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Periodic Structures of Rossby Wave under Influence of Dissipation*

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Abstract *A simple barotropic potential vorticity equation with the influence of dissipation is applied to investigate the nonlinear Rossby wave in a shear flow in the tropical atmophere. By the reductive perturbation method, we derive the rotational KdV (rKdV for short) equation. And then, with the help of Jacobi elliptic functions, we obtain various periodic structures for these Rossby waves. It is shown that dissipation is very important for these periodic structures of rational form.*

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Key words: rKdV equation, elliptic equation, Jacobi elliptic function, periodic structure, rational form

1 Introduction

Planet-scale Rossby waves in the atmosphere and the ocean are the main disturb related to weather changes. Its nonlinear effect is an important factor to the high-low index shift in the circulation of the atmosphere and the ocean. So, the kinematic and dynamic characters of Rossby wave have attracted much attention of climatic scientists. As one of the factors affecting Rossby wave, dissipation is very important to the generation and development of Rossby wave. Lindzen[1] found that the break of gravity wave can lead to wave damping in the middle atmosphere. Geller[2] gave further reference that the break of gravity wave will give most of the friction dissipation in troposphere, and he described this process using Rayleigh friction parameter. Holton[3] also used Reyleigh friction to investigate Rossby waves in large scale model. Tan[4] suggested in his study that dissipation is always made the amplitude of envelope solitary wave attenuate. Yi[5] showed the dynamics behaviors of gravity wave in high atmosphere are decided by both nonlinear and dissipation. Thus, in the study of the movement of Rossby wave, we need to discuss some dissipative nonlinear influence to Rossby waves.

In this paper, a simple barotropic potential vorticity equation with the influence of dissipation is applied to investigate the nonlinear Rossby wave in a shear flow, where the reductive perturbation method is used to derive rotational KdV equation, i.e. rKdV equation. And then the basic structures of this rKdV equation are obtained by using the knowledge of Jacobi elliptic functions and elliptic equation. We have taken elliptic equation as an intermediate transformation tool to solve nonlinear wave equations,[6–8] and have obtained some periodic solutions and solitary wave solutions. However, still more researches are needed to do in order to find more solutions of different forms. In Ref. [9], we derived periodic solutions of rational forms, which are due to external forcing. All these studies may help to learn about coherent structures such as atmospheric blocking events, long lived eddies in the ocean or coherent structures in the Jovian atmosphere such as the Great Red Spot.

2 Derivation of rKdV Equation with Dissipation

We take the potential vorticity equation with the effect of dissipation as our governing equation,[10]

$$\frac{\partial}{\partial x}\nabla^2\psi + J(\psi,\nabla^2\psi) + \beta\frac{\partial\psi}{\partial x} = -\lambda_*\nabla^2\psi\,, \tag{1}$$

where $\beta > 0$ is the planartary-vorticity gradient and λ_* is dissipation coefficient, ψ is the stream function.

The total stream function is written as

$$\psi_* = -\int^y [\bar{u}(r) - c]\,\mathrm{d}r + \psi'\,, \tag{2}$$

then equation (1) can be rewritten as

$$\Big[\frac{\partial}{\partial t} + (\bar{u}-c)\frac{\partial}{\partial x}\Big]\nabla^2\psi' + J(\psi',\nabla^2\psi') + (\beta-\bar{u}'')\frac{\partial\psi'}{\partial x} = -\lambda_*(\nabla^2\psi - \bar{u}')\,. \tag{3}$$

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Next we take Gardner–Morikawa transformation,[11] i.e.

$$X = \varepsilon^{1/2}x, \quad T = \varepsilon^{3/2}t, \quad y = y\,, \tag{4}$$

where $\varepsilon = U/f_{0L}$ is the non-dimensional small parameter. And the perturbation stream function is expanded as

$$\psi' = \varepsilon\psi_1 + \varepsilon^2\psi_2 + \varepsilon^3\psi_3 + \cdots\,, \tag{5}$$

and we consider that the effect of dissipation is weak, i.e. $O(\lambda_*) \sim O(\rho\varepsilon^{3/2})$.

Then from Eq. (3), we have

$$O(\varepsilon): \quad \wp(\psi_1) = \rho\bar{u}'\,, \tag{6}$$

$$O(\varepsilon^2): \quad \wp(\psi_2) = -\frac{\partial\psi_1}{\partial X}\frac{\partial^3\psi_1}{\partial y^3} + \frac{\partial\psi_1}{\partial y}\frac{\partial^3\psi_1}{\partial X\partial y^2} - \frac{\partial}{\partial T}\Big(\frac{\partial^2\psi_1}{\partial y^2}\Big) - \bar{u}\frac{\partial^3\psi_1}{\partial X^3} - \rho\frac{\mathrm{d}^2G}{\mathrm{d}y^2} \tag{7}$$

with the operator $\wp$ defined as

$$\wp(\quad) = (\bar{u} - c)\frac{\partial^3}{\partial X\partial y^2} + (\beta - \bar{u}'')\frac{\partial}{\partial X}\,. \tag{8}$$

ψ_1 satisfies Eq. (6), whose solution can be taken as

$$\psi_1 = A(X,T)G(y) + dX\,, \tag{9}$$

where d is a constant and $G(y)$ satisfies

$$\frac{\mathrm{d}^2G}{\mathrm{d}y^2} + QG = 0 \tag{10}$$

with

$$Q \equiv \frac{\beta - \bar{u}''}{\bar{u} - c} \tag{11}$$

and boundary condition

$$G|_{y=+\infty} = 0, \quad G|_{y=-\infty} = 0\,. \tag{12}$$

Substituting the first-order solution (9) into the second-order expansion equation (7), we have

$$\wp(\psi_2) = -\frac{\mathrm{d}^2G}{\mathrm{d}y^2}\frac{\partial A}{\partial T} - (\bar{u} - c)G\frac{\partial^3 A}{\partial X^3} + A\frac{\partial A}{\partial X}\Big(\frac{\mathrm{d}G}{\mathrm{d}y}\frac{\mathrm{d}^2G}{\mathrm{d}y^2} - G\frac{\mathrm{d}^3G}{\mathrm{d}y^3}\Big) - \rho\frac{\mathrm{d}^2G}{\mathrm{d}y^2}A\,, \tag{13}$$

which can be multiplied by G and integrated with respect to y to reach the following rKdV equation:

$$\frac{\partial A}{\partial T} + \alpha A\frac{\partial A}{\partial X} + \mu\frac{\partial^3 A}{\partial X^3} + \gamma A = 0 \tag{14}$$

with

$$\begin{aligned} &\alpha = \frac{I_1}{I_0}, \quad \mu = \frac{I_2}{I_0}, \quad \gamma = \frac{I_3}{I_0}\,, \\ &I_0 = -\int_{-\infty}^{+\infty}\frac{G}{\bar{u} - c}\frac{\mathrm{d}^2G}{\mathrm{d}y^2}\mathrm{d}y\,, \\ &I_1 = \int_{-\infty}^{+\infty}\Big[\frac{G}{\bar{u} - c}\Big(\frac{\mathrm{d}G}{\mathrm{d}y}\frac{\mathrm{d}^2G}{\mathrm{d}y^2} - G\frac{\mathrm{d}^3G}{\mathrm{d}y^3}\Big)\Big]\mathrm{d}y\,, \\ &I_2 = -\int_{-\infty}^{+\infty}G^2\mathrm{d}y, \quad I_3 = -\int_{-\infty}^{+\infty}\Big(\frac{\rho G}{\bar{u} - c}\frac{\mathrm{d}^2G}{\mathrm{d}y^2}\Big)\mathrm{d}y\,. \end{aligned} \tag{15}$$

3 Solutions to the rKdV Equation

Following the method mentioned in Ref. [12], we can decompose $A(X,T)$ as

$$\begin{aligned} &A(X,T) = W(\sigma)V(\xi), \quad \sigma = T\,, \\ &\xi = \delta(T)[X - \theta(T)]\,, \end{aligned} \tag{16}$$

where $W(\sigma)$, $V(\xi)$, $\delta(T)$, and $\theta(T)$ are certain functions for their argument to be determined. Here the decomposition (16) is equivalent to introducing Lagrangian variables[13] $\{\xi,\sigma\}$, which are related to the Eulerian ones $\{X,T\}$ through formula Eq. (16). Substituting Eq. (16) into rKdV equation (14) results in

$$\Big(\frac{W_\sigma}{W} + \gamma\Big)V + \frac{\delta_\sigma}{\delta}\xi V_\xi + \delta\big(-\theta_\sigma V_\xi + \alpha W^2VV_\xi + \mu\delta^2V_{\xi\xi\xi}\big) = 0\,. \tag{17}$$

Here we put some restrictions on Eq. (17), i.e.,

$$-\theta_\sigma V_\xi + \alpha W^2VV_\xi + \mu\delta^2V_{\xi\xi\xi} = 0\,, \tag{18}$$

$$\Big(\frac{W_\sigma}{W} + \gamma\Big)V + \frac{\delta_\sigma}{\delta}\xi V_\xi = 0\,. \tag{19}$$

Equation (18) can be integrated once with respect to ξ to arrive at

$$-\theta_\sigma V + \alpha W^2\frac{V^2}{2} + \mu\delta^2V_{\xi\xi} = B(\sigma)\,. \tag{20}$$

Case 1 $B = \text{const.}$

In Eq. (20), the highest nonlinear term is balanced with the highest dispersion term, so we get

$$V'' = b_0 + b_1z + b_2z^2\,, \tag{21}$$

where $z(\xi)$ satisfies elliptic equation

$$z'^2 = a_0 + a_1z + a_2z^2 + a_3z^3 + a_4z^4\,. \tag{22}$$

So we consider two special cases.

(i) $a_1 = 0$ and $a_3 = 0$

In this case, we have

$$a_0 = \frac{(\theta'(T)/\alpha W^2(\sigma))b_0 - (\alpha W^2(\sigma)/2\mu\delta^2(\sigma))b_0^2 + B/\mu\delta^2(\sigma)}{2b_2},$$

$$a_2 = \frac{\theta'(T)}{4\alpha W^2(\sigma)} - \frac{\alpha W^2(\sigma)}{4\mu\delta^2(\sigma)} b_0, \tag{23}$$

$$b_2 = -\frac{12\mu\delta^2(\sigma)}{\alpha W^2(\sigma)} a_4, \quad b_0 = 0, \quad b_1 = 0, \tag{24}$$

$$W(\sigma) = \pm\sqrt{-\frac{24a_0a_4\mu^2}{B\alpha}}\,\delta^2(\sigma). \tag{25}$$

From Refs. [6] $\sim$ [9], [12], and [14] we know that there are many kinds of solutions to elliptic equation (22). For example, when $a_0 = 1$, $a_1 = -(1+m^2)$, and $a_2 = m^2$, then $a_1^2 - 3a_0a_2 = 1 - m^2 + m^4 > 0$ and the solution is

$$z_1 = \mathrm{sn}(\xi, m), \tag{26}$$

where $0 \le m \le 1$ is called modulus of Jacobi elliptic functions and $\mathrm{sn}(\xi, m)$ is Jacobi elliptic sine function, see Refs. [12] and [15] $\sim$ [18]. Thus $V(\xi)$ can be determined as

$$V_1 = -\frac{12\mu\delta^2(\sigma)m^2}{\alpha W^2(\sigma)}\mathrm{sn}^2(\xi, m). \tag{27}$$

This result for $V(\xi)$ can be used to solve Eq. (19), which is firstly integrated with respect to ξ from $-l$ to l with l positive constant of a bounded value,

$$\Big(\frac{W_\sigma}{W} + \gamma - \frac{\delta_\sigma}{2\delta}\Big)\int_{-l}^{l} V^2 \mathrm{d}\xi + \frac{\delta_\sigma}{2\delta}\int_{-l}^{l}\mathrm{d}(V^2\xi) = 0. \tag{28}$$

Owing to the fact

$$\int_{-l}^{l} V_1^2 \mathrm{d}\xi = \frac{B^2}{4\mu\delta^2}\Big[\frac{(p^2+1)E(\xi)}{2p^2(p^2-1)(m^2-p^2)} + \frac{\xi}{2p^4(p^2-1)} + 2p^2(p^2-1)\pi(\xi,p^2)\Big]\Big|_{-l}^{l}, \tag{29}$$

where $E(\xi)$ and $\pi(\xi, p^2)$ are Legendre's incomplete elliptic integrals of the second kind and the third kind, respectively, with

$$p = \frac{4(1+m^2)\mu\delta^2 + \theta_\sigma}{12\mu\delta^2} \tag{30}$$

being bounded (actually, if V takes other elliptic functions, similar results can be reached, too), we take assumption that

$$\int_{-l}^{l}\mathrm{d}(V^2\xi) = s_1\int_{-l}^{l} V_1^2\mathrm{d}\xi, \tag{31}$$

where s_1 is a constant, its value depending on the specific form of V. Then we have

$$\frac{W_\sigma}{W} + \gamma + (s_1 - 1)\frac{\delta_\sigma}{2\delta} = 0. \tag{32}$$

From Eq. (25), one has

$$\frac{W_\sigma}{W} = \frac{2\delta_\sigma}{\delta}. \tag{33}$$

Combining Eq. (32) with Eq. (33) leads to

$$W = k_0\,\mathrm{e}^{[4\gamma/(3+s_1)]T},$$

$$\theta = -\frac{\delta_0(3+s_1)\alpha a_2 k_0^2}{2\gamma}\,\mathrm{e}^{[8\gamma/(3+s_1)]T}, \tag{34}$$

$$\delta = \pm\Big(\frac{B\alpha k_0^2\,\mathrm{e}^{[-8\gamma/(3+s_1)]T}}{24a_0a_4\mu^2}\Big)^{1/4}. \tag{35}$$

So $A(X,T)$ in Eq. (16) can be written as

$$A_1 = -\frac{12\mu\delta^2(T)m^2k_0}{\alpha W^2(T)}\,\mathrm{e}^{-[4\gamma/(3+s_1)]T}\mathrm{sn}^2(\xi, m), \tag{36}$$

with δ and θ given by Eqs. (34) and (35), and $\xi = \delta(T)[X - \theta(T)]$.

Similarly, when a_0, a_1 and a_2 take other sets of values, there are different forms of solutions for V, which will results in different s_1. This indicates different W, δ, θ and $\xi = \delta(T)[X - \theta(T)]$. For example, there are the following solutions.

(a) If $a_0 = 1 - m^2$, $a_2 = 2m^2 - 1$, and $a_4 = -m^2$, then the solution is

$$z_2 = \mathrm{cn}(\xi, m),$$

$$V_2 = \frac{12\mu\delta^2(T)m^2}{\alpha W^2(T)}\mathrm{cn}^2(\xi, m), \tag{37}$$

where $\mathrm{cn}(\xi, m)$ is Jacobi elliptic cosine function,[12,15–18] and

$$A_2 = \frac{12\mu\delta^2(T)m^2k_0}{\alpha W^2(T)}\,\mathrm{e}^{-[4\gamma/(3+s_1)]T}\mathrm{cn}^2(\xi, m). \tag{38}$$

(b) If $a_0 = m^2 - 1$, $a_2 = 2 - m^2$, and $a_4 = -1$, then the solution is

$$z_3 = \mathrm{dn}(\xi, m)\,,$$

$$V_3 = -\frac{12\mu\delta^2(T)}{\alpha W^2(T)}\mathrm{dn}^2(\xi, m)\,, \tag{39}$$

where $\mathrm{dn}(\xi, m)$ is Jacobi elliptic function of the third kind,[12,15−18] and

$$A_3 = \frac{12\mu\delta^2(T)k_0}{\alpha W^2(T)}\,\mathrm{e}^{-[4\gamma/(3+s_1)]T}\mathrm{dn}^2(\xi, m)\,. \tag{40}$$

(c) If $a_0 = m^2$, $a_2 = -(1 + m^2)$, and $a_4 = 1$, then the solution is

$$z_4 = \mathrm{ns}(\xi, m) \equiv \frac{1}{\mathrm{sn}(\xi, m)}\,,$$

$$V_4 = -\frac{12\mu\delta^2(T)}{\alpha W^2(T)}\mathrm{ns}^2(\xi, m)\,, \tag{41}$$

and

$$A_4 = -\frac{12\mu\delta^2(T)m^2k_0}{\alpha W^2(T)}\,\mathrm{e}^{-[4\gamma/(3+s_1)]T}\mathrm{ns}^2(\xi, m)\,. \tag{42}$$

(d) If $a_0 = -m^2$, $a_2 = 2m^2 - 1$, and $a_4 = 1 - m^2$, then the solution is

$$z_5 = \mathrm{nc}(\xi, m) \equiv \frac{1}{\mathrm{cn}(\xi, m)}\,,$$

$$V_5 = -\frac{12\mu\delta^2(T)}{\alpha W^2(T)}\mathrm{nc}^2(\xi, m)\,, \tag{43}$$

and

$$A_5 = -\frac{12\mu\delta^2(T)m^2k_0}{\alpha W^2(T)}\,\mathrm{e}^{-[4\gamma/(3+s_1)]T}\mathrm{nc}^2(\xi, m)\,. \tag{44}$$

(e) If $a_0 = -1$, $a_2 = 2 - m^2$, and $a_4 = m^2 - 1$, then the solution is

$$z_6 = \mathrm{nd}(\xi, m) \equiv \frac{1}{\mathrm{dn}(\xi, m)}\,,$$

$$V_6 = -\frac{12\mu\delta^2(T)}{\alpha W^2(T)}\mathrm{nd}^2(\xi, m)\,, \tag{45}$$

and

$$A_6 = -\frac{12\mu\delta^2(T)m^2k_0}{\alpha W^2(T)}\,\mathrm{e}^{-[4\gamma/(3+s_1)]T}\mathrm{nd}^2(\xi, m)\,. \tag{46}$$

(ii) $a_0 = 0$ and $a_1 = 0$

In this case, we have

$$b_0 = 1, \quad b_1 = -\frac{6\mu\delta^2(T)}{\alpha W^2(T)}a_3\,,$$

$$b_2 = -\frac{12\mu\delta^2(T)}{\alpha W^2(T)}a_4\,,$$

$$a_2 = \frac{\theta'(T)}{\mu\delta^2(T)} - \frac{\alpha W^2(T)}{\mu\delta^2(T)}\,. \tag{47}$$

Equation (22) can be rewritten as

$$z'' = a_2z^2 + a_3z^3 + a_4z^4\,, \tag{48}$$

and its solution and the solutions of $A(X, T)$ are

$$z = -\frac{a_2a_3\mathrm{sech}^2(\sqrt{a_2}/2)\xi}{a_3^2 - a_2a_4(1 - \tanh(\sqrt{a_2}/2)\xi)^2}\,, \tag{49}$$

$$A(X,T) = k_0\,\mathrm{e}^{-[4\gamma/(3+e)]T}\Big(1 + \frac{6\mu\delta^2(T)a_2a_3^2\mathrm{sech}^2(\sqrt{a_2}/2)\xi}{[a_3^2 - a_2a_4(1 - \tanh\sqrt{a_2}\xi)^2]\alpha W^2(T)}\Big) - \Big[\frac{a_2a_3\mathrm{sech}^2(a_2/2)\xi}{a_3^2 - a_2a_4(1 - \tanh(\sqrt{a_2}/2)\xi)^2}\Big]^2\frac{12\mu\delta^2(T)}{\alpha W^2(T)}a_4\,, \tag{50}$$

and

$$z = \frac{2a_3\mathrm{sech}\sqrt{a_2}\xi}{\sqrt{a_3^2 - a_2a_4} - a_3\mathrm{sech}\sqrt{a_2}\,\xi}\,, \tag{51}$$

$$A(X,T) = k_0\,\mathrm{e}^{-[4\gamma/(3+s_1)]T}\Big[1 + \frac{12a_2a_3\mu\delta^2(T)\mathrm{sech}\sqrt{a_2}\xi}{\alpha W^2(T)(\sqrt{a_3^2a_2a_4} - a_3\mathrm{sech}\sqrt{a_2}\,\xi)}\Big] - \Big[\frac{2a_2\mathrm{sech}\sqrt{a_2}\,\xi}{\sqrt{a_3^2 - a_2a_4} - a_3\mathrm{sech}\sqrt{a_2}\,\xi}\Big]^2\frac{12\mu\delta^2(T)}{\alpha W^2(T)}a_4\,. \tag{52}$$

Case 2 $B = B(T)$

In this case,we have

$$b_0 = 0, \quad b_1 = 0, \quad b_2 = -\frac{12\mu\delta^2(T)}{\alpha W^2(T)}a_4\,, \tag{53}$$

and

$$\frac{W'(T)}{W(T)} = 2\frac{\delta'(T)}{\delta(T)} - \frac{B'(T)}{2B(T)}\,, \tag{54}$$

so according to Eq. (17), equation (54) can be rewritten as

$$\frac{(3+s_1)W'(T)}{4W(T)}+\gamma+\frac{B'(T)}{8B(T)}=0\,. \tag{55}$$

And using Eqs. (54) and (55), we have some solutions:

$$W(T)=k_1\,\mathrm{e}^{-[4\gamma/(s_1+3)]T}+k_1B^{1/2(s_1+3)}(T)\,, \tag{56}$$

$$\delta(T)=\pm\frac{B(T)\alpha k_1}{-24a_0a_4\mu^2}\big[\,\mathrm{e}^{-[8\gamma/(s_1+3)]T}+B^{1/(s_1+3)}(T)\big]^{1/4}\,, \tag{57}$$

$$\theta'(T)=4a_2\alpha k_1^2\big[\,\mathrm{e}^{-[4\gamma/(s_1+3)]T}+B^{1/2(s_1+3)}(T)\big]\,. \tag{58}$$

z still satisfies Eq. (22). So, similarly we can obtain solutions just similar to solutions from A_1 to A_6:

(i) If $a_0=1-m^2$, $a_2=2m^2-1$, and $a_4=-m^2$, then the solution is

$$A_1'=-\frac{12\mu\delta^2(T)m^2k_1}{\alpha W^2(T)}\,\mathrm{e}^{-[4\gamma/(3+s_1)/T}\big[\,\mathrm{e}^{-[8\gamma/(s_1+3)/T}+B^{1/(s_1+3)}(T)]\mathrm{sn}^2(\xi,m)\,. \tag{59}$$

(ii) If $a_0=1-m^2$, $a_2=2m^2-1$, and $a_4=-m^2$, then the solution is

$$A_2'=\frac{12\mu\delta^2(T)m^2k_1}{\alpha W^2(T)}\,\mathrm{e}^{-[4\gamma(3+s_1)]T}\big[\,\mathrm{e}^{-[8\gamma/(s_1+3)T]}+B^{1/(s_1+3)}(T)\big]\mathrm{cn}^2(\xi,m)\,. \tag{60}$$

(iii) If $a_0=m^2-1$, $a_2=2-m^2$, and $a_4=-1$, then the solution is

$$A_3'=\frac{12\mu\delta^2(T)k_1}{\alpha W^2(T)}\,\mathrm{e}^{-[4\gamma/(3+s_1)]T}\big[\,\mathrm{e}^{-[8\gamma/(s_1+3)]T}+B^{1/(s_1+3)}(T)\big]\mathrm{dn}^2(\xi,m)\,. \tag{61}$$

(iv) If $a_0=m^2$, $a_2=-(1+m^2)$, and $a_4=1$, then the solution is

$$A_4'=-\frac{12\mu\delta^2(T)k_1}{\alpha W^2(T)}\,\mathrm{e}^{-[4\gamma/(3+s_1)]T}\big[e^{-[8\gamma/(s_1+3)]T}+B^{1/(s_1+3)}(T)\big]\mathrm{ns}^2(\xi,m)\,. \tag{62}$$

(v) If $a_0=-m^2$, $a_2=2m^2-1$, and $a_4=1-m^2$, then the solution is

$$A_5'=-\frac{12\mu\delta^2(T)k_1}{\alpha W^2(T)}\,\mathrm{e}^{-[4\gamma/(3+s_1)]T}\big[\,\mathrm{e}^{-[8\gamma/(s_1+3)]T}+B^{1/(s_1+3)}(T)]\mathrm{cs}^2(\xi,m)\,. \tag{63}$$

(vi) If $a_0=-1$, $a_2=2-m^2$, and $a_4=m^2-1$, then the solution is

$$A_6'=-\frac{12\mu\delta^2(T)k_1}{\alpha W^2(T)}\,\mathrm{e}^{-[4\gamma/(3+s_1)]T}\big[\,\mathrm{e}^{-[8\gamma/(s_1+3)]T}+B^{1/(s_1+3)}(T)\big]\mathrm{ds}^2(\xi,m)\,. \tag{64}$$

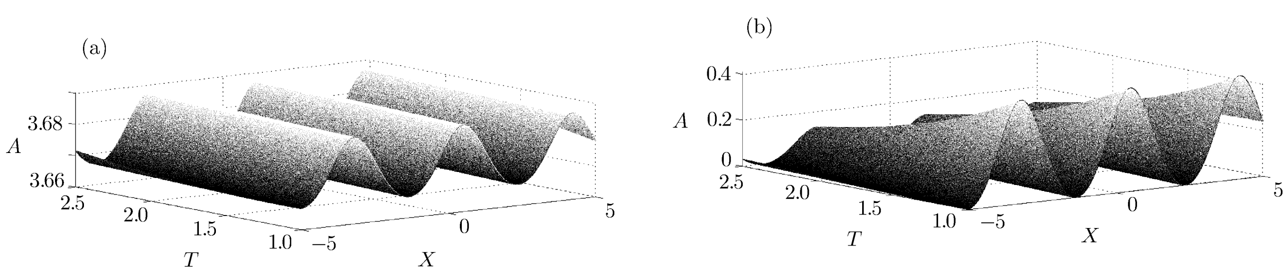

Fig. 1 The graphical representations of KdV equation (a) and rKdV equation (b).

We know that the solution of KdV equation is

$$A=c+4(1-2m^2)\mu k^2+12m^2\mu\,\mathrm{cn}^2[\xi,m]\,, \tag{65}$$

where c is wave velocity and k is wave number. In order to find the difference between the solutions of rKdV equation and KdV equation, we plot Fig. 1 to show one solution of rKdV equation, Eq. (38), and the solution of KdV equation to compare them. We choose $\mu=1/12$, $m=0.5$, $c=1$, $k=4$, $k_0=4$, and $\delta^2=W^2$.

From Fig. 1, we can see that the amplitude of rKdV equation decreases with T increasing, but that of KdV equation keeps constant.

4 Conclusion

A simple barotropic potential vorticity equation with the influence of dissipation is applied to investigate the nonlinear Rossby wave in a shear flow. By the reductive perturbation method, we derived the rKdV equation. And then we obtain various periodic structures for these equational Rossby waves with the help of Jacobi elliptic functions. So we know that dissipation is of great importance for these periodic structures of rational form. Of course, these periodic structures also contain solitons and solitary waves, which can be used in explaining different practical oceanic wave phenomena. This needs more further research.

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