mathrm{GeV})^{-2}$$

Loosely like QED, but zero range and non-diagonal

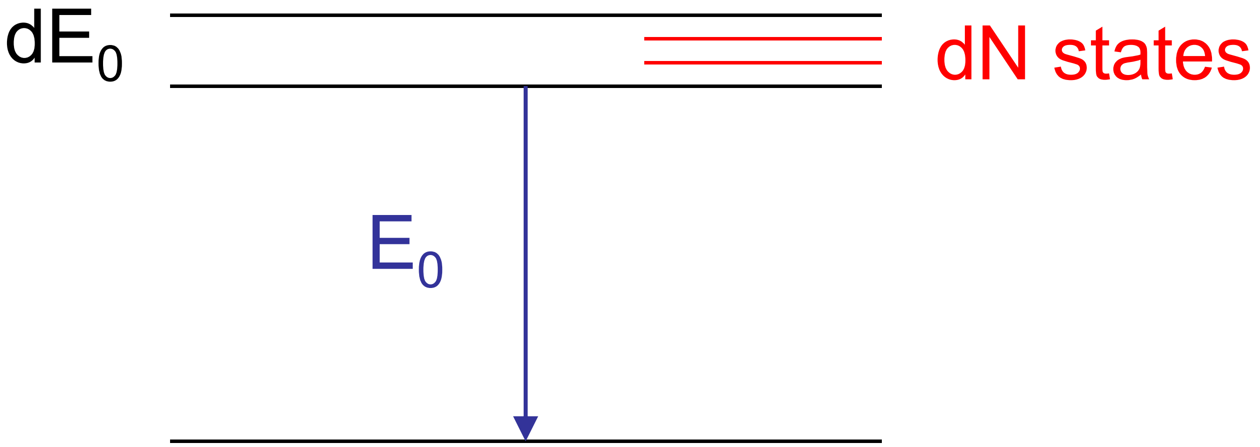
The interaction behind beta decay remains unknown in Fermi’s time.

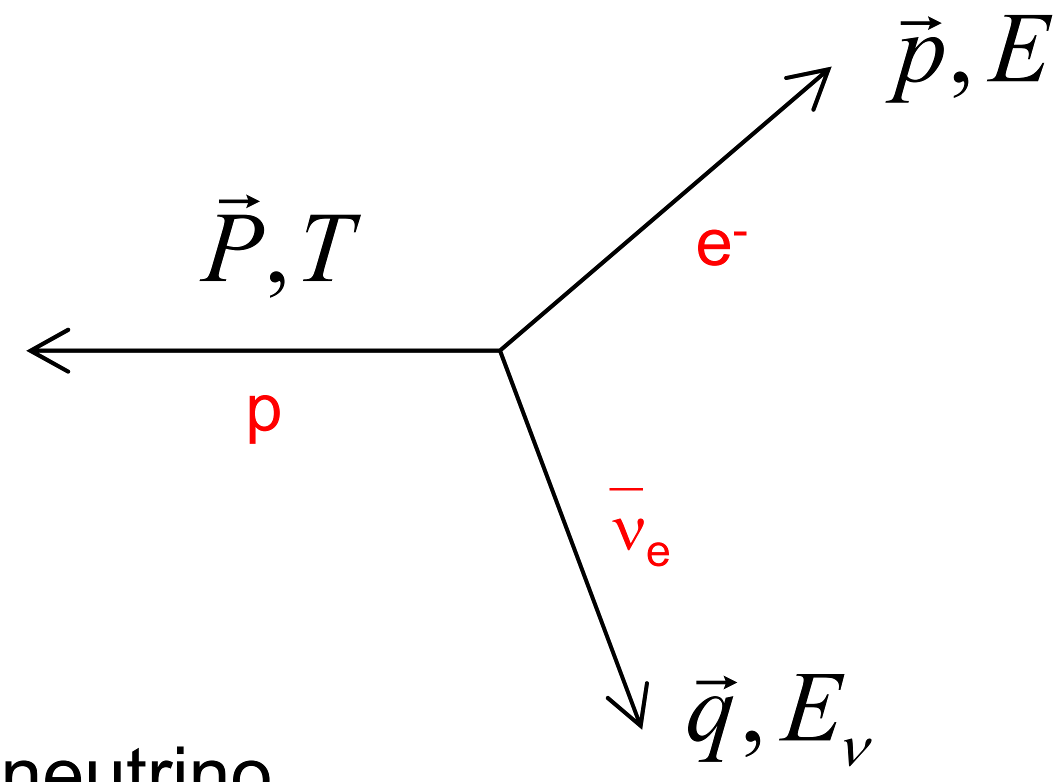
It took some 20 years of work to figure out a detailed model fitting the observation.

In Fermi theory the transition probability per unit time is given by:

$$W = \frac{2\pi}{\hbar} G^2 |M|^2 \frac{dn}{dE_0}$$

if J(leptons) = 0	$|M|^2 = 1$	Fermi transition
if J(leptons) = 1	$|M|^2 = 3$	Gamow-Teller transition





T, E, E_ν kinetic energies of proton, electron, antineutrino

Energy and momentum conservation

$$\vec{P} + \vec{p} + \vec{q} = 0$$

$$T + E_\nu + E = E_0$$

$$E_0 = m_n - m_p - m_e \approx 0.8\ MeV$$

Parity (reflection) Violation

Parity conservation had been assumed, almost without question

$\theta - \tau$ puzzle (1950's)

$$\theta^{+} \rightarrow \pi^{+}\pi^{0} \quad P = +1$$

$$\tau^{+} \rightarrow \pi^{+}\pi^{+}\pi^{-},\ \pi^{+}\pi^{0}\pi^{0}$$
$$P = -1$$

Two particles with same mass, charge, spin, lifetime, but different decay modes and parity

Lee, Yang (1956)

Need a pseudo-scalar to measure the parity violation effects.

$$\vec{\sigma} \cdot \vec{p}$$

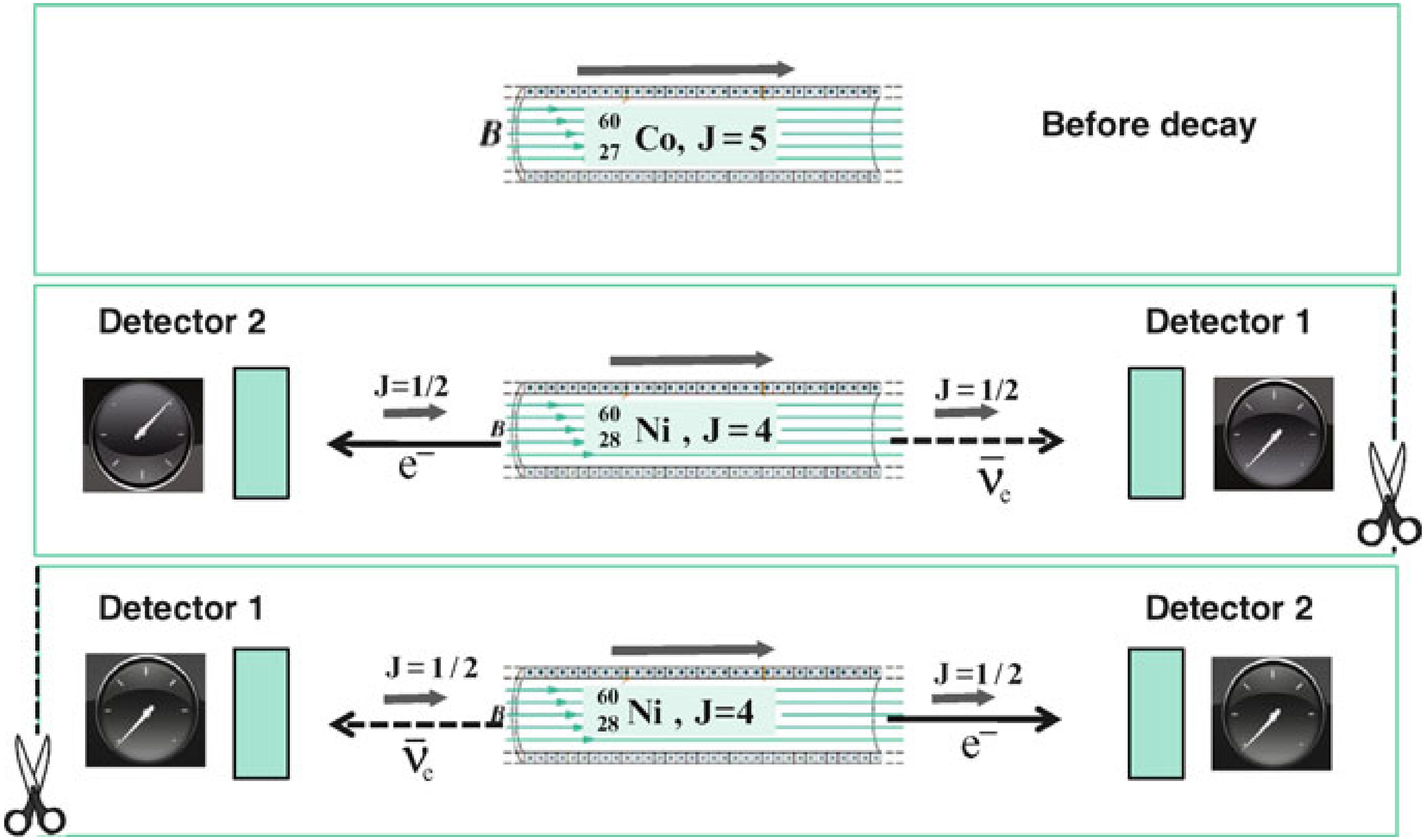
Parity Violation in Decay of Polarized Nuclei

$$^{60}_{27}Co(J = 5^+) \rightarrow^{60}_{28} Ni^*(J = 4^+)\, e^- \overline{\nu}_e$$

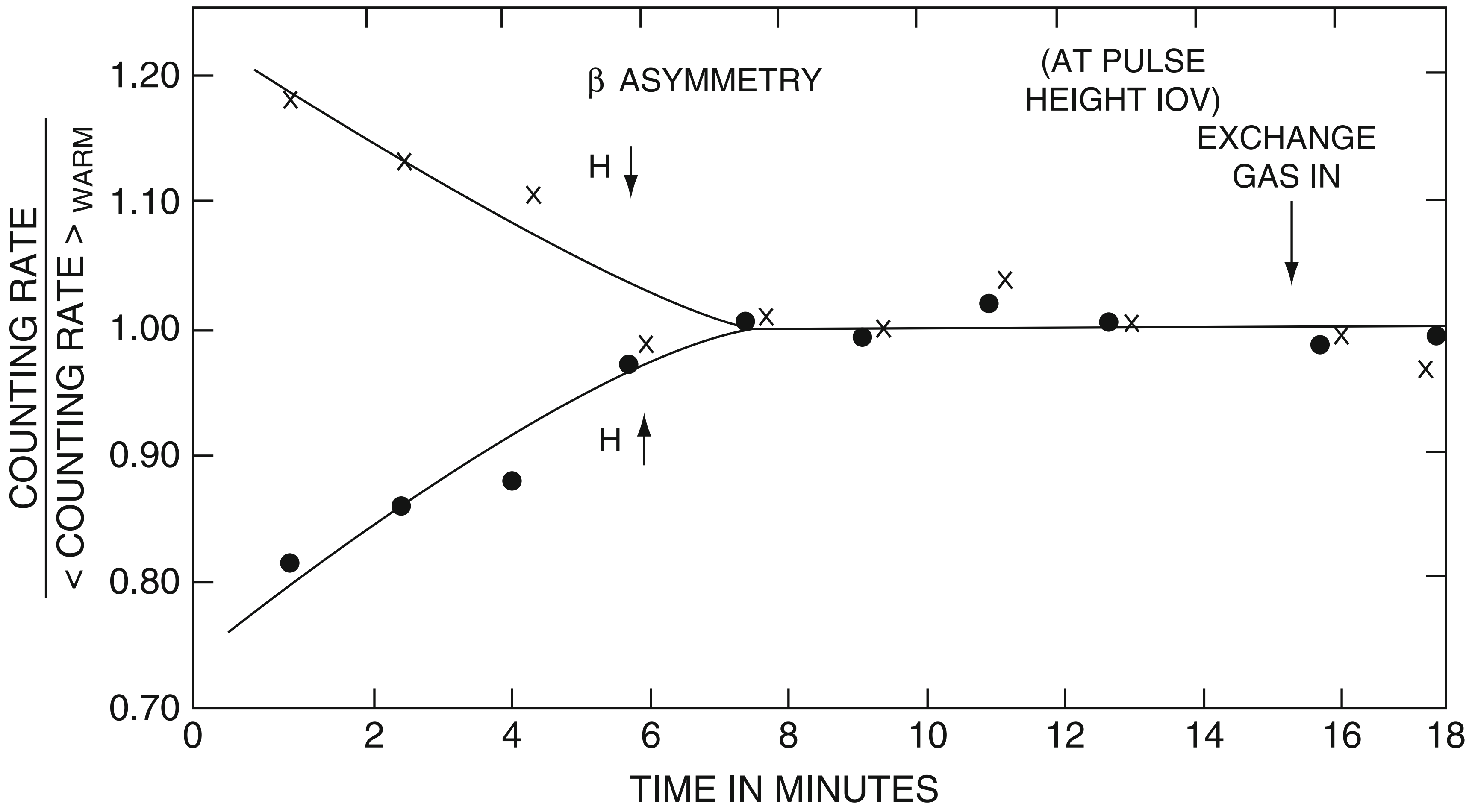
$$\boxed{\vec{\sigma}_{\rm Co} \cdot \vec{p}_e}$$

Gamow-Teller transition

Wu, et al
PRL 105, 1414
(1957)



Parity Violation in Beta Decay



T~0.01k

$$I(\theta) = 1 + \frac{\alpha \boldsymbol{\sigma}_{\mathrm{Co}} \cdot \mathbf{p}_e}{E_e} = 1 + \alpha \frac{v_e}{c} \cos\theta$$

$$\Lambda = \frac{I_+ - I_-}{I_+ + I_-} = \alpha \frac{v}{c}$$

$$e^- : \Lambda = -v_e/c$$

$$e^+ : \Lambda = +v_e/c$$

left-handed

right-handed

Two-Component Neutrino Theory

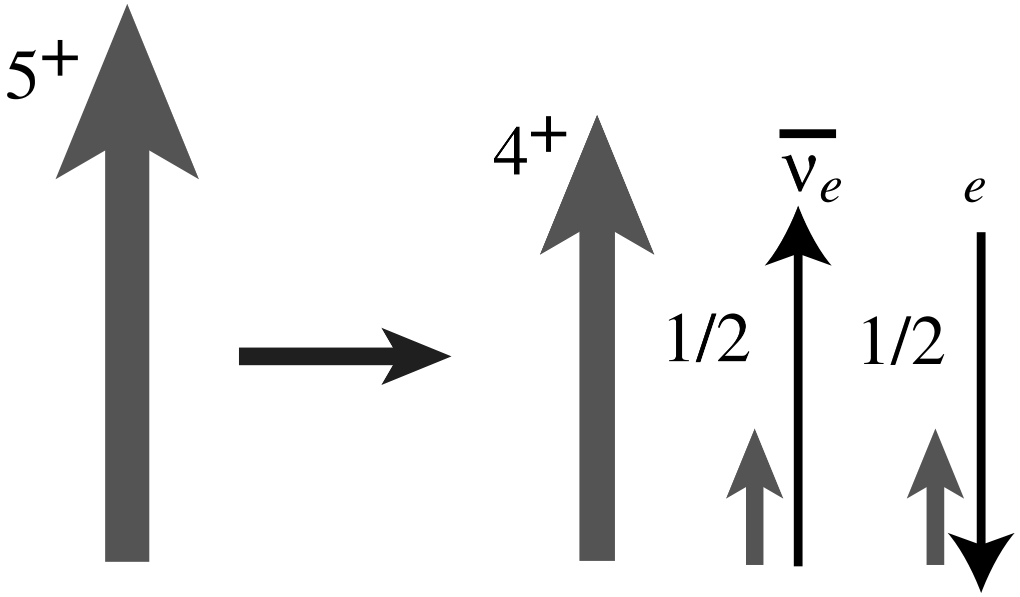
Goldhaber et al (1958)

Neutrino: Left-handed; Anti-neutrino: Right-handed

$\mathbf{p}_\nu \uparrow\Downarrow \sigma_\nu$ $\mathbf{p}_{\overline{\nu}} \uparrow\Uparrow \sigma_{\overline{\nu}}$

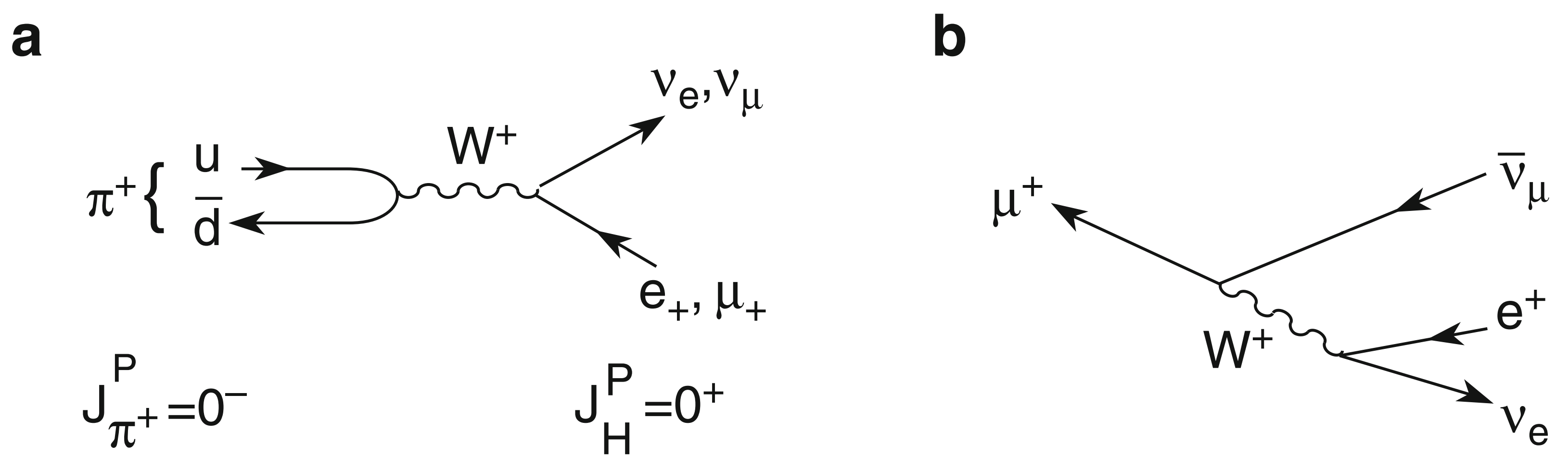
particle	ν_e	$\overline{\nu}_e$	e^-	e^+
helicity probability	-1	$+1$	$-v/c$	$+v/c$

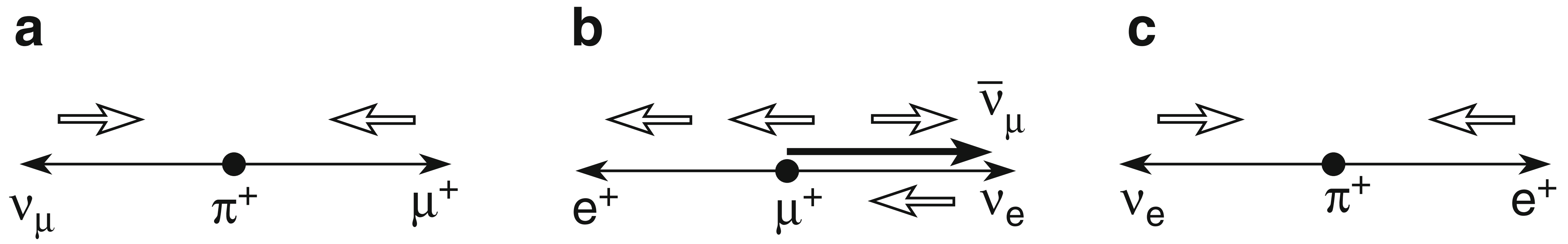
$$^{60}\mathrm{Co}\left(J^P=5^+\right)\Uparrow\rightarrow {}^{60}\mathrm{Ni}^{**}\left(J^P=4^+\right)\Uparrow + e^- + \bar{\nu}_e$$



Charged Pion Decay

Garwin, Lederman, Weinrich, PRL 1415 (1957)





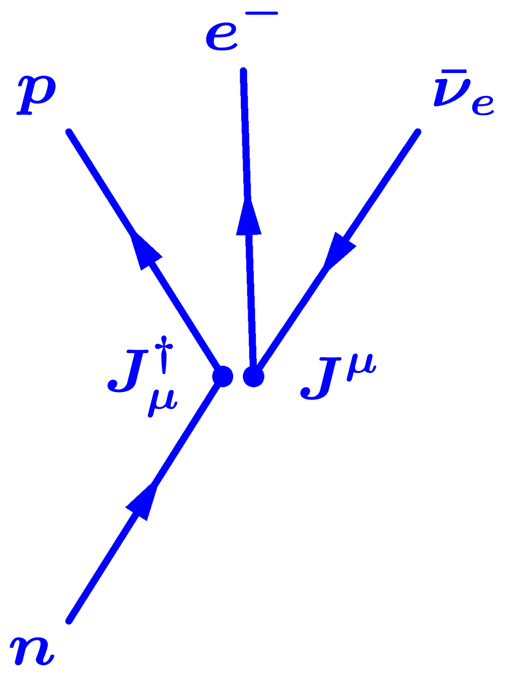
99.9%

10^{-4}

V-A Theory

(maximal violation of parity and charge conjugation)

Feynman & Gell-man; Sudarshan, Marshak (1958)



$$\mathcal{H} = \frac{G_F}{\sqrt{2}} J^\dagger_\mu J^\mu \qquad J_\mu = \underbrace{J^\ell_\mu}_{\text{leptonic}} + \underbrace{J^h_\mu}_{\text{hadronic}}$$

$$J^\ell_\mu = \bar{e}^- \gamma_\mu \left(1 - \gamma^5\right) \nu_e + \bar{\mu} \gamma_\mu \left(1 - \gamma^5\right) \nu_\mu$$
$$= 2 \left[\bar{e}_L \gamma_\mu \nu_{eL} + \bar{\mu}_L \gamma_\mu \nu_{\mu L}\right]$$

- $\psi_L = P_L \psi = \frac{1-\gamma^5}{2} \psi$ **(vector - axial)**
- $G_F \simeq 1.17 \times 10^{-5}\ \mathrm{GeV}^{-2}$ **(Fermi constant)**
- **Can extend third family, neutrino masses**
- **Hadronic current**

$$J^{h\dagger}_\mu \sim \bar{p} \gamma_\mu \left(1 - \gamma^5\right) n \cos \theta_c + \text{pion, strangeness, etc}$$

P, C and CP

- **$V - A \Rightarrow$ maximal violation of P, C**
  - **WCC acts of e^-_L and e^+_R** **(not on e^-_R or e^+_L)**
  - **$\psi_{L,R} \equiv \frac{1 \mp \gamma^5}{2}\psi$: spin opposite (along) momentum** **(helicity $= \mp\frac{1}{2}$)**

- **Under space reflection (P):**

$$
\begin{aligned}
J^\ell_\mu &\rightarrow \bar{e}^- \gamma^\mu \left(1 + \gamma^5\right) \nu_e + \bar{\mu} \gamma^\mu \left(1 + \gamma^5\right) \nu_\mu \\
&= 2 \left[\bar{e}_R \gamma^\mu \nu_{eR} + \bar{\mu}_R \gamma^\mu \nu_{\mu R}\right]
\end{aligned}
$$

  - **i.e., $J^\ell_{\mu L}(t, \vec{x}\,) \rightarrow J^{\ell\mu}_R(t, -\vec{x}\,)$**
  - **P violated maximally**

Homework-1

P, C and CP

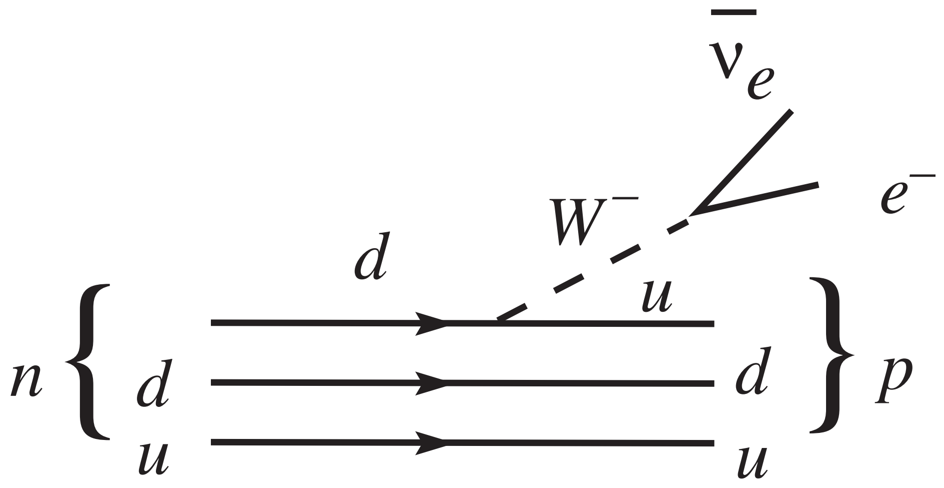
- **Under charge conjugation (C):**

$$J_\mu^\ell \to -\bar{\nu}_e \gamma_\mu \left(1+\gamma^5\right) e^- - \bar{\nu}_\mu \gamma_\mu \left(1+\gamma^5\right) \mu^-$$

  - **i.e., $J_{\mu L}^\ell \to -J_{\mu R}^{\ell\dagger}$**
  - **C violated maximally**
  - **However, $H = \int d^3\vec{x}\,\mathcal{H}$ invariant under CP**

Homework-2

‘V-A’ Theory: SM Picture



$$\mathcal{H} = \frac{G_F}{\sqrt{2}} J_\mu^\dagger J^\mu \qquad J_\mu = \underbrace{J_\mu^\ell}_{\text{leptonic}} + \underbrace{J_\mu^h}_{\text{hadronic}}$$

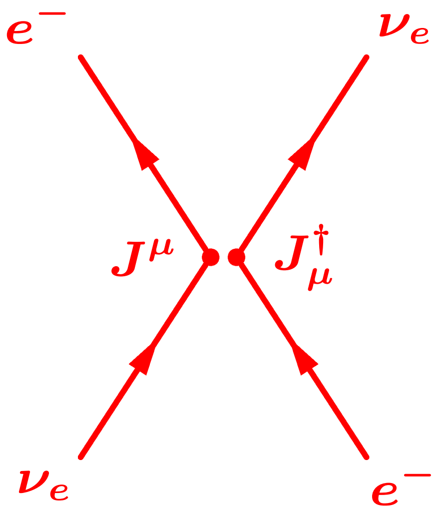
- **Leptonic current**

$$J_\mu^\ell = \bar{e}^- \gamma_\mu \left(1 - \gamma^5\right) \nu_e + \bar{\mu} \gamma_\mu \left(1 - \gamma^5\right) \nu_\mu$$

- **Quark form** $(p \sim uud, \ n \sim udd)$

$$J_\mu^{h\dagger} = \bar{u} \gamma_\mu \left(1 - \gamma^5\right) d' = 2 \bar{u}_L \gamma_\mu d'_L$$

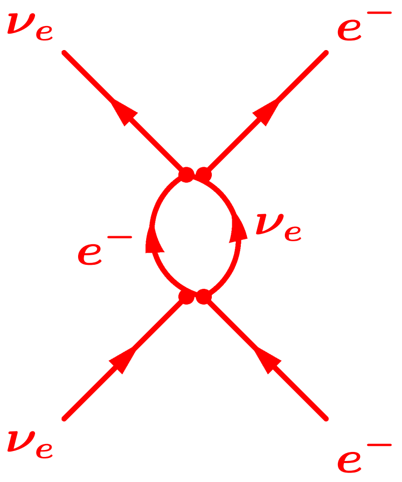
Fermi Theory Violates Unitarity at High Energy



- $\sigma(\nu_e e^- \longrightarrow e^- \nu_e) \longrightarrow \frac{G_F^2 s}{\pi}$
  $(s \equiv E_{CM}^2)$
- **pure S-wave unitarity:** $\sigma < \frac{16\pi}{s}$
- **fails for** $\frac{E_{CM}}{2} \geq \sqrt{\frac{\pi}{G_F}} \sim$ **500 GeV**

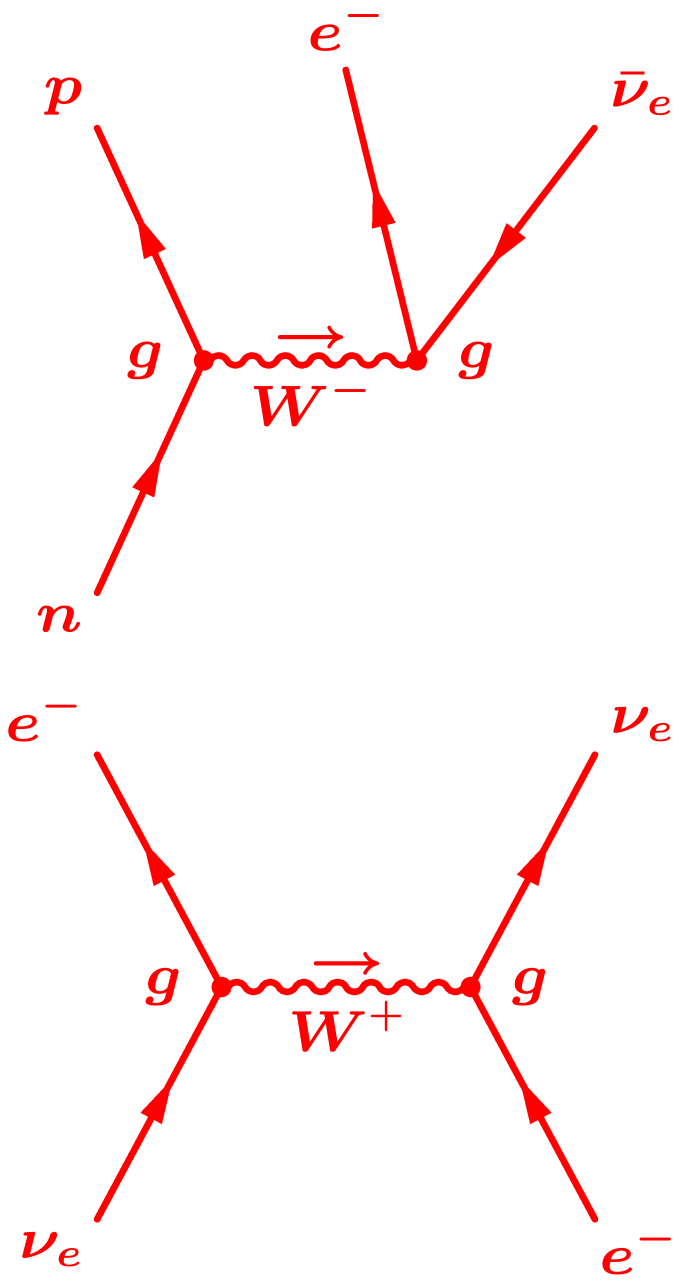
Fermi theory: divergent integrals

$$\int d^4k \left(\frac{\not{k}}{k^2}\right)\left(\frac{\not{k}}{k^2}\right)$$



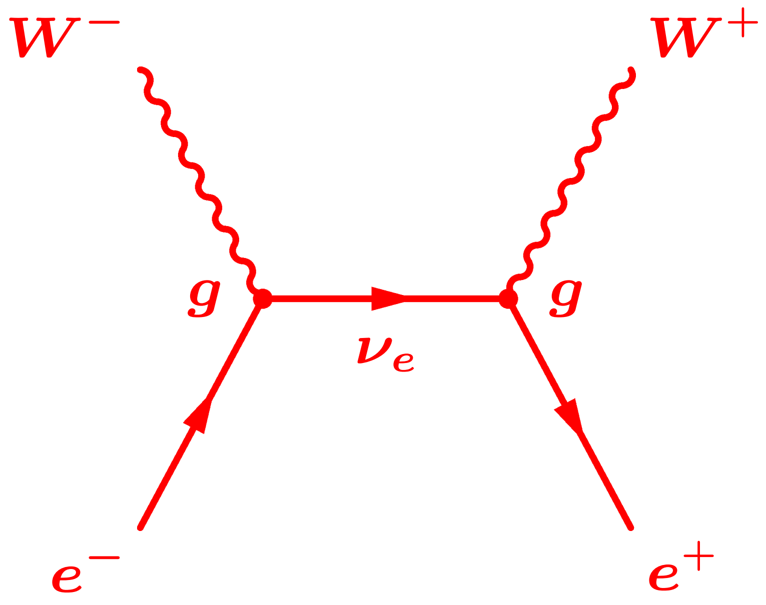
Intermediate Vector Boson Theory

Yukawa (1935); Schwinger (1957)



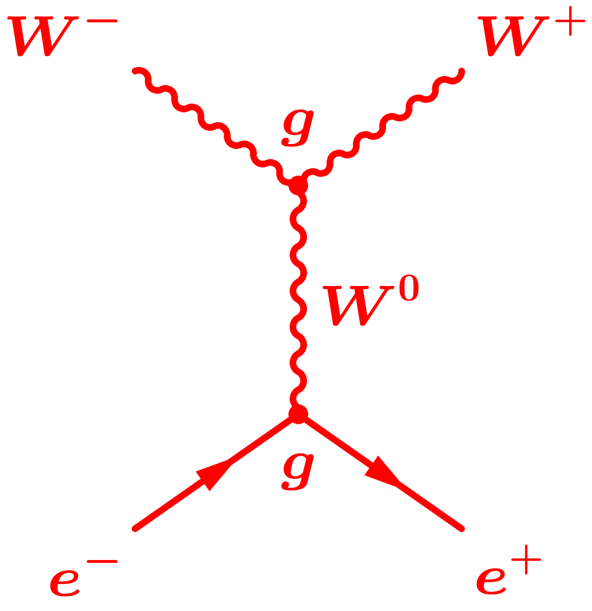
$$\frac{G_F}{\sqrt{2}} \sim \frac{g^2}{8M_W^2} \text{ for } M_W \gg Q$$

- **no longer pure S-wave** $\Rightarrow$
- $\nu_e e^- \longrightarrow \nu_e e^-$ **better behaved**



- $but, \quad e^+e^- \quad \rightarrow \quad W^+W^-$ violates unitarity for $\sqrt{s} \gtrsim 500$ GeV
- $\epsilon_\mu \quad \sim \quad k_\mu/M_W$ for longitudinal polarization (non-renormalizable)

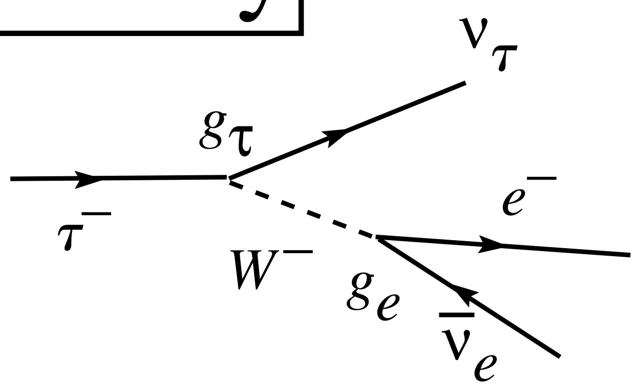
- introduce W^0 to cancel
- fixes $W^0W^+W^-$ and $e^+e^-W^0$ vertices
- requires $\left[J, J^\dagger\right] \sim J^0$ (like $SU(2)$)
- not realistic

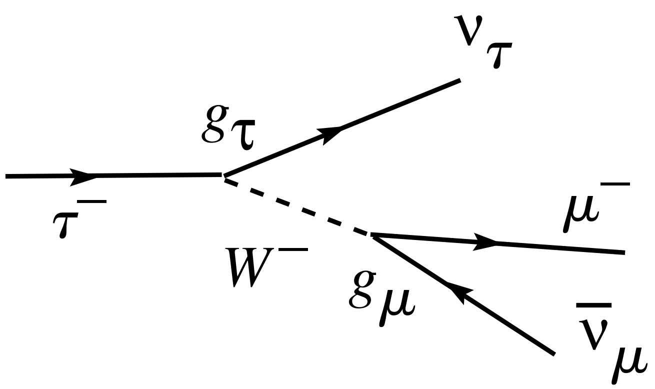


Glashow model (1961) (W,Z,gamma, but no mass term)

Lepton universality

e-μ universality





$$\Gamma\left(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau\right) \propto \frac{g_\tau^2}{M_W^2}\frac{g_\mu^2}{M_W^2} m_\tau^5 \qquad \Gamma\left(\tau^- \to e^- \bar{\nu}_e \nu_\tau\right) \propto \frac{g_\tau^2}{M_W^2}\frac{g_e^2}{M_W^2} m_\tau^5$$

$$\frac{\Gamma\left(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau\right)}{\Gamma\left(\tau^- \to e^- \bar{\nu}_e \nu_\tau\right)} = \frac{\mathrm{BR}\left(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau\right)}{\mathrm{BR}\left(\tau^- \to e^- \bar{\nu}_e \nu_\tau\right)} = \frac{g_\mu^2}{g_e^2}\frac{\rho_\mu}{\rho_e}$$

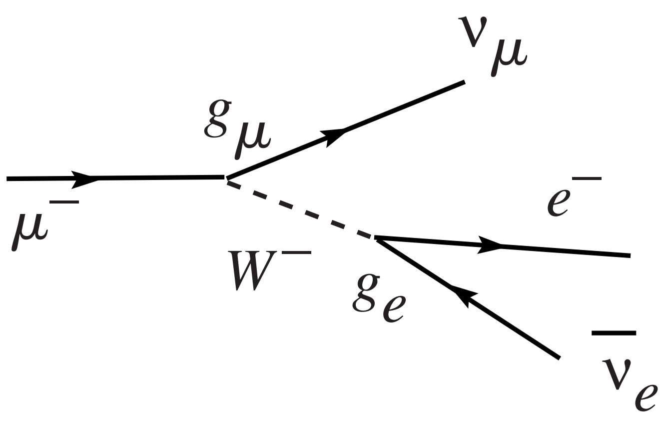
Phase space factor

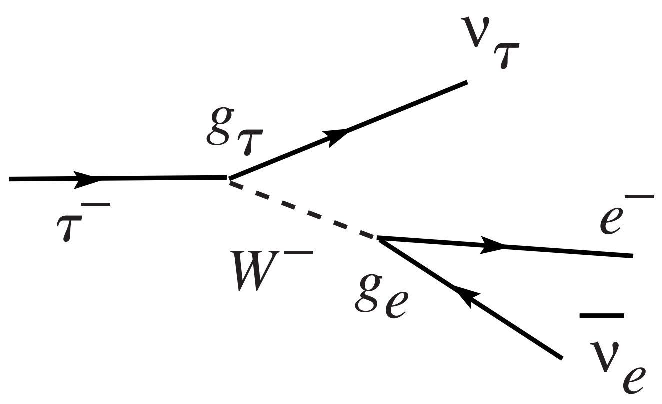
$$\frac{\mathrm{BR}\left(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau\right)}{\mathrm{BR}\left(\tau^- \to e^- \bar{\nu}_e \nu_\tau\right)} = \frac{(17.36 \pm 0.05)\%}{(17.84 \pm 0.05)\%} = 0.974 \pm 0.004$$

$$g_\mu / g_e = 1.001 \pm 0.002$$

Lepton universality

μ-τ universality





$$\frac{\Gamma(\mu^- \to e^-\bar{\nu}_e\nu_\mu)}{\Gamma(\tau^- \to e^-\bar{\nu}_e\nu_\tau)} = \frac{1}{\tau_\mu}\frac{\tau_\tau}{\mathrm{BR}(\tau^- \to e^-\bar{\nu}_e\nu_\tau)}$$

$$\frac{\Gamma(\mu^- \to e^-\bar{\nu}_e\nu_\mu)}{\Gamma(\tau^- \to e^-\bar{\nu}_e\nu_\tau)} = \frac{g_e^2 g_\mu^2}{g_e^2 g_\tau^2}\frac{m_\mu^5}{m_\tau^5}\frac{\rho_\mu}{\rho_\tau} = \frac{g_\mu^2}{g_\tau^2}\frac{m_\mu^5}{m_\tau^5}\frac{\rho_\mu}{\rho_\tau}$$

$$\Rightarrow \quad \frac{g_\mu^2}{g_\tau^2} = \frac{1}{\tau_\mu}\frac{\tau_\tau}{\mathrm{BR}(\tau^- \to e^-\bar{\nu}_e\nu_\tau)}\frac{m_\tau^5}{m_\mu^5}\frac{\rho_\tau}{\rho_\mu}$$

$$\Rightarrow \quad g_\mu/g_\tau = 1.001 \pm 0.003$$

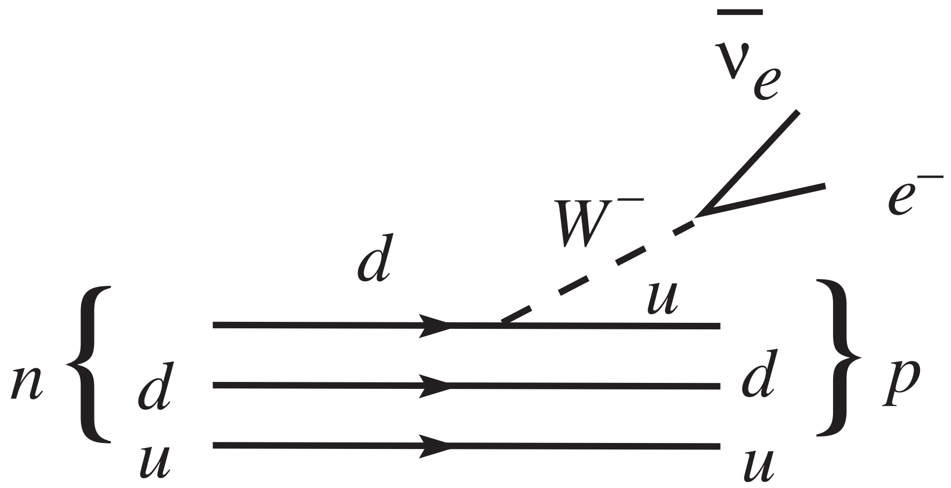
Quark Mixing

Charged current is universal in lepton sector
but not in quark sector

Universality requires

$$M \propto G_{\mathrm{F}} \cdot \bar{\nu}_{eL}\gamma_{\alpha}e_{eL} \cdot \bar{d}_{L}\gamma^{\alpha}u_{L} \qquad M \propto G_{\mathrm{F}} \cdot \bar{\nu}_{eL}\gamma_{\alpha}e_{eL} \cdot \bar{s}_{L}\gamma^{\alpha}u_{L}$$

BUT



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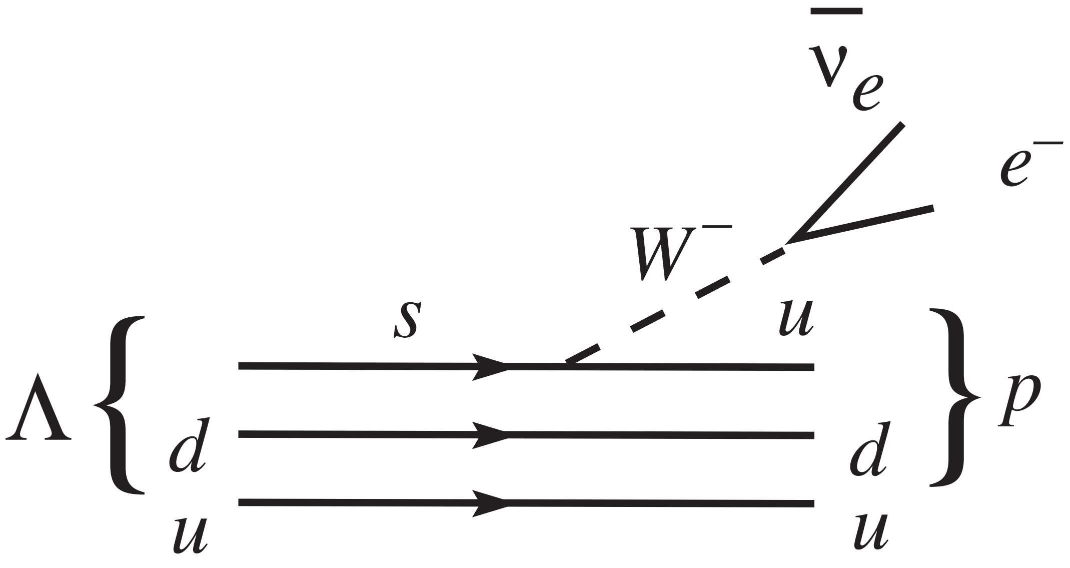
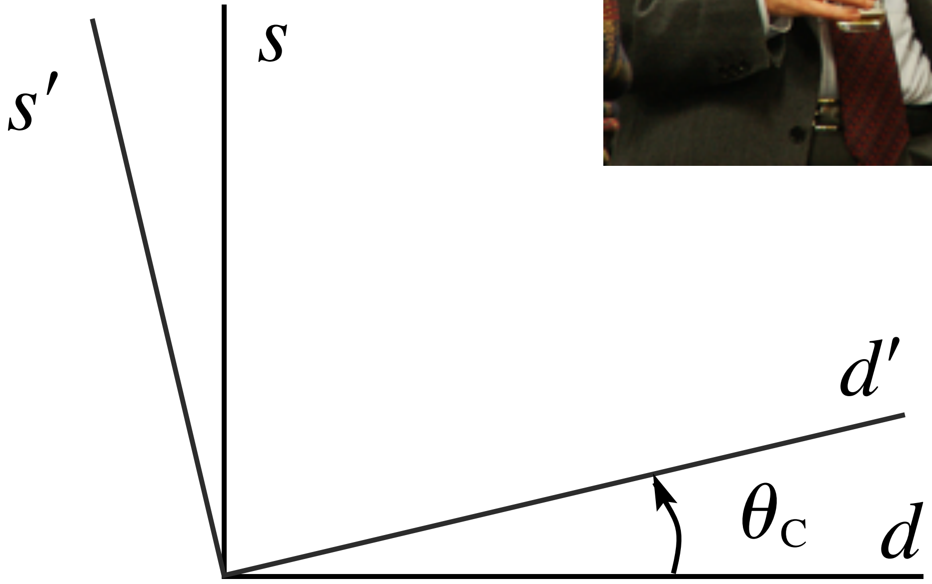


$\Delta S = 0$ $\qquad$ $\Delta S = 1$

Cabibbo Mixing

1963

Introducing a mixing



$$d' = d\cos\theta_C + s\sin\theta_C$$

$$M \propto G_F \cdot \bar{e}_L\gamma_a\nu_{eL} \cdot \bar{d}'_L\gamma^a u_L$$

$$M \propto G_F\cos\theta_C \cdot \bar{e}_L\gamma_a\nu_{eL} \cdot \bar{d}_L\gamma^a u_L \quad \text{for } \Delta S = 0$$

$$M \propto G_F\sin\theta_C \cdot \bar{e}_L\gamma_a\nu_{eL} \cdot \bar{s}_L\gamma^a u_L \quad \text{for } \Delta S = 1.$$

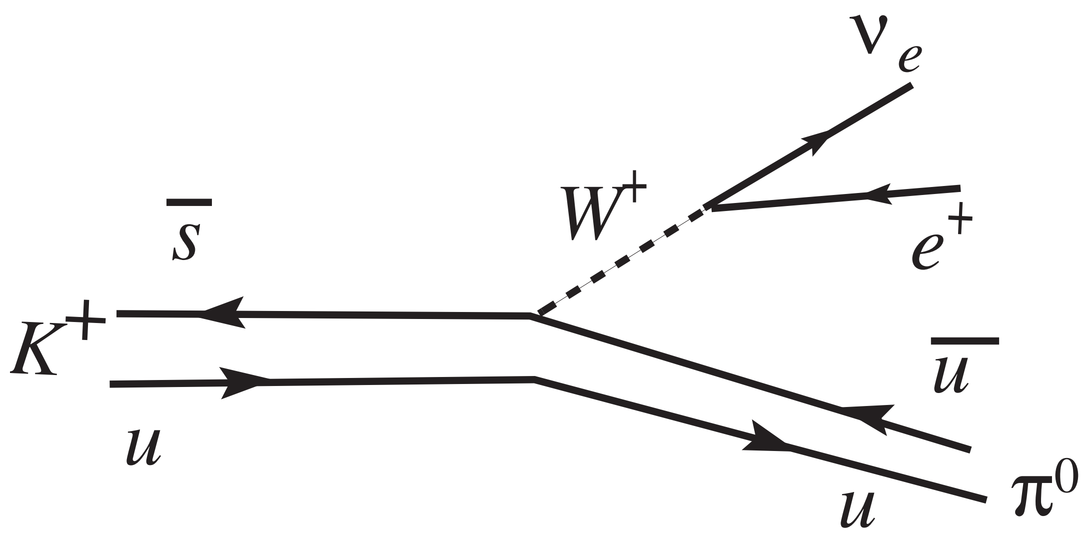
suppression factor $\sin\theta_C = 0.221$

The charged weak interaction are also universal in quark sector, provided quark-mixing.

Drawback of Cabibbo mixing

Flavor changing neutral current

$$\bar{d}'_L \gamma_a d'_L = \cos^2\theta_C \bar{d}_L \gamma_a d_L + \sin^2\theta_C \bar{s}_L \gamma_a s_L$$
$$+ \cos\theta_C \sin\theta_C [\bar{d}_L \gamma_a s_L + \bar{s}_L \gamma_a d_L]$$



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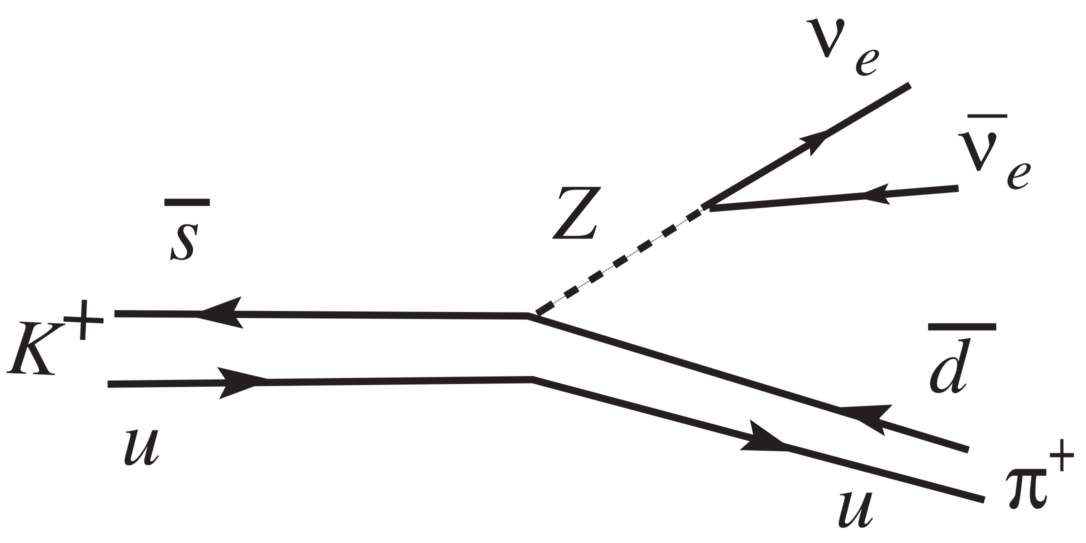


$\mathrm{Br} \sim 10^{-2}$

$\mathrm{Br} \sim 10^{-10}$

$G_F \sin\theta_c$

$G_F \cos\theta_c \sin\theta_c$

GIM Mechanism

1970

Glashow, Illipoulos, Maiani

$$\begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix} \qquad s' = -d\sin\theta_\mathrm{C} + s\cos\theta_\mathrm{C}$$

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_\mathrm{C} & \sin\theta_\mathrm{C} \\ -\sin\theta_\mathrm{C} & \cos\theta_\mathrm{C} \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$$\bar{s}'_L\gamma_a s'_L = \sin^2\theta_\mathrm{C}\bar{d}_L\gamma_a d_L + \cos^2\theta_\mathrm{C}\bar{s}_L\gamma_a s_L - \cos\theta_\mathrm{C}\sin\theta_\mathrm{C}[\bar{d}_L\gamma_a s_L + \bar{s}_L\gamma_a d_L]$$

$$\bar{d}'_L\gamma_a d'_L = \cos^2\theta_\mathrm{C}\bar{d}_L\gamma_a d_L + \sin^2\theta_\mathrm{C}\bar{s}_L\gamma_a s_L + \cos\theta_\mathrm{C}\sin\theta_\mathrm{C}[\bar{d}_L\gamma_a s_L + \bar{s}_L\gamma_a d_L]$$

$$\Rightarrow \quad \bar{s}'_L\gamma_a s'_L + \bar{d}'_L\gamma_a d'_L = \bar{d}_L\gamma_a d_L + \bar{s}_L\gamma_a s_L$$

Flavor changing neutral currents cancel out,
but Flavor conserving neutral currents remains.

1973
CERN

Gauge Theories

Standard Model is remarkably successful gauge theory of the microscopic interactions

Symmetry

- A symmetry follows from the assumption that a certain quantity is not measurable.
- That implies the existence of conserved quantities.

—— Noether's theorem

1) 不可观测

无法观测的物理量

绝对位置	$\vec{p}$
绝对时间	E
绝对方位	$\vec{L} = \vec{r} \times \vec{p}$
绝对左右	P
绝对未来	T
绝对电荷	C

2) 无法区分

一个物体变换为另一个物体

整体对称性：同位旋

时空对称性 等价性

3) 无序

Quantum Mechanics

- Group operations represented by unitary operators (u) in a linear vector space of state vector $|\alpha\rangle$

vector transformation: $|\alpha\rangle \rightarrow |\alpha'\rangle = u\,|\alpha\rangle$

operator transformation: $\theta \rightarrow \theta' = u\theta u^{-1}$

- If system is symmetric under group, $[H, u] = 0$
- Of particular interest are symmetry groups with representation like

$$u(\epsilon) = e^{-i\sum_j \epsilon^j Q^j}$$

infinitesimal parameters

Generators of the group & operators having quantum #'s as eigenvalues

- Connection through ‘charge’ & conserved ‘current’

$$Q \equiv \int d^3x j^0(x) \qquad \partial_\mu j^\mu(x) = 0$$

Quantum Field Theory

$\phi(x)$ is an operator

$$\begin{aligned} \phi \to \phi' &= u\phi u^{-1} \\ &= (1 - i\sum_j \epsilon^j Q^j)\phi(1 + i\sum_j \epsilon^j Q^j) \\ &= \cdots = \phi + i\sum_j \epsilon^j [Q^j, \phi(x)] \end{aligned}$$

so $[Q^j, \phi(x)] = 0$ → symmetry → conservation law

Note: often $u\phi u^{-1} = \exp(i\sum_j \epsilon^j q^j)\phi(x)$

eigenvalues of Q^j

Internal Symmetry

- Symmetries whose transformation parameters do not affect the point of space and time x
- It is more natural in QM and QFT. For example, the phase of the wave function. Equation of Motion (Dirac or Schrodinger), normalization condition are invariant under the transformation:

$$\Psi(x) \to e^{i\theta}\Psi(x)$$

- It implies the conservation of the probability current.

Heisenberg Isospin Theory

- Assume the strong interaction are invariant under a group of SU(2) transformation in which the proton and neutron form a doublet N(x)

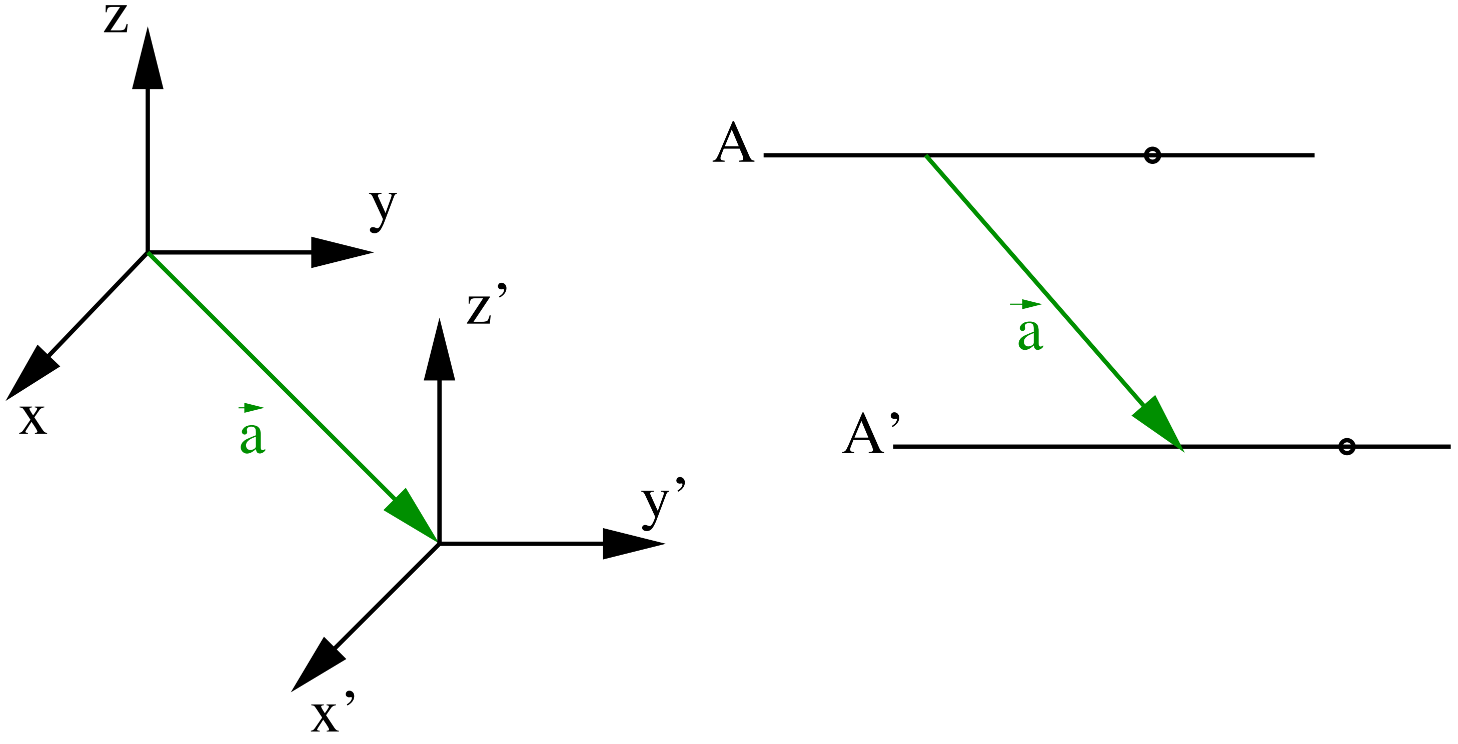
$$N(x) = \begin{pmatrix} p(x) \\ n(x) \end{pmatrix} \quad ; \quad N(x) \rightarrow e^{i\vec{\tau}\cdot\vec{\theta}} N(x)$$

$\vec{\tau}$ are proportional to Pauli matrices

$\vec{\theta}$ are the three angles of a general rotation in a three dimensional Euclidean space

Global Symmetry

$$\vec{x} \longrightarrow \vec{x} + \vec{a}$$



A is trajectory of a free particle in the (x,y,z) system

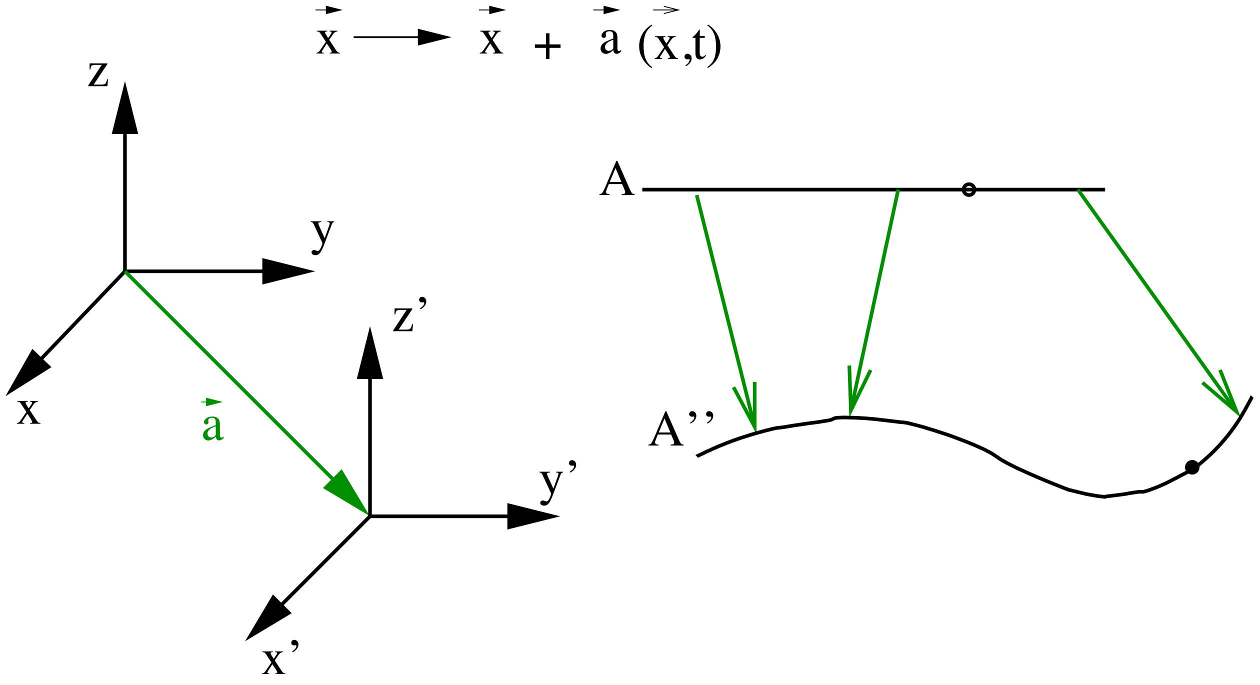
A' is also a possible trajectory of a free particle
in the new system

The dynamics of free particles is invariant under space translations by a constant vector

Gauge Transformation

The transformation parameters are functions of the space-time point x

A free particle dynamics is not invariant under translations in which $\vec{a}$ is replaced by $\vec{a}(x)$.



For A’’ to be a trajectory, the particle must be subject to external forces

Weyl's Gauge Transformation

- Soon after GR was written by Einstein, Weyl proposed a modification …

He added invariance with respect to

a) $g'_{\mu\nu} = \lambda(x) g_{\mu\nu}$

b) $A'_{\mu} = A_{\mu} - \frac{\partial \lambda(x)}{\partial x^{\mu}}$

same $\lambda(x)$ phase

b) is the regular ambiguity required of EM potentials

a) is weird → $ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} \rightarrow \lambda ds^2$

Lengths are re-'gauged'

Weyl’s Gauge Transformation

- suggests an invariance even though space & time can change over all space and time
- the mediator which holds the space-time structure together would be the electromagnetic field

An early attempt to unify gravitation with electromagnetism

The brilliant idea did not work but the name stuck.

In 1927 London revived the idea … but the symmetry isn’t the scale of space-time, rather the phase of the wave function.

Symmetry= Force

- Neither Dirac nor Schrodinger equation are $\theta(x)$ invariant under a local change of phase

Free Dirac Lagrangian

$$\mathcal{L} = \bar{\Psi}(x)(i\not{\partial} - m)\Psi(x)$$

is not invariant under the transformation

$$\Psi(x) \to e^{i\theta(x)}\Psi(x) \quad \Longrightarrow \quad \partial_\mu \theta(x)$$

In order to restore invariance, we must modify free Dirac Lagrangian such that it is no longer describe a free Dirac Field.

➡ Invariance under gauge symmetry leads to the introduction of interactions.

QED Interaction

- Local U(1) symmetries $u(\theta) = e^{i\theta(x)Q}$

$$\psi(x) \to \psi'(x) = e^{i\theta(x)q}\psi(x)$$

$$\begin{aligned}\mathcal{L}(\psi) \to \mathcal{L}(\psi') &= e^{-i\theta(x)q}\bar{\psi}(x)\left[i\gamma^\mu\partial_\mu - m\right]e^{i\theta(x)q}\psi(x) \\ &= \bar{\psi}(x)\left[i\gamma^\mu\partial_\mu - m\right]\psi(x) - q\partial_\mu\theta(x)\bar{\psi}(x)\gamma^\mu\psi(x) \\ &\neq \mathcal{L}(\psi)\end{aligned}$$

Derivative term causes trouble —> define a new divergence operator to cancel the unwanted term!

$$D_\mu \equiv \partial_\mu + X_\mu$$

as-yet unnamed vector operator

QED Interaction

- The goal is to get the gradient term to transform simply

$$(D_\mu \psi) \to (D_\mu \psi)' = e^{iq\theta(x)}\,(D_\mu \psi)$$

Start out with

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(x)\,[i\gamma^\mu D_\mu - m]\,\psi(x)\\ &= \bar{\psi}(x)\,[i\gamma^\mu \partial_\mu + i\gamma^\mu X_\mu - m]\,\psi(x)\end{aligned}$$

Transform $\psi \to \psi'$

$$\mathcal{L}(\psi) \to \mathcal{L}(\psi') = \bar{\psi}'(x)\,\{i\gamma^\mu\,[\partial_\mu + X_\mu - iq\partial_\mu\theta(x)] - m\}\,\psi'(x)$$

Still not right!

One must simultaneously transform

$$X_\mu \to X'_\mu = X_\mu - iq\partial_\mu\theta(x)$$

Bingo!

QED Interaction

- Denote $X_\mu \equiv iqA_\mu(x)$ so the gradient looks like

$$D_\mu \equiv \partial_\mu + iqA_\mu$$

and total transformation necessary to leave $\mathcal{L}$ along is

$$\psi(x) \to \psi'(x) = e^{iQ\theta(x)}\psi(x)$$

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \partial_\mu\theta(x)$$

gauge function

- Add free gauge field

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu\partial_\mu - m\right) - qA_\mu\bar{\psi}\gamma^\mu\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

free ψ interaction free A_μ

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

Utilizing Symmetry

If invariance with respect to local U(1) symmetry is of paramount important …

—-> one is forced to invent the photon

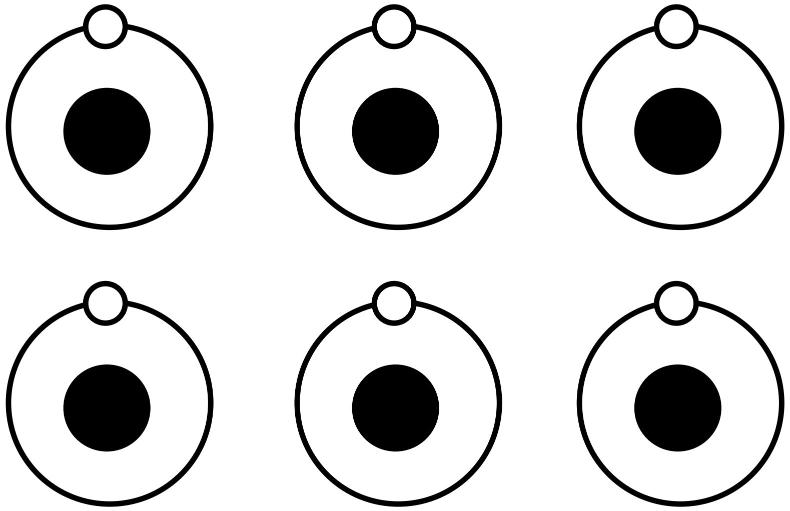
Demand of a symmetry … Get new fields AND dynamics!!

Other symmetries —> New spin 1, 2, .. fields?

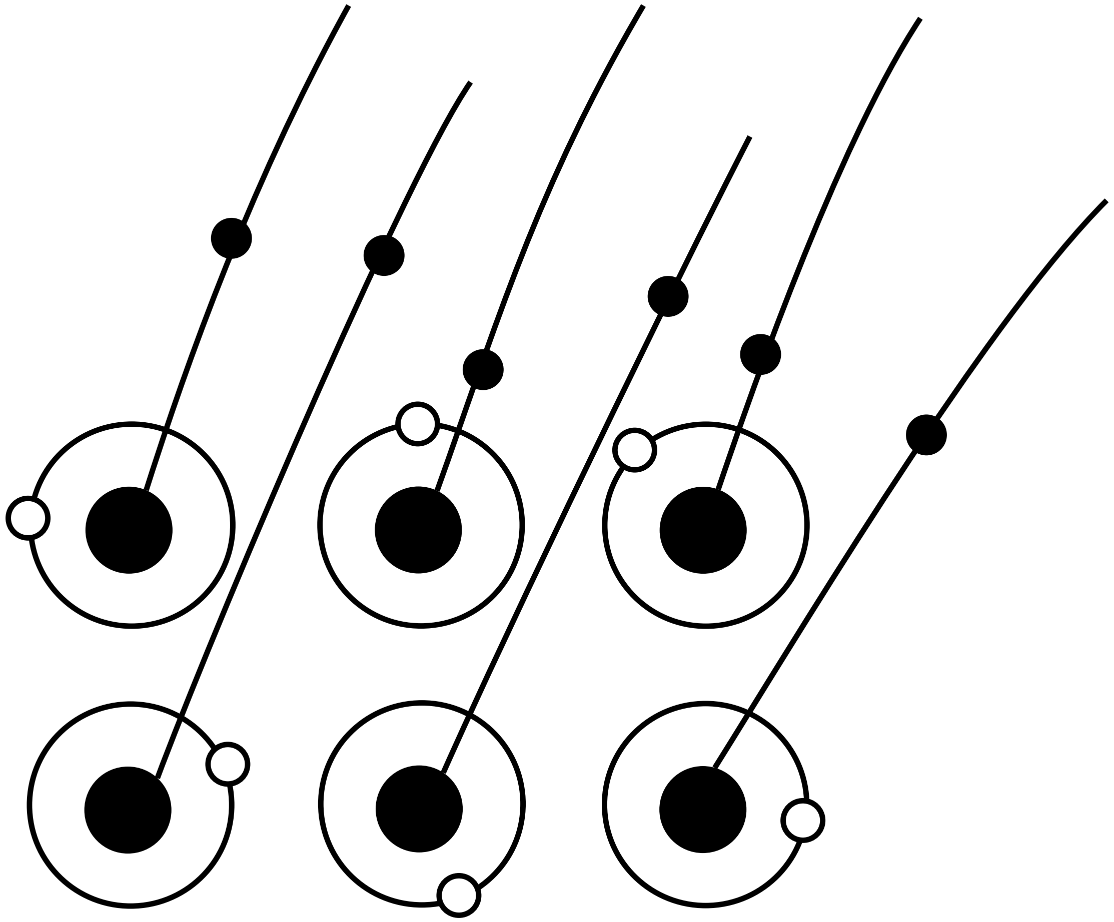
The intriguing research project in 1954 of
Yang & Mills … and independently by Shaw

Local SU(2) symmetry —> isotriplet of spin-1 fields

Global versus Local



Global U(1) gauge transformation



Local U(1) gauge transformation

Non-Abelian Gauge Theory

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

Now $\psi = \left(\begin{array}{c} \psi_1 \\ \psi_2 \end{array} \right)$ as bases for SU(2) operators

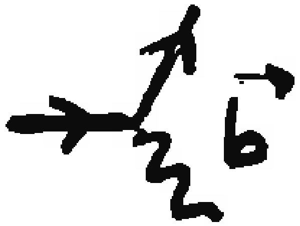
Define a new covariant derivative

$$D_\mu \equiv \partial_\mu + ig\vec{W}_\mu \cdot \frac{\vec{\tau}}{2}$$

⬇

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - \frac{g}{2}\bar{\psi}\gamma^\mu\psi \cdot \vec{W}_\mu - \frac{1}{4}\vec{f}_{\mu\nu} \cdot \vec{f}^{\mu\nu}$$

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - \frac{g}{2}\bar{\psi}\gamma^\mu\vec{\tau}\psi\cdot\vec{b}_\mu \underbrace{- \tfrac{1}{4}\vec{f}_{\mu\nu}\cdot\vec{f}^{\mu\nu}}_{\text{complicated}}$$



$$-\tfrac{1}{4}\vec{f}^{\mu\nu}\cdot\vec{f}_{\mu\nu} = -\frac{1}{2}(\partial_\nu\vec{b}_\mu - \partial_\mu\vec{b}_\nu)\cdot\partial^\nu\vec{b}^\mu$$

$$+ g\,\vec{b}_\nu\times\vec{b}_\mu\cdot\partial^\nu\vec{b}^\mu$$

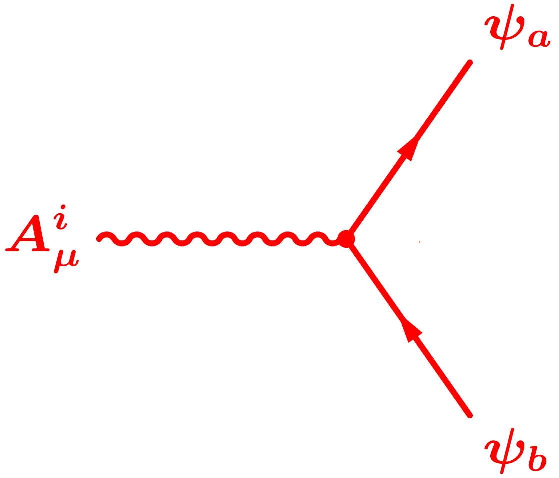
$$-\tfrac{1}{4}g^2\left[(\vec{b}_\nu\cdot\vec{b}^\nu)^2 - (\vec{b}_\nu\cdot\vec{b}_\mu)(\vec{b}^\mu\cdot\vec{b}^\nu)\right]$$

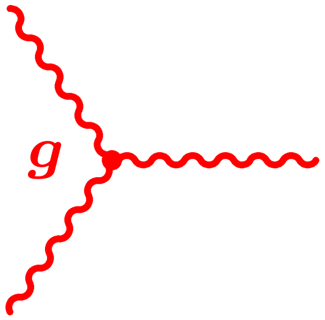
– get self-couplings for $\vec{b}$'s.

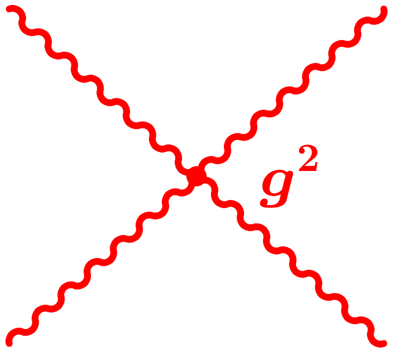
Non-Abelian Gauge Theory

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - \frac{g}{2}\bar{\psi}\gamma^\mu\psi \cdot \vec{W}_\mu - \frac{1}{4}\vec{f}_{\mu\nu} \cdot \vec{f}^{\mu\nu}$$

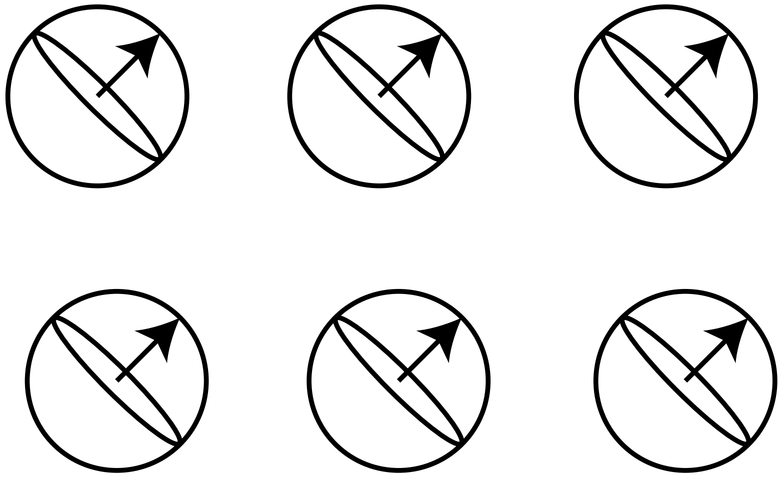
- **Gauge invariance implies:**
  - N **(apparently) massless gauge bosons** A^i_μ
  - **Specified interactions (up to gauge coupling** g**, group, representations), including self interactions**



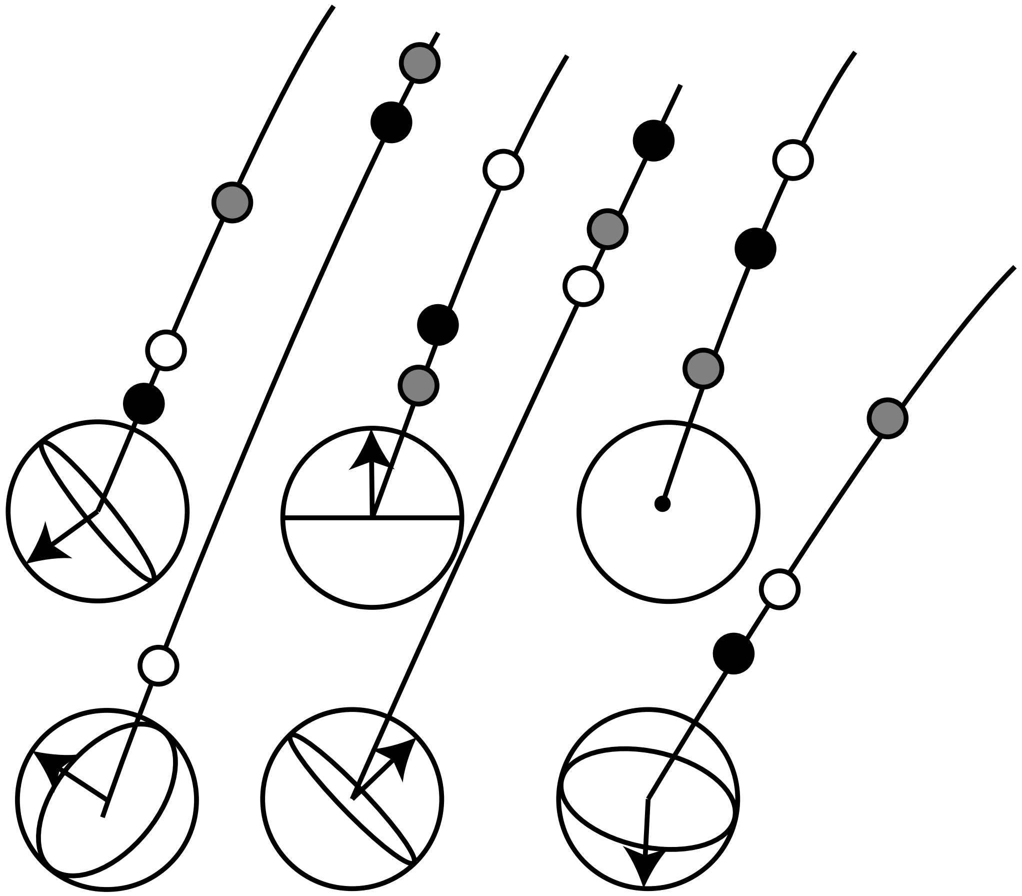




SU(2): Global versus Local



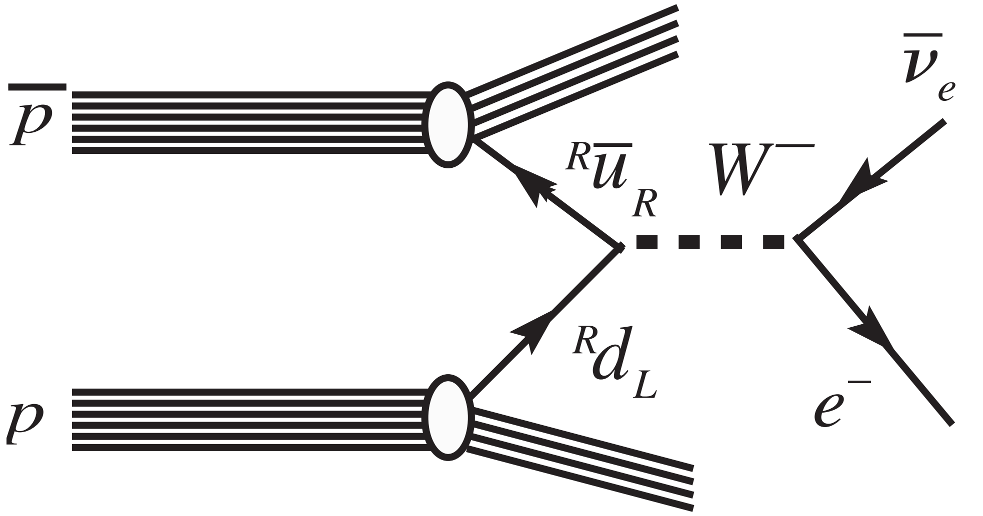
Global SU(2) gauge transformation

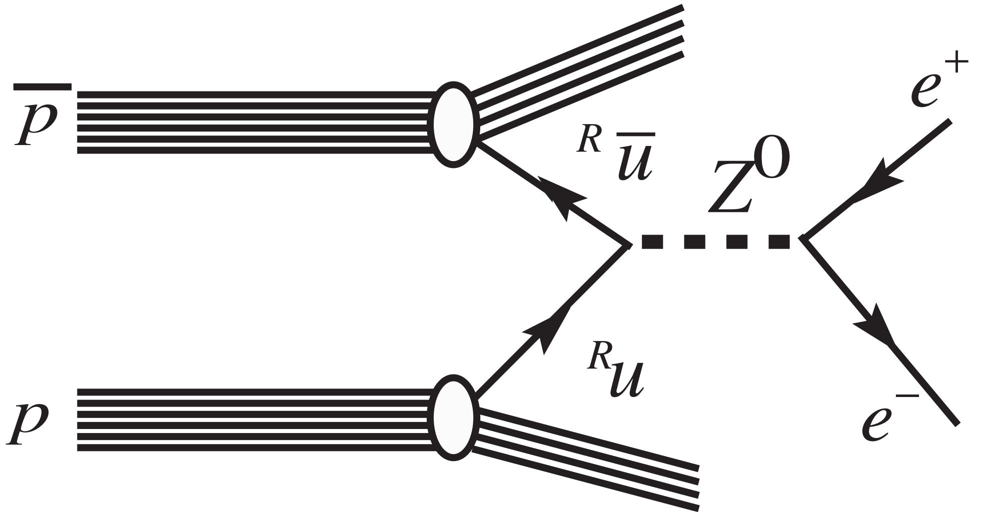


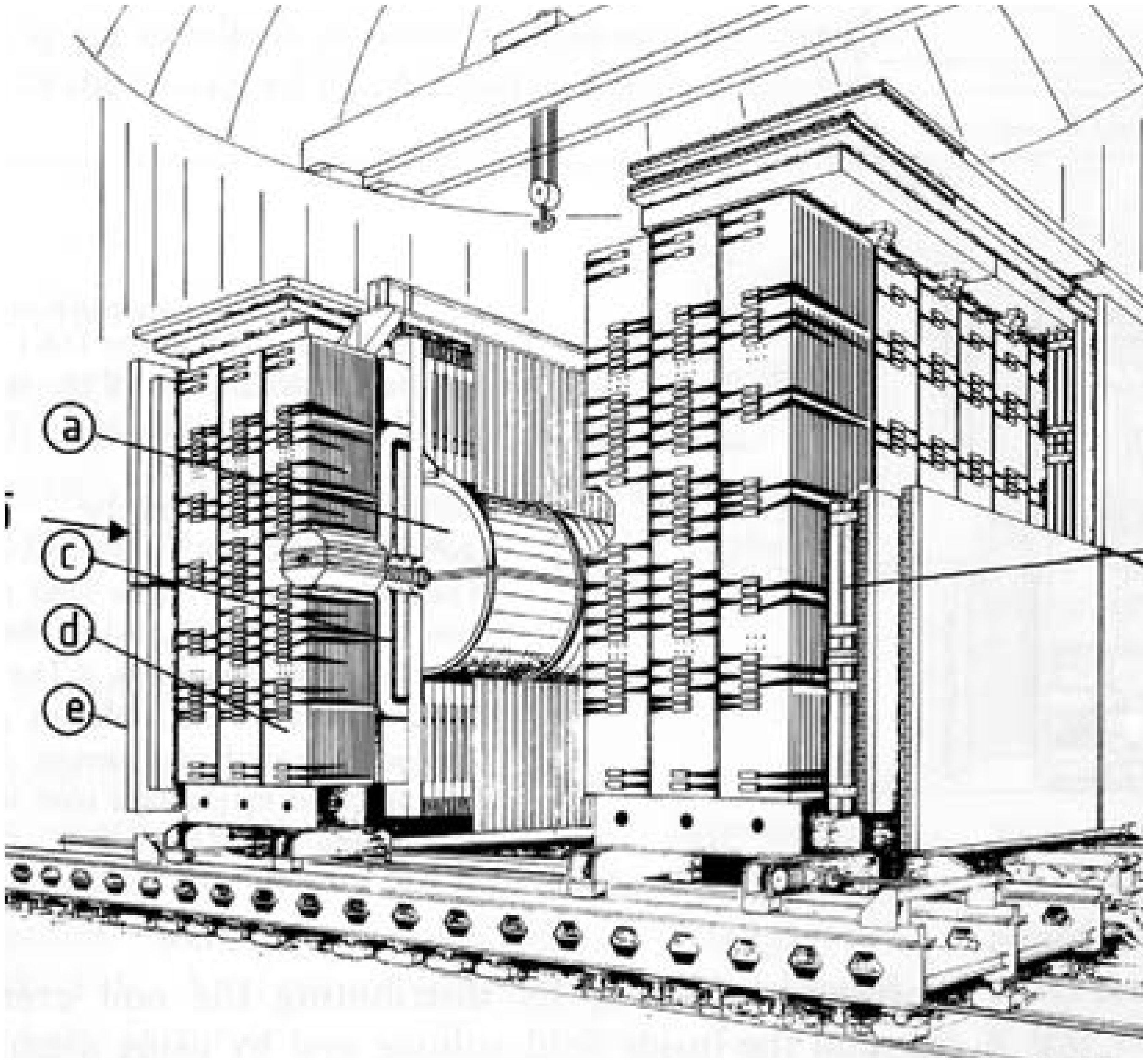
Local SU(2) gauge transformation

W and Z discovery

- UA1 experiment (1976, Rubbia, Cline, McIntyre)

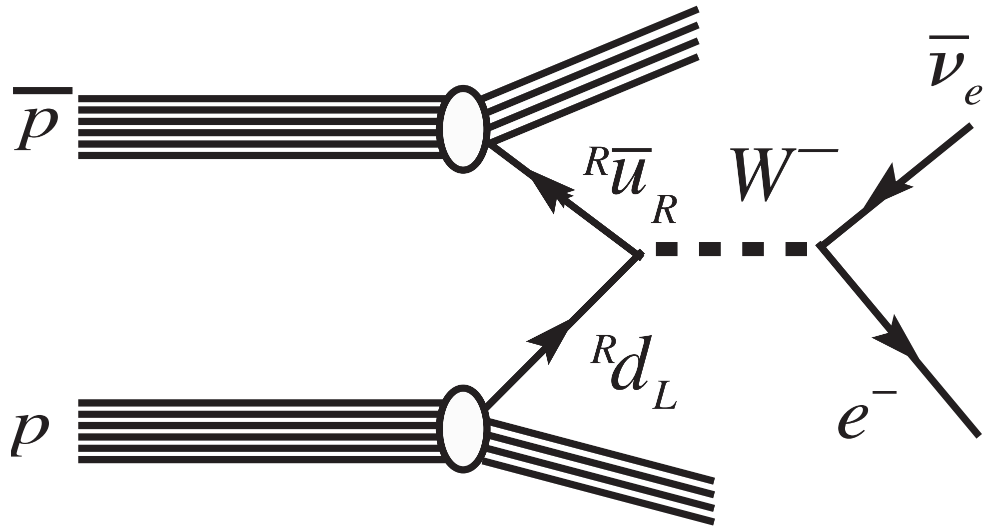


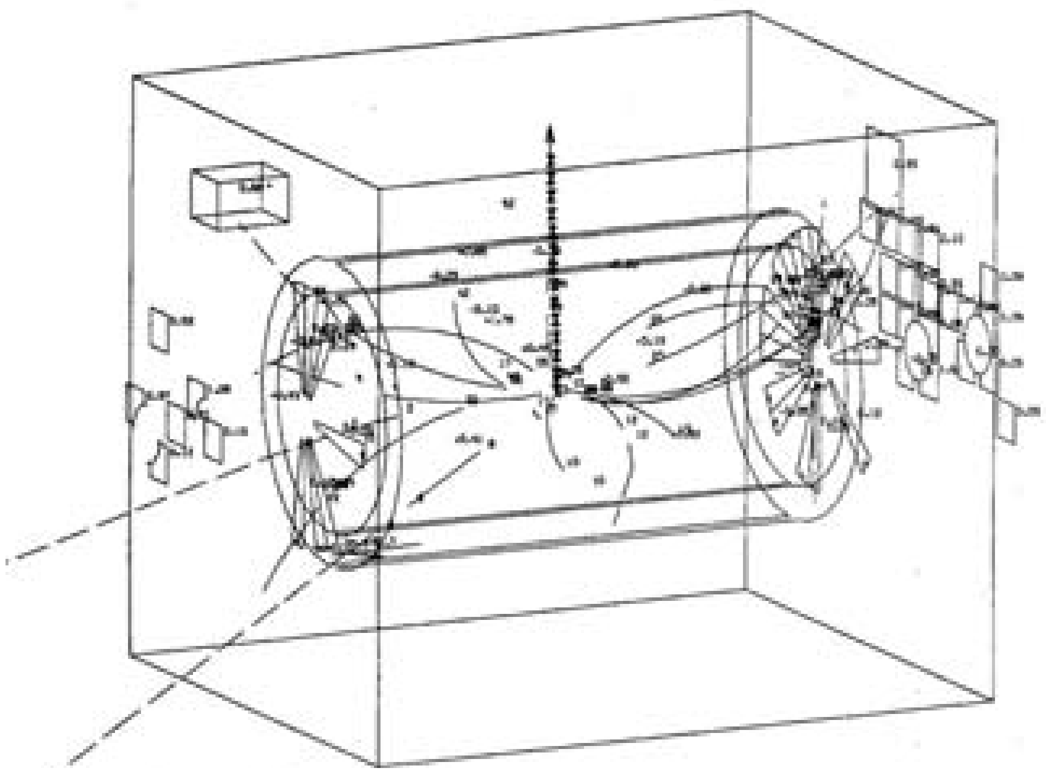


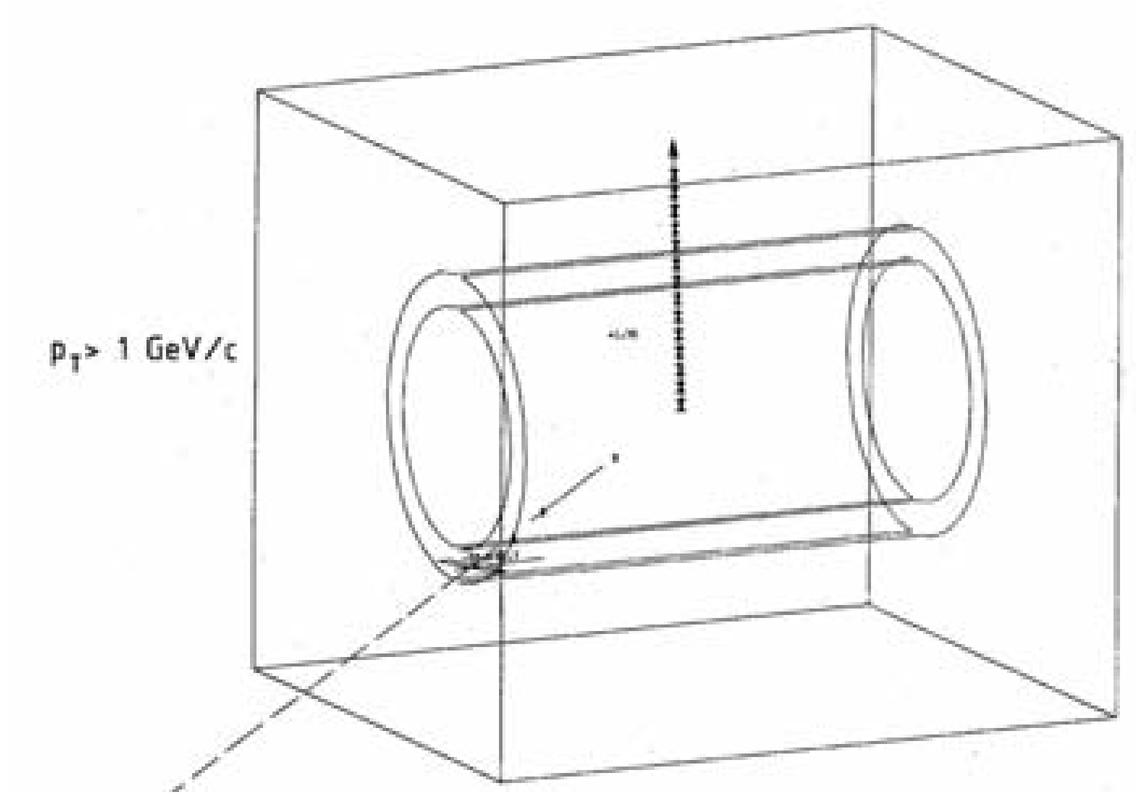


W-boson Discovery (1983)

- UA1 experiment

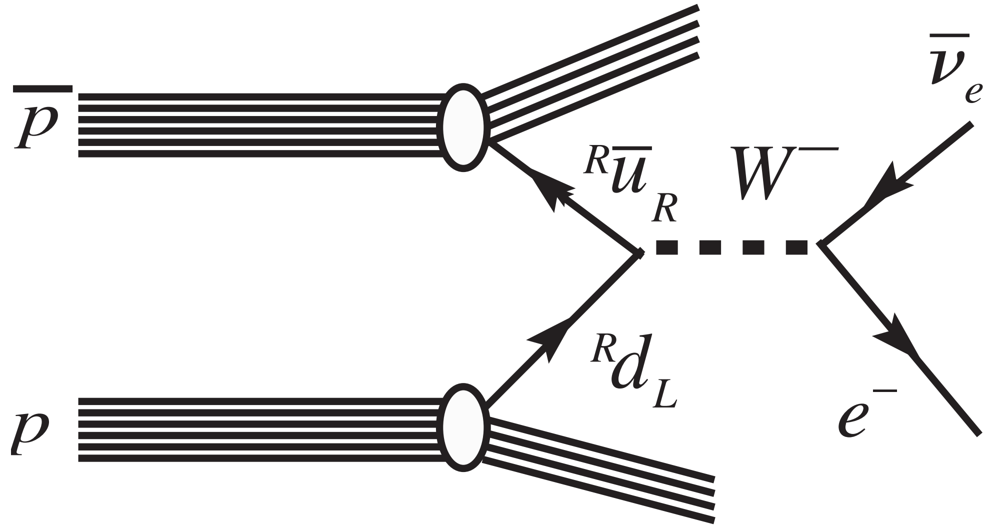




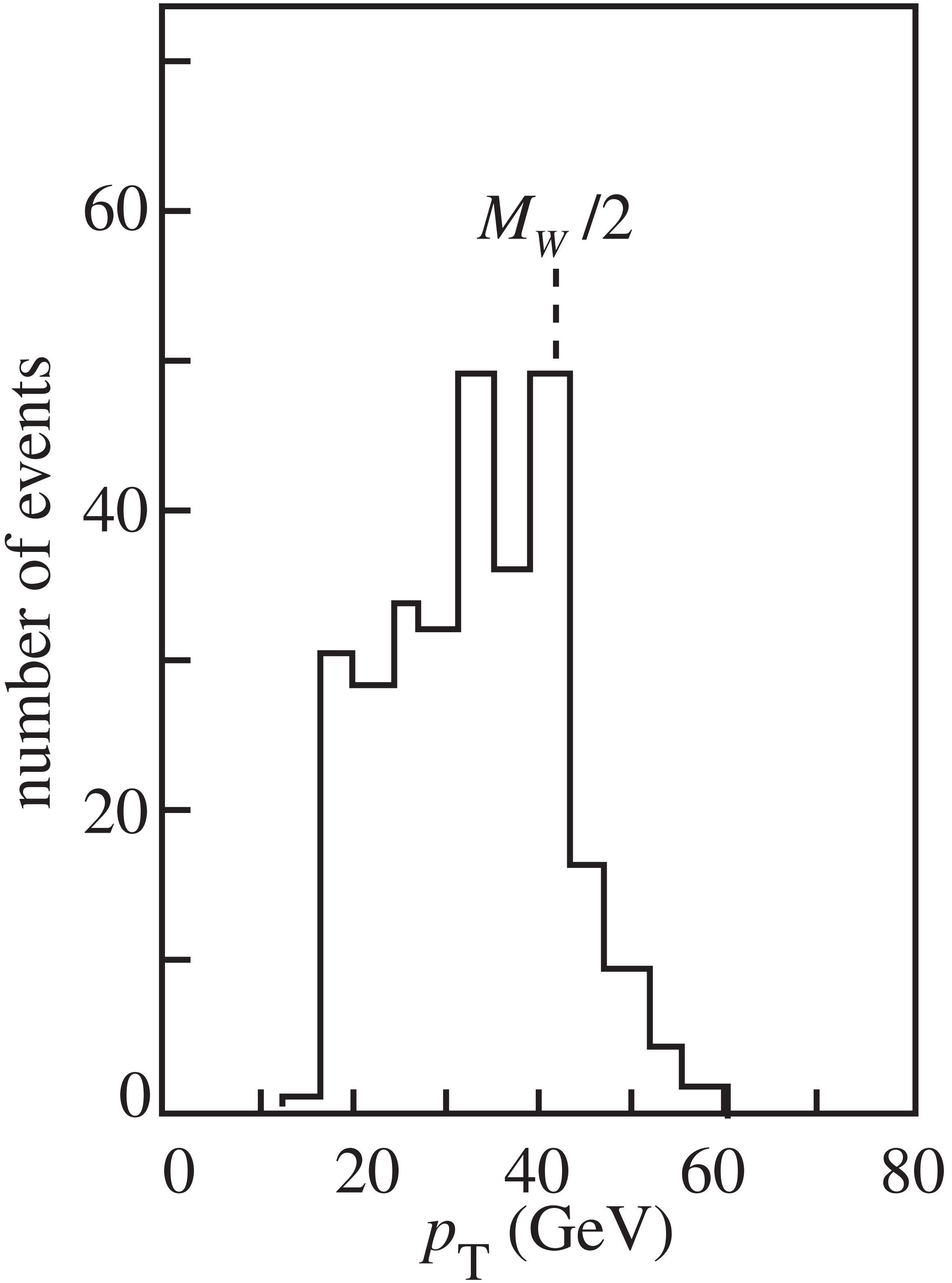


W-boson Discovery (1983)

- UA1 experiment

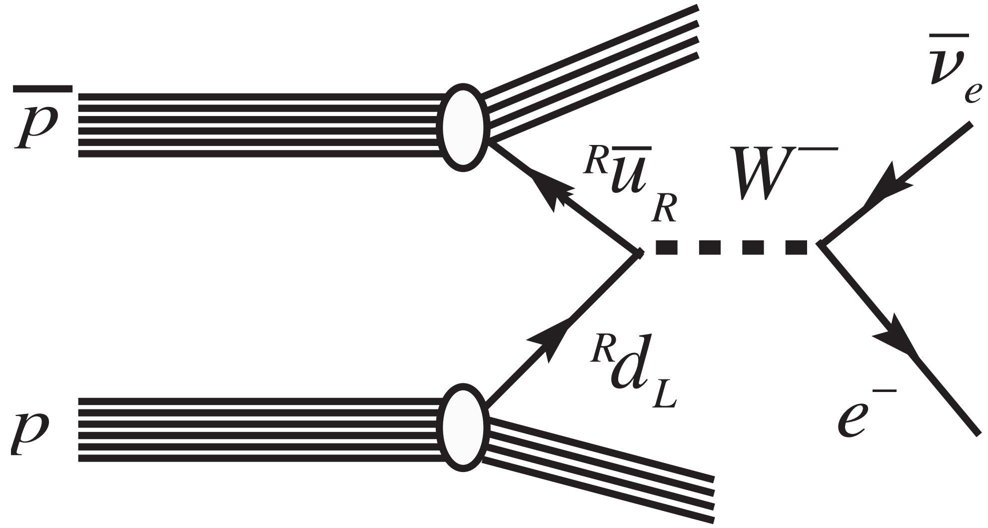


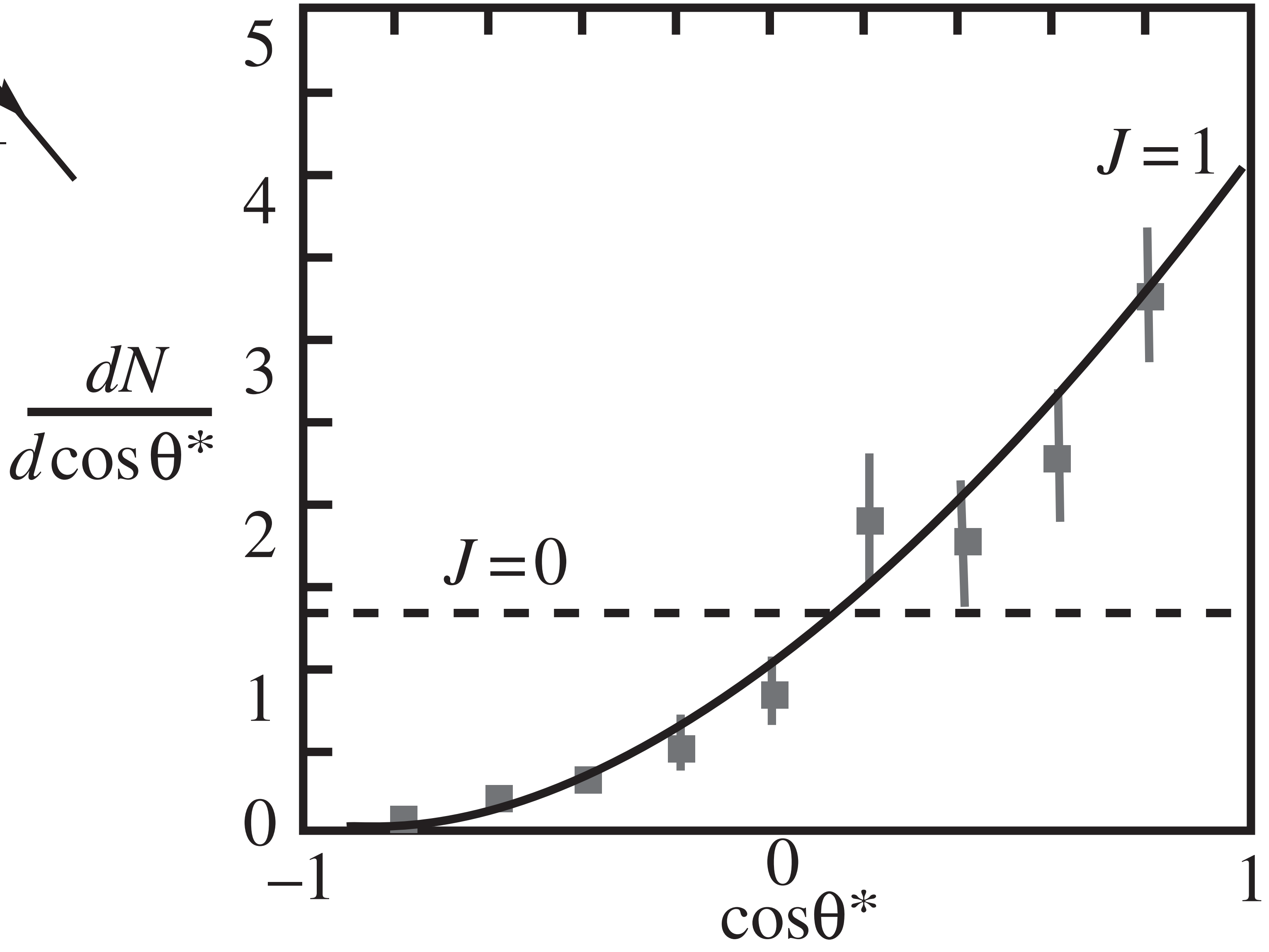
$$\frac{dn}{dp_{\mathrm{T}}} = \frac{1}{\sqrt{(M_{\mathrm{W}}/2)^2 - p_{\mathrm{T}}^2}} \frac{dn}{d\theta^*}$$



W-boson Discovery (1983)

- UA1 experiment

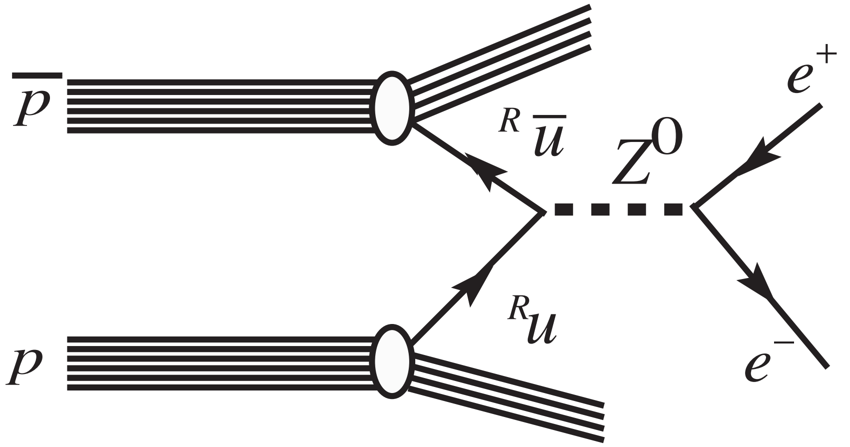


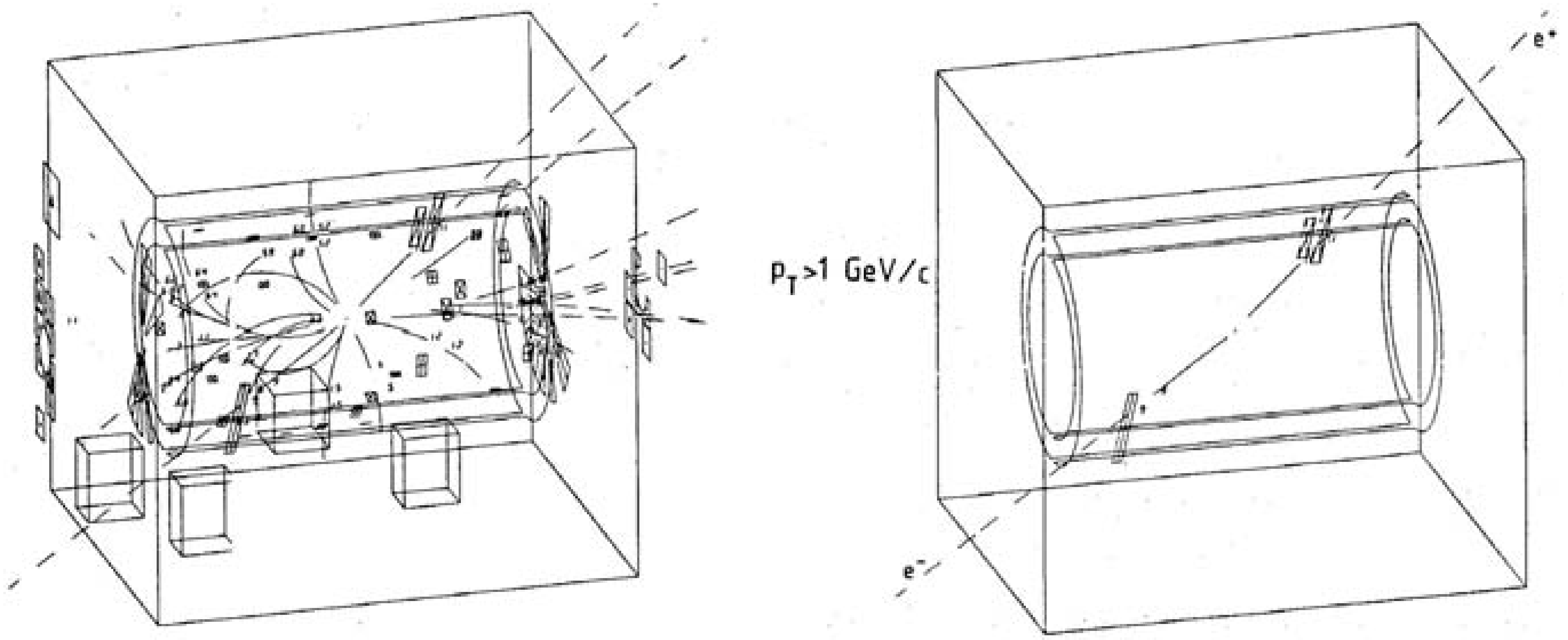


W-boson spin is one.

Z-boson Discovery (1983)

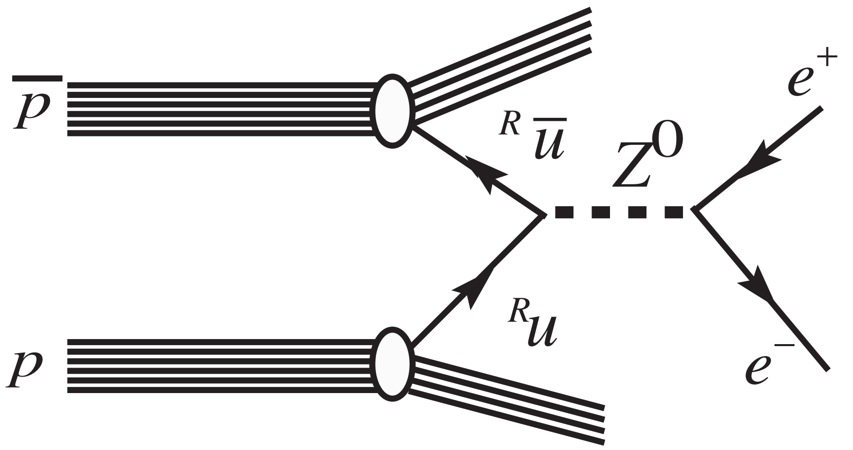
- UA1 experiment

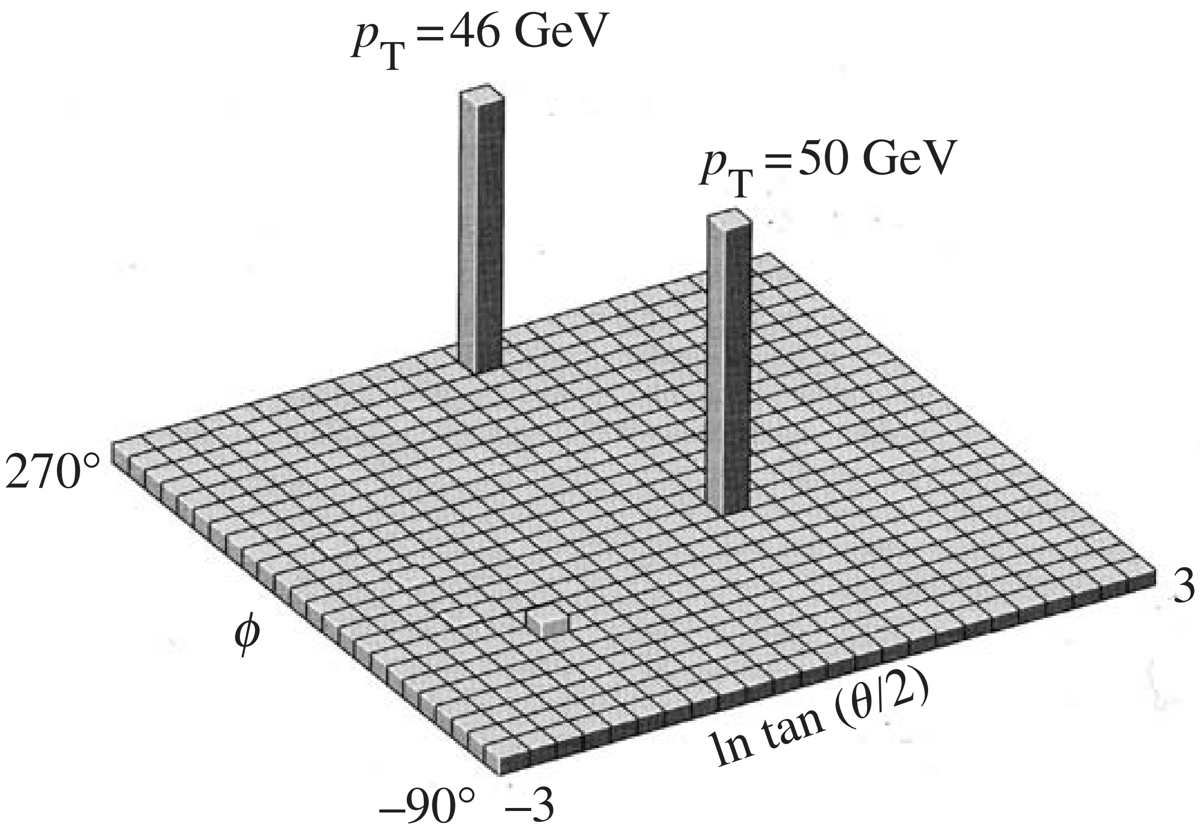


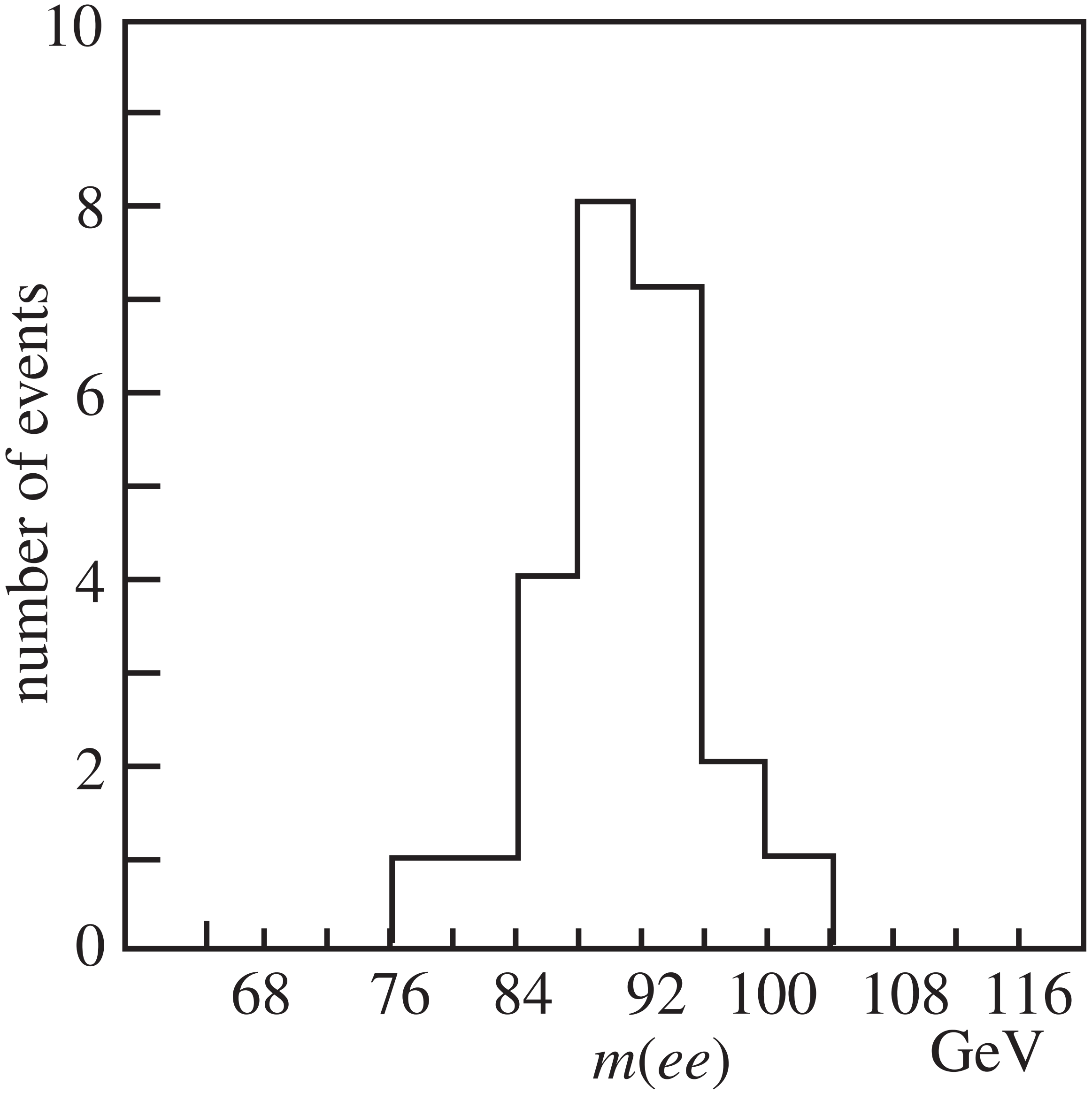


Z-boson Discovery (1983)

- UA1 experiment







Summary

- Beta decay; neutrino
- Fermi Theory
- Parity violation; two-component neutrino theory
- V-A theory; Quark mixing
- Gauge theory; QED; Non-Abelian SU(2)
- W-boson and Z-boson discovery

Next Lecture

The origin of W-boson and Z-boson masses

Afternoon Lecture

Confirming the W-boson and Z-boson event experimentally