Hierarchical structures in climate and atmospheric turbulence*

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Abstract The climatic time series encompass many time scales, forming multi-scale climatic variations and hierarchies. The periods and magnitudes of cooling or warming, especially for extreme climatic events, vary with the time scales. On the other hand, high-frequency atmospheric turbulent motions, which are certainly related to the climatic changes, are also typical multi-scale phenomena. In this paper, certain properties of the fluctuations are examined that are invariant with the change of the time scales. Specifically, the existence of a statistical similarity law is demonstrated in climatic time series and in atmospheric turbulent motions. This is the hierarchical similarity law. Further analyses show that the hierarchical similarity law is a general property of multi-scale fluctuation systems. Therefore, the study on the hierarchical similarity law is important for understanding the inherent universal physical mechanism of complex systems in the atmospheric motions.

Keywords: climate, multi-scale, hierarchical structure, atmospheric turbulence.

The global warming is a pressing issue in the current study of climate changes[1]. The climate variation is, however, a multi-scale phenomenon; the periods and magnitudes of its warming or cooling vary with time scales; long warming periods contain shorter warming and cooling periods, and so on[2], so it is meaningless to discuss the climate warming and cooling without referring to time scales. In addition to multi-scales property, the climate variations encompass also a wide range of fluctuation amplitudes. This is a general characteristic feature of multi-scaling phenomenon. For instance, the precipitation is intermittent on scale of days; some day there is no precipitation, but the next day there may be 10 mm to 10 cm precipitation or more. On the scale of years, some year there may be as much as 500 mm precipitation, but a year later there may be only 200 mm precipitation or less. Furthermore, there are from time to time large fluctuations called extreme events that occur within a short period of time or within a small region in space. This is referred to as intermittency effects (in time and in space). The duration and the magnitude of the extreme events vary with scales as well. In summary, the change of characteristics of the climate variations with scales is the general feature of multi-scale climate phenomena, forming some kind of hierarchy in climate dynamics[3,4]. On the other hand, the motions in the atmospheric boundary layer are generally of high-frequency, and turbulent because of high Reynolds number. Turbulent motions are obvious on multi-scales: the statistical characteristics for the motions change with the space and time scales. Then, it is interesting to ask whether there is the same universal statistical law that is valid for both climate variations and atmospheric turbulent motions while the time scales for the two motions are so different. In this paper, we will show that both variations satisfy the same hierarchical similarity law. What is common behind the two motions is the hierarchical self-organization, for which we will give some explanation about their physical mechanism.

1 Cooling or warming in climate dynamics

The climate state is characterized by warming and cooling, or by drought and flood; and it varies with the time scales. So it is important to discuss the climate state under the consideration of the scales. An example is the calculated monthly-averaged surface temperature over the Northern Hemisphere since 1851[5] by using wavelet transformation. One found that the cooling or warming variation magnitudes depend explicitly on the time states (the period of the average), see Fig. 1[6].

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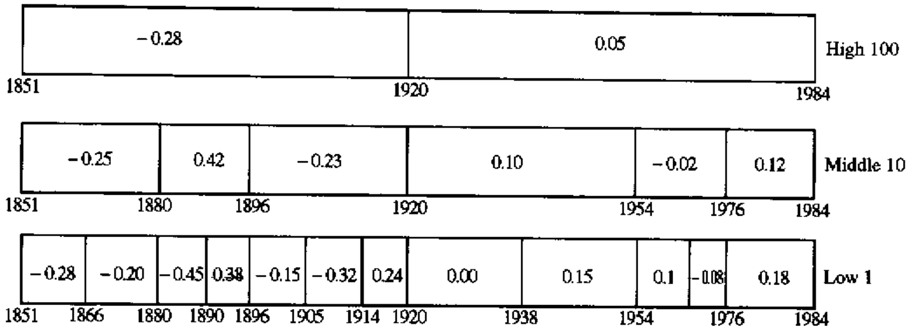

Fig. 1. Cooling and warming magnitudes at three scales measured by its departures from the mean obtained from climatic time series over the Northern Hemisphere.

From Fig. 1, it is obvious that the climate changes vary with the time scales. For example, on the scale of hundred years, the period from 1954 to 1976 is warming, but on the scale of ten years, it is cooling. Moreover, the number of transition points N from cooling to warming state or from warming to cooling state increases with the decrease of the duration of each constant climate state (time scales) a_i (unit: year). In other words, more variations in climate states are observed in shorter time scales. Table 1 shows the number N varying with a_i[7].

Table 1. The duration of each constant climate state and the number of temperature transitions obtained by a wavelet analysis of temperature time series over Northern Hemisphere

i	1	2	3	4	5	6	7	8
$a_i(a)$	190	124	85	65	50	30	20	13
N_i	1	2	3	5	8	12	21	33

From Table 1, we notice that the time scale decreases each time approximately 1.5 times, i.e.

$$\frac{a_i}{a_{i-1}} = \delta \approx 1.5, \qquad (1)$$

and the climate transition number increases approximately 1.6 times, i.e.

$$\frac{N_{i+1}}{N_i} = \alpha \approx 1.6. \qquad (2)$$

So there is the relation between the transition number $N(a)$ on the scale a_i and the transition number on the scale δa:

$$N(\delta a) = \frac{1}{\alpha} N(a) = \delta^{-\tau} N(a). \qquad (3)$$

Eq. (3) is a similarity relation, if $1/\alpha$ is rewritten as $\delta^{-\tau}$, then the solution to (3) is

$$N(a) = a^{-\tau}, \qquad (4)$$

where

$$\tau = \frac{\log\alpha}{\log\delta} = \frac{\log 1.6}{\log 1.5} \sim 1.2. \qquad (5)$$

Eq. (4) indicates that cooling or warming state varies with the time scales: when the time scale decreases approximately 1.5 times, the number of transitions increases approximately 1.6 times, and there is an invariant value $\tau \approx 1.2$, which does not vary with the time scale a. It is called the scaling exponent of climate cooling or warming state. The scaling is a law that does not change, while the climate varies with the time scales.

2 Hierarchical similarity of multi-scale fluctuations

The multi-scaling and hierarchy are two related characteristics for both a long-term climate change and a short-term (high-frequency) turbulent fluctuation. This is the central topic of this section. Recently, in the study of the properties of hydrodynamic turbulence, a theory called hierarchical structure theory[4,8~10] is demonstrated to be a powerful tool for describing such multi-scale characteristics, especially the similarity relation between fluctuations at different time scales and at different magnitudes (hierarchy). The hierarchical structure analysis is based on the study of the increment (variation) of

$$\delta\nu_{il} = | \nu_i(x_j + l) - \nu_i(x_j) |$$

over a scale l for an arbitrary time series or a spatial variation (where ν_i can be any physical quantity such as three components of the velocity, the temperature, the humidity and so on, x_j can be a coordinate in space and in time). Then one studies the p-order structure function written as $S_p(l) = \langle | \delta\nu_{il} |^p \rangle$. The hierarchical similarity analysis first introduces p-order hierarchical structure quantity $F_p(l)$, which is defined by

$$F_p(l) = S_{p+1}(l) / S_p(l), \qquad (6)$$

where $F_p(l)$ represents the magnitude of the fluctuations at pth level. Then the theory makes an important assumption[10]: there is a statistical similarity between the dimensionless hierarchical structure quantities at different scales:

$$\frac{F_{p+1}(l)}{F_\infty(l)} = A_p \left[\frac{F_p(l)}{F_\infty(l)}\right]^\beta, \qquad (7)$$

where $0<\beta<1$ is a universal constant independent of p or l, which is called hierarchical similarity parameter; A_p is a constant dependent on p and independent of l. $F_\infty(l)$ is defined as:

$$F_\infty(l) = \lim_{p\to\infty} F_p(l),$$

and called the most singular structure of the fluctuations (or the most intermittent structure). Often, there is a scaling law of the form $F_\infty(l) \propto l^\gamma$, where γ is called the most singular scaling exponent or scaling exponent of the most intermittent structure. Under these considerations, the hierarchical structure theory predicts a formula for the scaling exponents of the pth-order structure function, that is

$$\zeta_p = \gamma p + C(1-\beta^p), \qquad (8)$$

this is the She-Leveque (SL) scaling law widely used to describe the fluctuation structure of hydrodynamic turbulence, where C is a constant.

This model, after being proposed in 1994, has received wide support from experiments and numerical simulation[11~20], and many studies show that the hierarchical structure theory is applicable to other systems with complex multi-scale variations[21~25].

Let us normalize (7) to eliminate A_p and $F_\infty(l)$, and get

$$H_{p+l}(l) = H_p(l)^\beta, \qquad (9)$$

where

$$H_{p+1}(l) = \left(\frac{F_{p+1}(l)}{F_2(l)}\right) \cdot \left(\frac{F_{p+1}(l_0)}{F_2(l_0)}\right)^{-1}.$$

Eq. (9) allows us to do calculation with any time series and, with a plot of the result on the log-log scale, to check whether the hierarchical similarity law is satisfied, which is the so-called β-test of the hierarchical structure theory.

3 Climatic variation state and extreme climate events

The climatic time series usually exhibits intermittent fluctuations at multiple scales[26]. Fig. 2 is the temperature time series in Dali and Xiamen over fifty years since 1951. We have studied its pth-order structure function to explore the law that climatic fluctuations obey at different scales:

$$\langle \delta T_l^p \rangle \sim l^{\zeta_p}, \qquad (10)$$

where $\delta T_l = |T(t+l) - T(t)|$ is the temperature increment from the climate time series across the time scale l. ζ_p is the scaling exponent of p-order structure function, which, just like that in formula (4), does not vary with the scale.

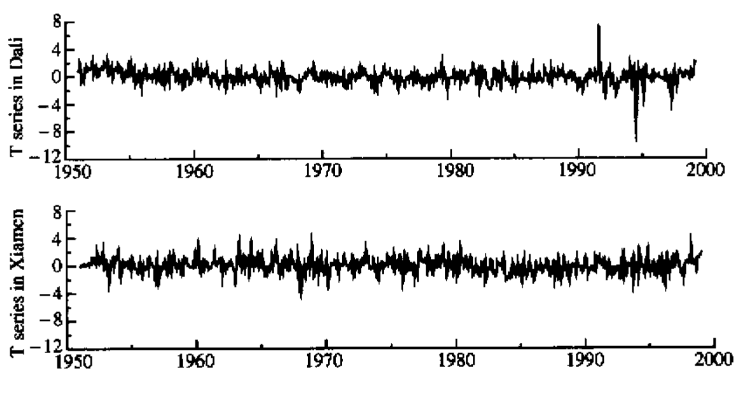

Fig. 2. Monthly-averaged temperature time series in Dali and Xiamen since 1951.

Applying the hierarchical structure theory to the analysis of the climatic temperature time series, one finds that the scaling exponents in formula (8) can be rewritten as

$$\zeta_p = \gamma p + \nu_\infty(1-\beta^p), \quad 0 \leqslant \beta \leqslant 1, \qquad (11)$$

where γ and ν_∞ are parameters related to the properties of the most intense fluctuations (or extreme climate events):

$$\delta T_l \mid_{\text{extreme climate}} \sim l^\gamma, \quad \gamma \ll 1, \qquad (12)$$

$$P(l) \mid_{\text{extreme climate}} \sim l^{3-D_\infty}, \quad 3 - D_\infty = \nu_\infty. \qquad (13)$$

Since the most intense fluctuation δT_l has the largest magnitude and occupies the smallest volume (or has the smallest probability) $P(l)$, then the parameter γ in (12) and (13) takes a minimum value, as well as the parameter D_∞, but the parameter ν_∞ takes a maximum value[27]. The two parameters calculated from the climatic time series in the Northern Hemisphere are[1)]

$$\gamma = 0.003, \quad \nu_\infty = 1.51 (\text{or } D_\infty = 1.49). \tag{14}$$

The paper[2)] analyzed the temperature time series over the Northern Hemisphere and 160 cities in China and found that the hierarchical similarity law is verified with these time series. The parameter calculated from the Northern Hemisphere is $\beta=0.7$, but in Dali and Xiamen they are $\beta=0.49$ and $\beta=0.77$, respectively. According to the hierarchical structure theory, the parameter β reflects the degree of intermittency of the time series: the smaller the parameter β is, the more intermittent the most intense events appear to be. This is qualitatively verified from a visual inspection of Fig. 2.: Dali's temperature variations contain several very outstanding intense (extreme) events and have a much smaller value of β. This is an example that the hierarchical structure analysis can help to deduce certain characteristics of the fluctuations from the time series by a statistical calculation. More importantly, the hierarchical structure analysis relates the property of the fluctuations to the properties of the extreme events, the most intense fluctuations. These rare events dominate the property because of an inherent self-organization.

4 Hierarchical structures in atmospheric turbulence

Compared to the long-term climatic variations, the atmospheric turbulence is composed of fluctuations at much shorter time scales with quite different magnitudes and degree of coherence. It is interesting to know whether the hierarchical similarity still holds in high-frequency atmospheric turbulence. Our careful analysis of data from an atmospheric boundary layer indicates that there is good hierarchical similarity in some quantities in the atmospheric boundary-layer under certain conditions, but there is no hierarchical similarity for some other fluctuation quantities. Specifically, we have applied the hierarchical similarity analysis to time series of temperature and velocity (with two horizontal components and one vertical component) fluctuations obtained from an atmospheric boundary layer measurement. The data set used here was collected at the State Key Laboratory of Atmospheric Boundary Layer Physics and Atmospheric Chemistry (LAPC), Institute of Atmospheric Physics, the Chinese Academy of Sciences, using an ultrasonic anemometer (type: SAT-211/3K) with sampling frequency 10 s^{-1} and 4 meters above the ground of a flat rice field. The data can be regarded as a typical sample of characteristic variations in a turbulent flow near the ground.

We found that the hierarchical similarity generally holds in the temperature fluctuations in the atmospheric boundary layer; however, the situation for the velocity fluctuations is more complex, which depends on whether the atmosphere is stably stratified or not and on which component (horizontal or vertical) is studied. In the situation of unstable stratification, the vertical velocity fluctuations generally satisfy the hierarchical similarity, but under the stable stratification, it generally does not. The horizontal components of velocity usually do not satisfy the hierarchical similarity under either stable or unstable stratification. Fig. 3 shows the result of the hierarchical similarity analysis for temperature fluctuations, and Fig. 4 for velocity fluctuations.

The linear correlation in the log-log plot of Fig. 3 shows that the temperature fluctuations satisfy the hierarchical similarity under both the unstable and the stable stratification. Under the stable stratification, the parameter measured is $\beta = 0.62$ (hollow dot), while under the unstable stratification, it is $\beta=0.64$ (solid dot), they almost have the same value. So the results here demonstrate that high-frequency (small-scale) temperature fluctuations are governed by the hierarchical similarity law. The value of hierarchical similarity parameter is close to the one obtained from the long-term climatic temperature variations. So there seems to be an invariant quantity here independent of the scales in the temperature fluctuations across a wide range of time scales.

Fig. 4 shows the results about the vertical velocity fluctuations. Under the unstable stratification, the plot shows a good linearity (see solid dot), which indicates the existence of the hierarchical similarity. However, no linearity appears under the stable strati-

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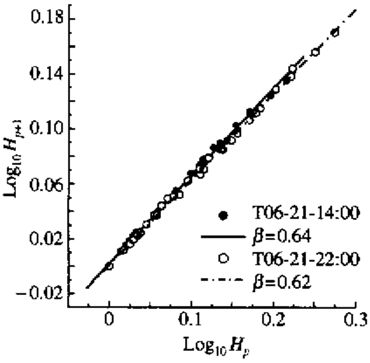

Fig. 3. Hierarchical similarity analysis of temperature fluctuations at stable stratification (hollow dot) and unstable stratification (solid dot).

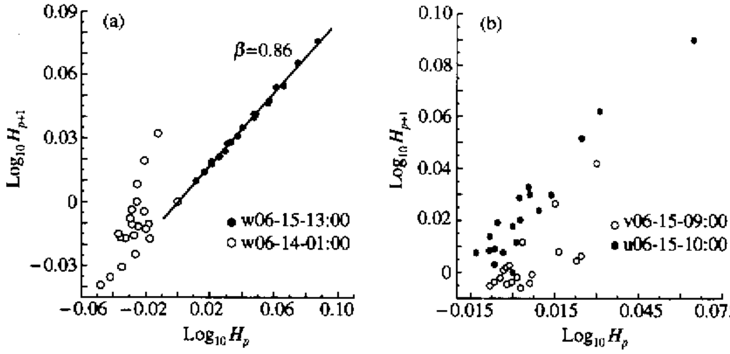

Fig. 4. Hierarchical similarity analysis of velocity fluctuations for vertical component (a) (stable stratification is indicated by hollow dot and unstable stratification by solid dot.) and horizontal components (b) (unstable stratification).

fication (see hollow dot in (a)). Furthermore, the results are different for horizontal velocity. The hierarchical similarity analysis under both the unstable and the stable stratification are similar and neither displays good linearity in the hierarchical similarity analysis plot. Fig. 4 (b) shows the results of horizontal velocity fluctuations under the unstable stratification; it is apparent that the points scatter everywhere and can hardly be fitted with a straight line. Our results are consistent with previous studies[28,29], indicating that the statistical properties of the vertical velocity fluctuations are different under stable and unstable stratifications. For example, Gallego et al. [28] found that the vertical velocity fluctuations have different correlation dimensions under stable and unstable stratifications, the value of correlation dimension of vertical velocity under the unstable stratification is smaller than that under the stable stratification. Our analysis here shows that the degree of the global organization is different.

The success of the hierarchical similarity analysis in describing experiment turbulence has widely been reported in the literature. The results obtained here demonstrate that some fluctuations with poor self-organization within long-time intervals (for example, horizontal velocity fluctuations) may not have the hierarchical similarity property. So, the hierarchical similarity analysis is an important method to measure the degree of self-organization in a complex system.

5 Conclusions

In this paper, we discuss a new method for characterizing the multi-scale fluctuation signal and apply it to the analysis of long-term climate variations and high-frequency turbulent variations in atmospheric boundary layers. We show that temperature variations in climate scales (years to hundreds of years) and in atmospheric turbulent scales (seconds to hours) both satisfy the hierarchical similarity law, which is an indication of the degree of the self-organization. We believe that this is because the basic mechanism for the generation of temperature fluctuations is the same across the wide range of scales, that is the convective instability. This kind of universality in the multi-scale temperature variations deserves further studies in the future. On the other hand, the analysis on the velocity fluctuations in an atmospheric turbulent boundary layer shows that fluctuations have various degrees of self-organization and different degree of intermittency for horizontal and vertical fluctuations, and the hierarchical similarity analysis provides a quantitative way to this measurement. Under the unstable stratifications, the instability mechanism is strong to produce turbulent fluctuations and the nonlinear interaction is stronger, so that the internal self-organization of the multi-scale fluctuations of vertical velocity component displays a strong linearity, while the horizontal component fluctuations display a wide variety of variations which may be un-related and form no hierarchy. It is worthwhile to point out that in both long-term climate variations and short-period turbulent fluctuations, the hierarchical similarity law finds its validity under certain conditions. In this case, they provide a way to quantify the system with, for example, the hierarchical similarity parameter. A more interesting issue to explore in the future is the universal mechanism for the measured invariant such as the hierarchical similarity parameter.

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