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A brief description to different multi-fractal behaviors of daily wind speed records over China

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ABSTRACT

Scaling behaviors of the long daily wind speed records of four selected weather stations over China were analyzed by using Multi-Fractal Detrended Fluctuation Analysis (MF-DFA). The results indicated that all these four stations are characterized by long-range power-law correlations, but MF-DFA results showed non-universal multi-fractal behaviors over China. We fitted generalized Hurst exponent $h(q)$ via a modified generalized binomial multiplicative cascade model, and different widths of the multi-fractal spectrum are estimated.

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1. Introduction

The long-range power law correlations of many physiological, physical signals and meteorological variables have been a well known phenomenon [1–5]. Such a characterization has led to data classification [6] and also to persistence in the data [1–5]. The existence of this power law scaling in the statistics used to describe the patterns of temporally fluctuating systems indicates the presence of a fractal behavior. Meteorological variables, as indicators of fluctuations in general atmospheric circulation and climate system, show the fractal phenomenon by taking the self-similar structure over a wide range of time scales.

Quantitatively, the long-range correlations indicate that meteorological variables of very distant time interval are correlated with each other, which can be captured by the auto-correlation function (ACF) or equivalently, the power spectrum [7,8]. The power spectrum $S(f)$ exhibits a power-law decay of the form $S(f) \sim 1/f^\beta$, while the ACF $C(s) \sim s^{-\gamma}$ decays slowly to zero. Such features are characteristic of statistically self similar processes with well defined power-law correlations. Hurst exponents are also used to

quantify long-term correlations in plasma turbulence [9], finance [10], and network traffic [11].

But there still exist some limitations of traditional techniques such as spectral analysis and Hurst estimators, since meteorological variable is governed by a variety of physical processes and exhibits fluctuations at various spatial and temporal scales, time series are susceptible to non-stationary effects, so we explore an alternate method known as Detrended Fluctuation Analysis (DFA) to qualify the scaling exponents [6]. Recently, the DFA method has been successfully applied to study scaling properties of meteorological data such as temperature [12], relative humidity [13,14], high frequency sampled wind speed [15,16], and cloud breaking [17].

Scaling exponent approaches mentioned above can be merely used to estimate a single scaling exponent from a time series. However, these techniques are insufficient to fully characterize a complex process which may be governed by more than one scaling exponent. Therefore, aforementioned methods are appropriate only for the estimation of mono-fractal signals which characterized by a single scaling exponent but cannot be used to capture the characteristics of multi-fractal signals characterized by more than one exponent completely [18].

To further characterize the daily wind speed record, we extend the study by using a fairly robust and powerful method called Multi-Fractal Detrended Fluctuation Analysis (MF-DFA) [19]. This

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method provides a systematic means to identify and more importantly quantify the multiple scaling exponents in the data [19], and it has been applied to study multi-fractal properties of many fields [20–23]. Some early studies also extract multi-fractal properties of high frequency sampled wind speed by using MF-DFA [15,16,24].

We study the multi-fractal properties of daily wind speed records from four representative stations over China during 1951–2000, the scaling exponents of the data are estimated under the assumption of a binomial multiplicative cascade model. This Letter is organized as follows. In Section 2, we briefly describe the method MF-DFA and the wind speed data used in this Letter. In Section 3, the results of MF-DFA and the calculation of the multi-fractal spectrum are provided. And the conclusions are summarized and discussed in Section 4.

2. Methodology and data

2.1. Methodology outline

In this Letter, we consider a fluctuating time series x_i ($i = 1, \dots, N$) of daily average wind speed with equidistant time $i\Delta t$, and the seasonal cycle is removed.

In the MF-DFA procedure [19], we build the profile time series by integrating the anomaly of wind speed series with removed seasonal cycle firstly,

$$Y(i) = \sum_{k=1}^i (x_k - \bar{x}). \quad (1)$$

This profile time series preserves variability characteristics of the origin time series [19] but degrades the noise level by removing non-stationary effects.

In order to perform the fluctuation analysis, profile series is divided into two non-overlapping $N_s = N/s$ segments of equal length s for data integrity since N is not always an integer multiple of s [25]. Then in every segment ν , k th order polynomial is applied to fit P_ν^k to eliminate the local trend, the order of the polynomial fixes the order of the DFA ($k = i$, DFA(i)). In the end, we get the detrended time series

$$Y_s(i) = Y(i) - P_k^\nu, \quad (2)$$

and corresponding square fluctuation $F_s^2(\nu, s)$, which defined as the variance of $Y_s(i)$, is calculated in each segment

$$F_s^2(\nu) = \langle Y_s^2(i) \rangle = \frac{1}{s} \sum_{i=1}^s Y_s^2[(\nu - 1) + i]. \quad (3)$$

Finally, the root-mean-square deviation gives the DFA fluctuation function by averaging over all segments

$$F(s) = \left[\frac{1}{2N_s} \sum_{k=1}^{2N_s} F_s^2(k)^{q/2} \right]^{1/q}. \quad (4)$$

For long-range power-law correlated data, $F_q(s)$ increases asymptotically with s and follows the power-law: $F_q(s) \sim s^{h(q)}$, where the exponent $h(q)$ describes the scaling behavior of the q -th order fluctuation function. This scaling exponent displays self-similar fractal behavior over a broad range of time scales. For positive values of q , $h(q)$ describes the scaling behavior of segments with large fluctuations while for negative values of q , $h(q)$ describes the scaling behavior of segments with small fluctuations [19]. For stationary time series, the exponent $h(2)$ is identical to the well-defined Hurst exponent. Thus we call the exponent $h(q)$ the generalized Hurst exponent [19]. For mono-fractal time series

Table 1

Detailed information of four selected stations.

Station (No.)	Longitude (E)	Latitude (N)	Height (m)	Series length (days)
50915 Dongwuzhumuqinqi	116.58	45.31	838.9	1955.11.1–2005.12.31
51156 Hebukesai	85.43	46.47	1291.6	1953.7.1–2005.12.31
56739 Tengchong	98.30	25.01	1654.6	1951.1.1–2005.12.31
59632 Qinzhou	108.37	21.57	4.5	1952.10.1–2005.12.31

characterized by a single exponent over all time scales, $h(q)$ is independent of q , whereas for multi-fractal time series, $h(q)$ varies with q . This dependence is considered to be a characteristic of multi-fractal process [19].

Traditional way to characterize a multi-fractal time series is to calculate the multi-fractal spectrum $f(\alpha)$ [7]. This singularity spectrum can be related to $h(q)$ via a Legendre transform [7,19]

$$\alpha = h(q) + q \frac{dh(q)}{dq} \quad (5)$$

and

$$f(\alpha) = q(\alpha - h(q)) + 1, \quad (6)$$

where α is the singularity strength or Holder exponent which characterizes the singularities (cusps, ridges, chirps, spikes) in a process $X(t)$ at time t [26]. The multi-fractal spectrum $f(\alpha)$ denotes the singularity content of the process, i.e. the dimension of the subset of the series that is characterized by α .

2.2. Data sets

The records used in this Letter were obtained from a high-quality daily surface climatic data set, processed by Chinese National Meteorological Information Center (NMIC), of 194 Chinese meteorological stations taking part in international exchange. This data set has been used in many studies to analysis climate change over China in recent years [27,28]. We selected four representative weather stations to study power-law scaling properties for daily wind speed records. The detailed information about the four stations is provided in Table 1. Since we don't have enough data of wind direction in the same period, we didn't involve wind direction in this study.

3. Results

A number of studies in recent years reveal that surface wind speed records show self-similarity properties or, more generally, fractal behavior manifested in a power-law scaling analysis. In [29] and [30], the strength and direction of the daily wind records over more than 100 years are considered as a two-dimensional random walk via numerical simulations and numerical analysis, they show verification of persistence law governing wind speed records with an exponent of 1 over short time scales. By application of MF-DFA to eleven surface wind speed series over a span of 24 hours, Govindan and Kantz [15] studied the correlations in wind speed data with a fluctuation exponent of $\alpha \sim 1.1$ along with a broad multi-fractal spectrum. The other examples given by several studies focus on wind speed data over North Dakota of America. In [24] and [31], a group of weather stations' records also admitted similar broad multi-fractal spectrum over a regional area, under the assumption of a binomial multiplicative cascade model.

Since high frequency sampled records are complicated by small scale effects resulted from topography [32], they usually fail to capture memory effect of large scale atmospheric circulation. Do long period daily wind speed variations show different scaling characteristics? To answer this question, by considering daily wind

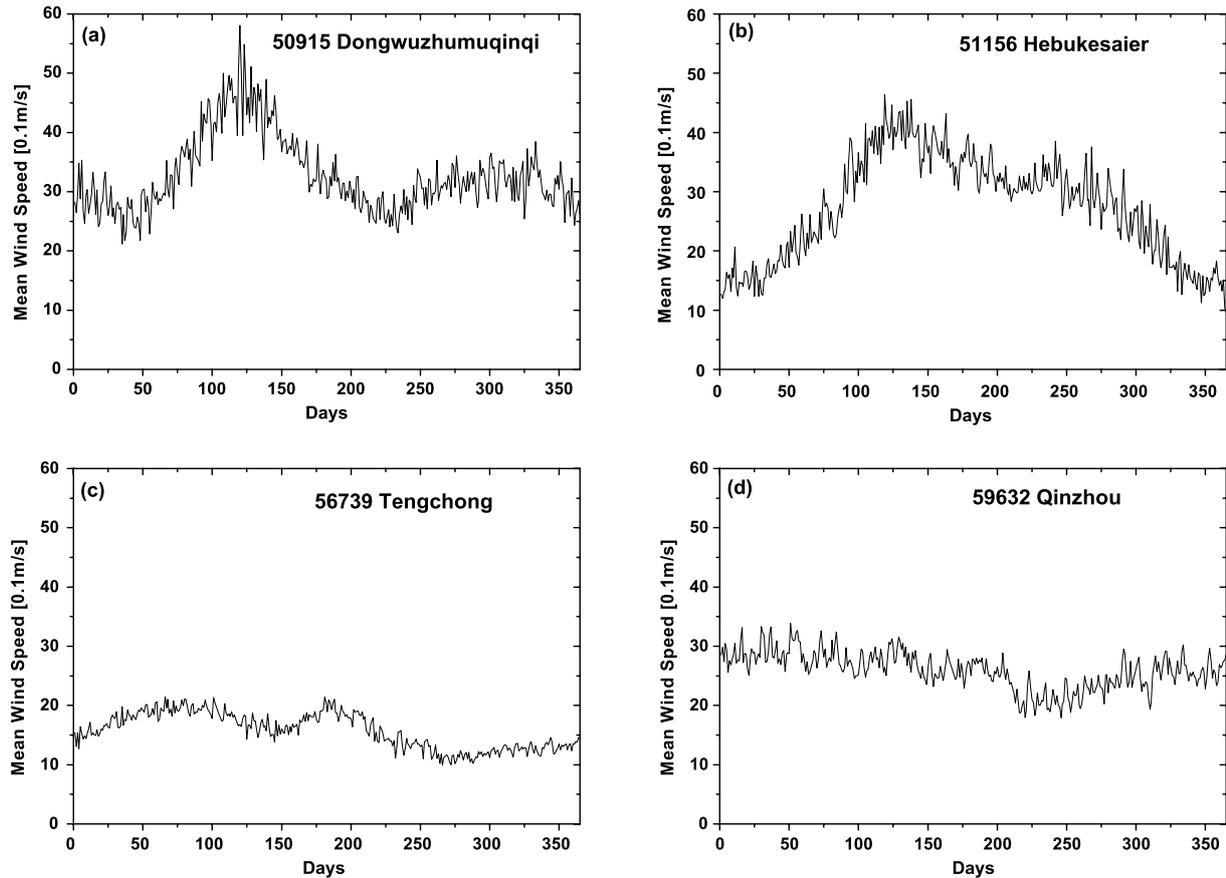


Fig. 1. Annual cycle of four representative weather stations used in this study.

speed records with relatively long time periods about fifty years, we extend analysis of long-range correlations up to time scale of several thousand of days.

Furthermore, to study local or some regional measurements can estimate temporal correlations only, but fails to reveal temporal-spatial diversities of scaling properties. A direct motivation for the present work is to seek the fractal phenomenon for daily wind speed over a broad geographic distributions. Our study involves measurements at four selected locations representing the most typical climate characteristics over China, which include mid-latitude frontal cyclone, disturbance in the westerlies, low-latitude plateau detouring flow and South China Sea monsoon. Dongwuzhumuqinqi is a city located in mid-latitude and North-east China where the climate character is cold, with strong wind in wintertime and springtime; and the Mongolian high pressure is an important influence factor. The strong wind characteristics in spring and fall exist in Hebukesaiier where belongs to the continental dry climate belt. The characteristics of climate of Tengchong in low latitude with plateau area exhibiting little change in a whole year, and monitored wind speed is in good agreement with this pattern. The city of Qinzhou has a sub-tropical monsoonal climate with long summers and warm winters, plentiful rainfall, and frequent rainstorms and typhoons in summer and autumn.

The seasonal cycle of daily wind speed for four selected stations are shown in Fig. 1, which appears as background climatic mode of four selected stations.

The log-log plots of the fluctuation function $F_q(s)$ versus s for the daily mean wind speed records at four weather stations are shown in Fig. 2, by using fourth order polynomial detrending (MF-DFA4) to eliminate cubic trends in the origin data. For stations Qinzhou and Hebukesaiier, the slopes $h(q)$ decrease as the mo-

ment is increased from negative to positive values, the curve shifts steeper and steeper vertically for clarity which indicates the multi-fractal behavior in wind speed records. But the curves' group not takes similar shape in four different sites, for stations Dongwuzhumuqinqi and Tengchong the slope of the curve changes little from the top to the bottom.

Quantitatively, the scaling exponent $h(q)$ can be obtained by analyzing log-log plots of $F_q(s)$ versus s for each q . Time scale in the range [100–1000 days] are selected for its consistency with a given q . The generalized Hurst exponents $h(q)$ estimated via the MF-DFA4 procedure with varying moments ($q = -6, -5, -4, -3, -2, -1, 1, 2, 3, 4, 5, 6$) are shown in Fig. 3. Daily wind speed time series of stations Qinzhou and Hebukesaiier show multi-fractal behavior as indicated by the strong q dependence of the generalized Hurst exponents. While the value of $h(q)$ seems to be constant for all q in stations Tengchong and Dongwuzhumuqinqi, possibly indicating the phenomena of the mono-fractals, and average scaling exponent of Tengchong is higher than that of station Dongwuzhumuqinqi.

Generally, the observed multi-fractal scaling behavior is due to a fatness of the probability density function (PDF) of the time series or different long-range correlations in small and large scale fluctuations. Randomized shuffled surrogates are applied to origin time series, by comparing MF-DFA4 results of origin data to shuffled series, we can distinguish multi-fractality due to long-range correlations from multi-fractality due to a broad probability density function. Since all long-range correlations are destroyed by the shuffling procedure, if the multi-fractality belongs only to the long-range correlation, we should find $h_{\text{shuf}}(q) = 0.5$. If both types of multi-fractality are present, the shuffled series will show weaker multi-fractality than the original series. As shown in Fig. 4, it is

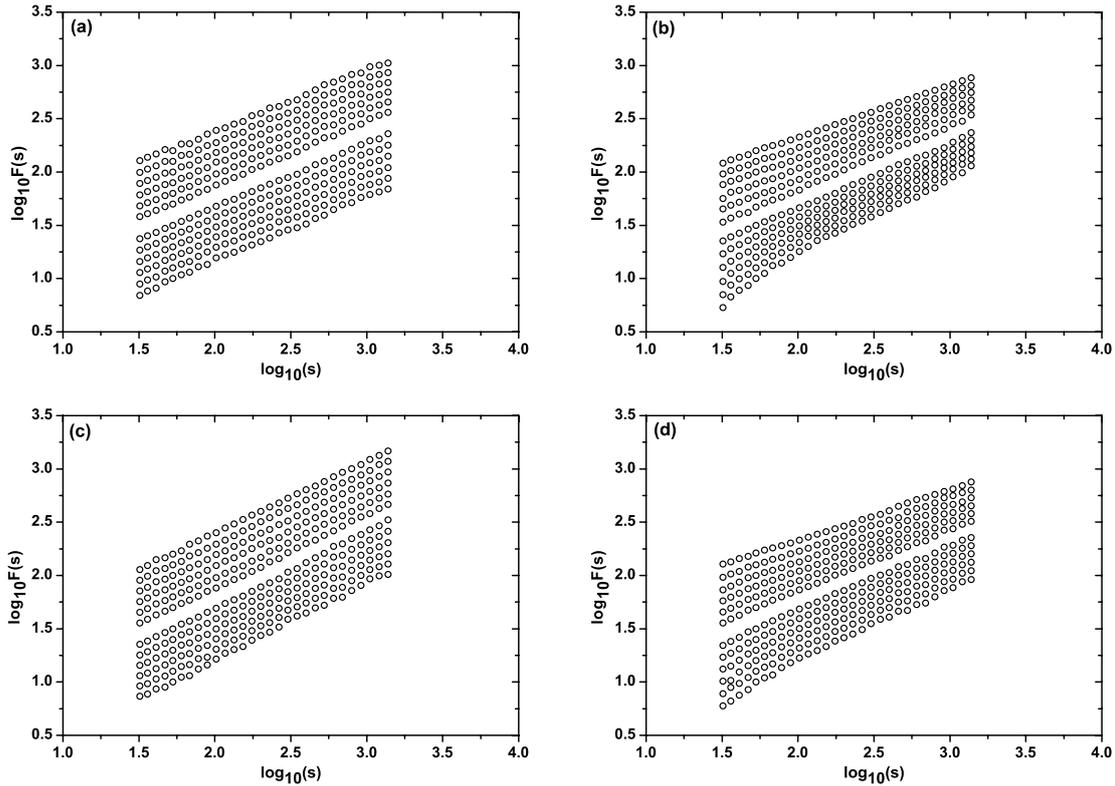


Fig. 2. Log-log plots of the MF-DFA4 curves of daily wind speed for (a) Dongwuzhumuqinqi, (b) Hebukeisaier, (c) Tengchong and (d) Qinzhou. From the top to the bottom curves correspond to different q (from $q = -6$ to $q = 6$) and are shifted vertically for clarity.

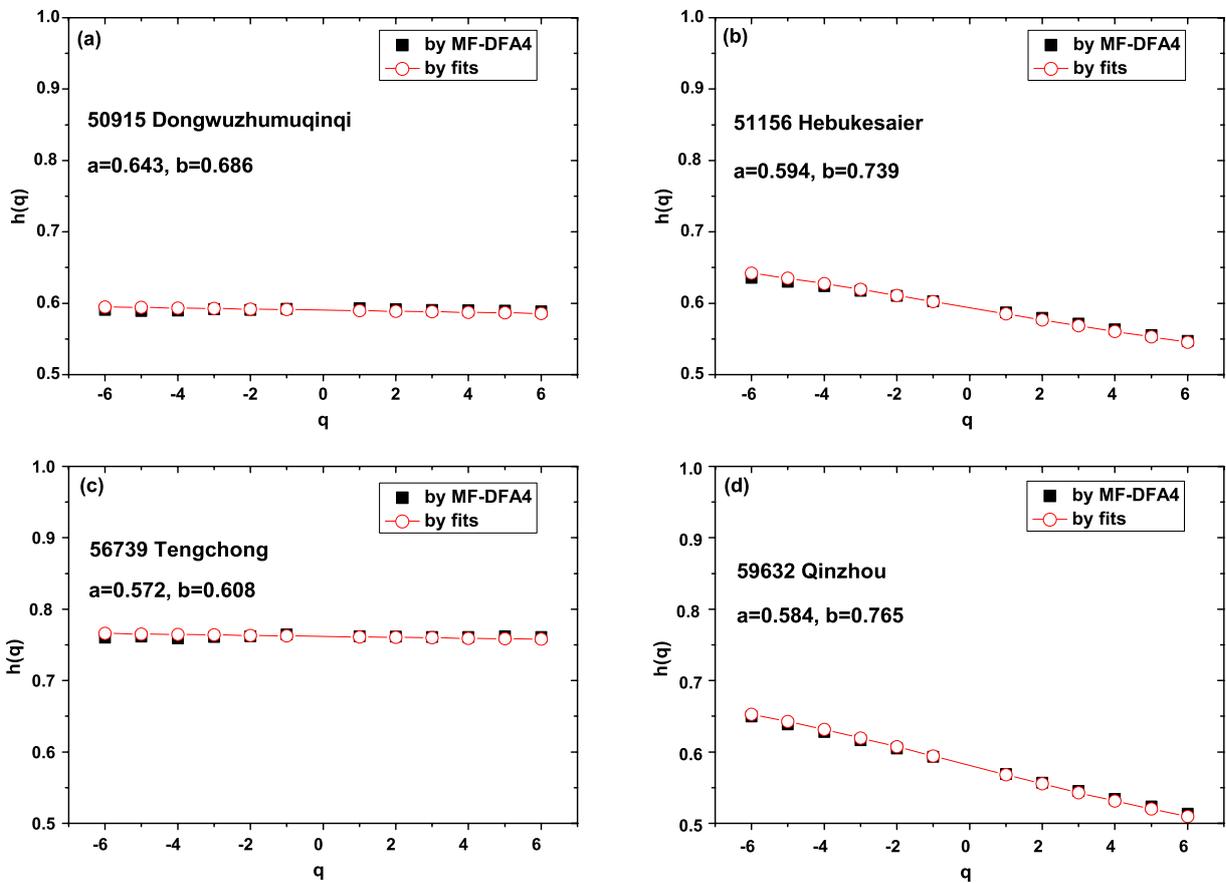


Fig. 3. (a) $h(q)$ versus q plots for Dongwuzhumuqinqi. Solid squares: estimated from MF-DFA4 results in Fig. 2(a); open circles: obtained by fits of the two-parameter binomial model. The analog to (a) but for Hebukeisaier (b), Tengchong (c) and Qinzhou (d).

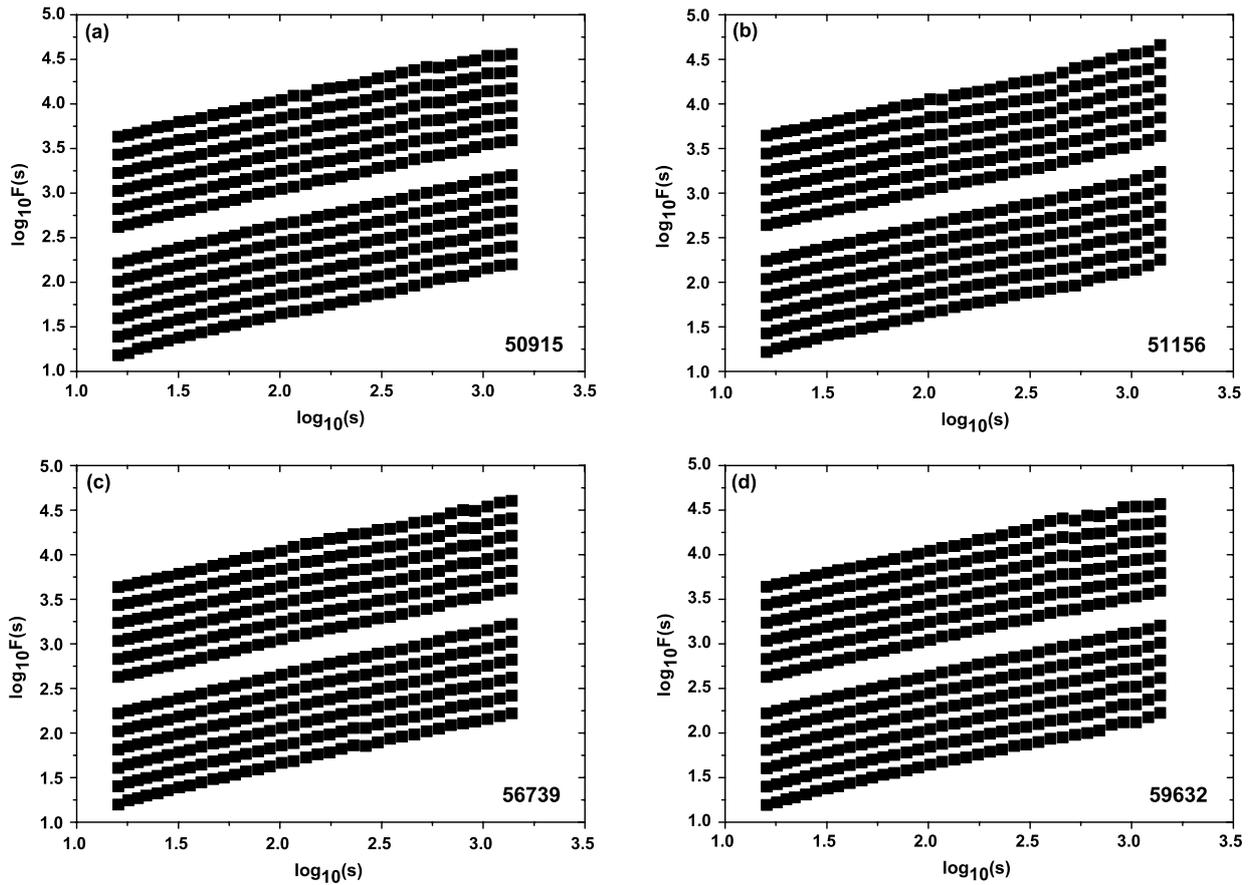


Fig. 4. Log-log plots of the MF-DFA4 curves of daily wind speed for the shuffled data of Dongwuzhumuqinqi (a), Hebukesaiier (b), Tengchong (c) and Qinzhou (d).

obvious that the multi-fractality of four selected stations is due to long-range correlations, since $h_{shuf}(q) = 0.5$ characterizes a loss of correlation.

The generalized Hurst exponent $h(q)$ can be estimated according to the formula

$$h(q) = \frac{1}{q} - \frac{\ln(a^q + b^q)}{q \ln 2}, \quad (7)$$

assuming a binomial multiplicative cascade model [7], which has served as one of the standard paradigms to describe multi-fractal scaling behavior (see Ref. [19] for details). Finally, the width of the multi-fractal spectrum $f(\alpha)$ at $f = 0$ given by

$$\Delta\alpha = h(-\infty) - h(\infty) = \frac{\ln(b) - \ln(a)}{\ln 2}, \quad (8)$$

is computed to characterize the different strength of multi-fractality for four selected stations over China. Fig. 3 illustrates the generalized Hurst exponents of the daily mean wind speed and nonlinear least-squares fits for these stations. We find that for all of these stations the generalized binomial multiplicative cascade model fits the $h(q)$ curves very well.

Using the values of a and b obtained from Eqs. (5), (6), (7), the multi-fractal spectrum can be estimated from the generalized binomial multiplicative cascade model. Four stations' generalized Hurst exponents $h(q)$ and the width of the multi-fractal spectrum obtained from this model are shown in Fig. 5. The width $\Delta\alpha$ of the multi-fractal spectrum $f(\alpha)$ is correspond to the vertical width of calculated $h(q)$ from Eq. (7). The figure shows that stations Hebukesaiier and Qinzhou are characterized with strong multi-fractal behavior, while stations Dongwuzhumuqinqi and Tengchong exhibit weakly multi-fractal fluctuations. Specific values of a , b and the width of the multi-fractal spectrum $\Delta\alpha$ are shown in Table 2.

Table 2

Specific values of a , b and the width of the multi-fractal spectrum $\Delta\alpha$.

Station (No.)	a	b	$\Delta\alpha$
50915 Dongwuzhumuqinqi	0.643	0.686	0.093
51156 Hebukesaiier	0.594	0.739	0.315
56739 Tengchong	0.572	0.608	0.088
59632 Qinzhou	0.584	0.765	0.389

Since the multi-fractal spectrum $f(\alpha)$ denotes the singularity content of the process, the difference of width of $f(\alpha)$ shown in Fig. 5 indicates that the strength of the multi-fractal behavior of daily mean wind speed appears not universal, but has great relevance to the singularity characteristics for the different areas. Time series of anomaly daily wind speed records of four representative stations over three years are shown in Fig. 6, we can find that the fluctuations of Qinzhou's record and Hebukesaiier's record show more furious behavior than that of Dongwuzhumuqinqi's record and Tengchong's record. This difference reflects the different atmospheric circulation patterns and processes controlling the area around selected stations. For example, station Hebukesaiier is located in the Northwest region of China, where the activity of cold wave cyclone is frequent in winter, while station Qinzhou is under control of irregular frequent tropical cyclones (Typhoon) in summer. Both climate modes accompanied with abrupt strong wind show strongly asymmetry and singularity attribute to high frequency of extreme cyclone event. Since both Hebukesaiier and Qinzhou exhibit intense multifractality which is due to a variety of long-range correlations for small and large scale fluctuations. We definitely approve of the key role of extreme wind event in intense multifractality, which contributes to the different statistical properties between large scale and small scale fluctuations, and

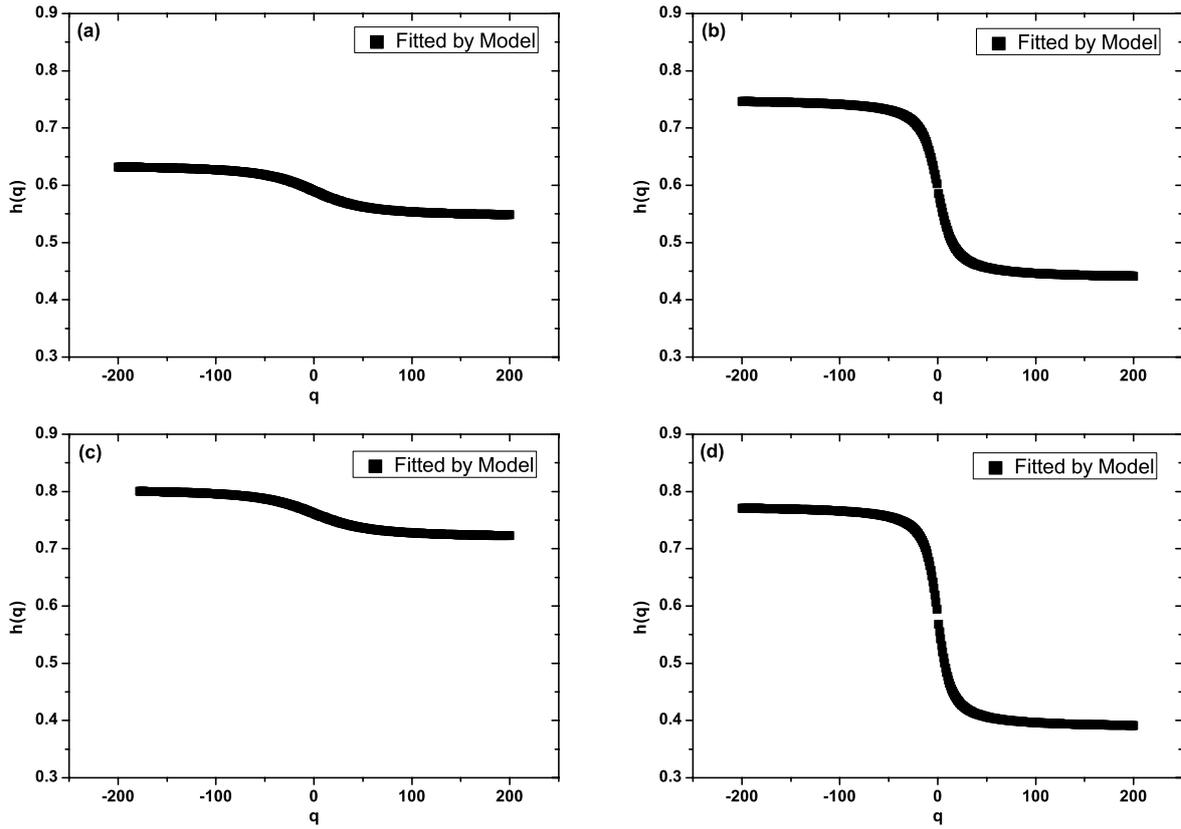


Fig. 5. Generalized Hurst exponent $h(q)$, as a function of q for the daily wind speed data, obtained by fits of the two-parameters binomial model. For station Dongwuzhumuqinqi (a), $\Delta\alpha = 0.093$; for station Hebukeisaier (b), $\Delta\alpha = 0.315$; for station Tengchong (c), $\Delta\alpha = 0.088$; for station Qinzhou (d), $\Delta\alpha = 0.389$.

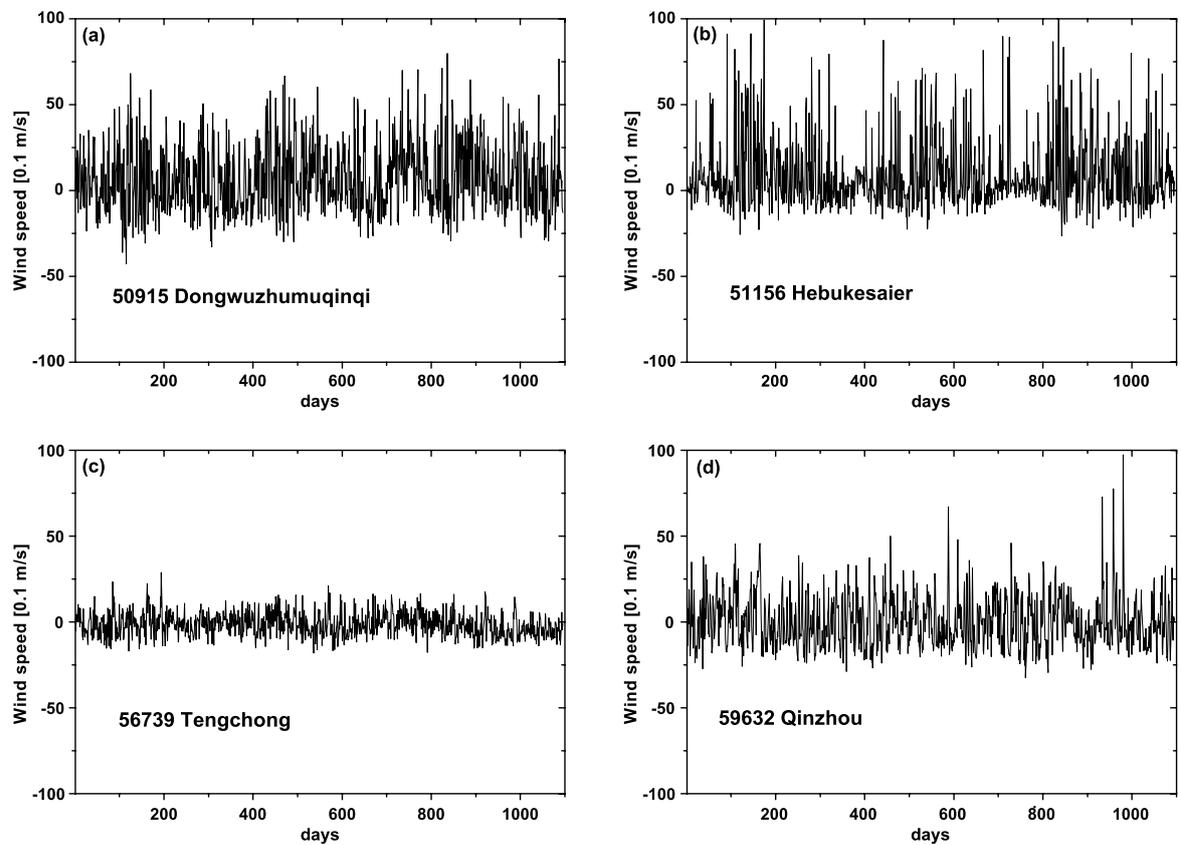


Fig. 6. Time series of anomaly daily mean wind speed record for the station Dongwuzhumuqinqi (a), Hebukeisaier (b), Tengchong (c) and Qinzhou (d), as derived from the same period over three years.

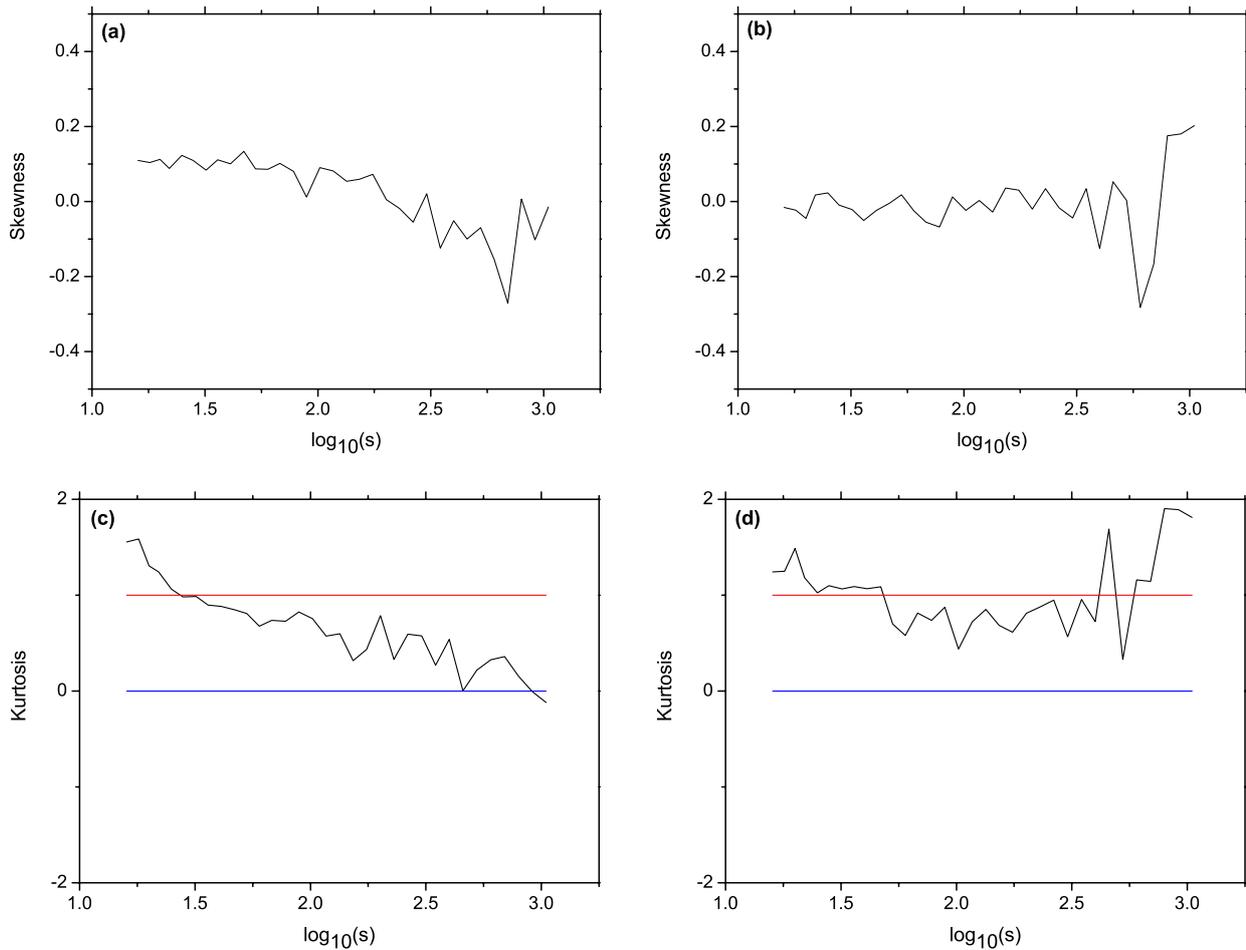


Fig. 7. The relationship between skewness, kurtosis and detrend time window: (a) skewness of Hebukesaiier; (b) kurtosis of Hebukesaiier; (c) skewness of Tengchong; (d) kurtosis of Tengchong.

finally leads to the distinct power-law scaling for small and large scale fluctuations. To confirm this idea, we perform an analysis of skewness and kurtosis of detrended time series over segments from small scale to large scale.

Statistically, skewness and kurtosis are measurements of the nonlinearity. The moment coefficient of skewness and kurtosis are defined as the normalized third and fourth statistical moment,

$$\text{skewness} = \frac{m_3}{m_2^{3/2}}, \quad (9)$$

$$\text{kurtosis} = \frac{m_4}{m_2^2} - 3 \quad (10)$$

where m_k is the k th moment,

$$m_k = \sum_{i=1}^N \left(\frac{x_i - \bar{X}}{N} \right)^k \quad (11)$$

and where x_i is the i th observation, \bar{X} the mean, and N the number of observations.

Fig. 7 shows results from stations Hebukesaiier and Tengchong. The two stations present definitely different behaviors on the scale from 100 days to 1000 days. For station Hebukesaiier which represents intense multifractality, skewness and kurtosis show a clearly change with different time windows. But for station Tengchong which represents monofractality, skewness and kurtosis show coherent value. Stations Dongwuzhumuqinqi and Qinzhou also show very similar results, but figures are not posted here. Such results

agree with that extreme wind events lead to strongly asymmetry in wind speed records, and then contribute to the different statistical properties between large scale and small scale fluctuations, and finally lead to the distinct power-law scaling for small and large scale fluctuations.

Additionally, if we apply multi-fractal detrended fluctuation analysis (MF-DFA) to eleven equal-length subseries where annual cycle has been removed and we choose a subseries to calculate the width of the multi-fractal spectrum by MF-DFA, then move subseries gradually without changing the length of subseries, and repeat this operation until the end of the original series, we will reach almost the same results. This indicates that the stationarity of the wind dynamics during the considered period for both records in stations Dongwuzhumuqinqi and Hebukesaiier play weak impact on the conclusion given in the study.

4. Conclusion and discussion

In this study, we analyzed the power-law scaling behavior of the daily mean wind speed records from four selected weather stations using multi-fractal detrended fluctuation analysis (MF-DFA) method, and different multi-fractal behaviors have been identified. We indeed found that the presence of multi-fractal behavior of daily wind speed records, but the strength of the multi-fractal behavior exhibits non-universal law over China. Station Hebukesaiier located in Northwest of China and station Qinzhou in coastal areas of southern China, records for these two stations show very strong multi-fractal behavior. While station Tengchong located in

Southwest of China and station Dongwuzhumuqinqi in Northeast of China records for these two stations exhibit weak multi-fractal behavior. This difference may be due to some external forces.

And we found that the type of multi-fractal behavior in daily wind speed records over China exhibits consistence with a generalized binomial multiplicative cascade model [20]. The width $\Delta\alpha$ of the multi-fractal spectrum $f(\alpha)$ can be estimated from this modified model, and difference of multi-fractal spectrum width corresponds to singularity characteristics of representative stations over very wide geographical distribution. This difference reflects the different atmospheric circulation patterns and processes controlling the area around representative stations. But the origin of this diversity in fractal behavior and related atmospheric mechanism are still unknown, we will discuss these problems further in the future.

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