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The most intensive fluctuation in chaotic time series and relativity principle

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Abstract

Relativity principle in mechanics and principle of invariant speed of light lead to Einstein theory. The exponent of p-order momentum, derived from a piece of multi-scale chaotic time series, varies with the order p and cannot exceeds a maximum, so there exists the principle of scale relativity. Its special case is the same one as Lorenz transformation from Einstein theory.

1. Introduction

Abundant information is contained in any piece of chaotic time series. The classical statistical method for analyzing time series lies in calculating mean value and probability density, i.e. dominant component in the time series and large probability density near mean value are regarded as important. But the fractal and multi-fractal method for time series lies in finding the self-similar structure of different scales, i.e. the invariant quantities, such as scaling exponent and fractal dimension, are taken to be important.

Single scaling is the case where only one scaling exponent can be found in these simpler multi-scale systems. For example, the scaling exponent for the famous cantor set is $\alpha = \lg 2/\lg 3 = 0.6309$ [1], Brown motion is $\alpha = 1/2$ [2], Kolmogorov turbulence is $\alpha = 1/3$ [3], and a random cantor set of $\epsilon^{(\infty)}$ theory has a golden mean $(5^{1/2} - 1)/2 = 0.618033$ scaling.

But multi-scaling exponent and multi-dimension are needed for inhomogeneous multi-scale systems [4]. In these cases, the states of time series, such as climatic warming and cooling, length of seacoast line, vortex number of turbulence, variate with different scales [5]. And the intensity fluctuations of state variate also with different scales [6]. We cannot say that "now climate becomes warming", "length of seacoast line is xxxkm", "the number of turbulent eddy is xxx", for these quantities variate with different scales, too, i.e. there are different results for different scales. At the same time, the stochastic fluctuating intensity of the time series variate with different scales, there are the most intensive fluctuations in each scale. It is called the most intermittent structures in turbulence by She [7].

Fractal space–time physics refers to a set of ideas on the role of scale in microphysics. Because space–time is a notion taken from relativity, so the $\epsilon^{(\infty)}$ cantorian theory of space–time have close connection with geometry and the topological theory [8] of four manifolds as well as heterotic string theory [9–11]. In this article I shall briefly apply fractal space–time theory to the structure of chaotic time series.

Relativity principles in mechanics [12] say that the velocity is different in different inertial systems. The physical quantity is different for different scales in chaotic time series. Einstein relativity theory [13] deals with the invariance

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principle of speed of light (maximum velocity). There are the most intensive fluctuations for each scale in chaotic time series, so there exist scale relativity principles, too.

2. Essence of the most intensive fluctuations

Because of the intermittence of chaotic time series [14], there are strong and weak fluctuations, or dense and thin point sets, which denote inhomogeneous distribution of chaotic time series. So there are infinite scaling exponents $\alpha(p)$ and dimensions D_p [15] $(-\infty < p < +\infty)$.

Multi-fractal theory has shown that the most intensive fluctuations (or the most dense point set) occur when $p \to +\infty$, i.e. α_∞ and D_∞ [16]. So it is called the most intensive fluctuation.

For example, the scaling behavior of velocity difference $\delta v_l = v(x+l) - v(x)$ is expressed as

$$\langle \delta v_l^p \rangle \sim l^{\zeta_p} \tag{1}$$

where

$$\zeta_p = \gamma p + v_\infty (1 - \beta^p), \quad 0 \leqslant \beta \leqslant 1 \tag{2}$$

which is obtained by She [17] in 1994. Here γ is the scaling exponent for most intensive fluctuations, i.e.

$$\delta v_l|_{\text{maximal-intensity}} \sim l^\gamma \tag{3}$$

So γ has the minimum value in infinite scaling exponents.

The most fluctuations will occupy the smallest volume (or probability), there is

$$p(l) \sim l^{v_\infty} \tag{4}$$

So the value of v_∞ must be maximum. According to multi-fractal theory, the volume occupied by the vortex of scale l is [18]

$$p(l) \sim l^{3-D} \tag{5}$$

where

$$v_\infty = 3 - D \tag{6}$$

is called co-dimension.

Because v_∞ is maximum, so D_∞ is the minimum in infinite D_p. According to multi-fractal theory [19], the relation between α and ζ_p is

$$\alpha = \frac{\mathrm{d}\zeta_p}{\mathrm{d}p} = \gamma - v_\infty \beta^p \lg \beta \tag{7}$$

it is obvious $\lg \beta < 0$, for $\beta < 1$. When $p \to \infty$, $\alpha \to \gamma$, γ is the minimum among infinite $\alpha(p)$, i.e.

$$\gamma = \alpha_{\min} \tag{8}$$

(8) is just same as (3).

In addition, from the point of multi-fractal, there is

$$\zeta_p = \inf_\alpha [\alpha p + 3 - D(\alpha)] \tag{9}$$

For She-model (2), it is

$$3 - D(\alpha) = \zeta_p - \alpha p = (\gamma - \alpha)p + v_\infty (1 - \beta^p) \tag{10}$$

Obviously, when $p \to \infty$, $3 - D(\alpha) \to v_\infty$. Because $\beta < 1$, v_∞ is maximum in the right hand of (10). Therefore, D_∞ is minimum among infinite D_p, which is same as (4).

In brief, there is the most intensive fluctuation in each scale l for chaotic time series. They have the maximum intensity and the minimum volume. And they can be denoted by $\alpha_{\min}$ and $D_{\min}$, which may be calculated from given chaotic time series [20].

3. Relativity principle

From above, we see that if $\alpha_{\min}$ and $D_{\min}$ are known for a piece of chaotic time series, the scaling exponent ζ_p of p order structure function can be formulated as

$$\zeta_p = \alpha_{\min} p + (3 - D_{\min}) f(p) \tag{11}$$

where $f(p)$ must satisfy the following three conditions

$$f(0) = 0, \quad f(\infty) = 1, \quad f'(\infty) = 0 \tag{12}$$

The first condition demands for $\zeta_0 = 1$, the second for $(3 - D)$ maximum as $p \to \infty$, the third for α minimum as $p \to \infty$.

From scaling exponent (11), we see that if ζ_p is taken as velocity v_p, then different p is corresponding to different inertial systems. If there is only the first term in the right hand of (11), then v_p can be written as

$$v_p = \gamma p \tag{13}$$

which indicates single scaling fractal field, in which there is no most intensive fluctuations, so

$$v_{p+q} = \gamma(p + q) = v_p + v_q \tag{14}$$

(14) is just the Galilean transformation for relativity principle in mechanics.

If there is nonlinear term of p in (11), it implies multi-scaling, i.e. infinite scaling exponents and dimensions exist. Certainly, there is most intensive fluctuation. Set

$$v_p = v_\infty f(p) \tag{15}$$

where v_∞ is the co-dimension of most intensive fluctuation, it is corresponding to invariance of speed of light in mechanics. So the relativity principle, which is similar to Einstein relativity theory, occurs. We will show this by some examples in the following:

Example 1. She-model

From (2), we can see that the form of $f(p)$ is

$$f(p) = 1 - \beta^p, \quad \beta < 1 \tag{16}$$

which satisfies the condition (12), so there is

$$v_{p+q} = v_\infty(1 - \beta^{p+q}) = v_\infty(1 - \beta^p) + v_\infty(1 - \beta^q) + v_\infty(1 - \beta^p - \beta^q - \beta^{p+q}) = v_p + v_q - \frac{v_p v_q}{v_\infty} \tag{17}$$

(17) is the relativity principle of nonGalilean transformation [21].

Example 2

$$f(p) = \frac{p}{p+1} \tag{18}$$

which satisfies the condition (12), too. So there is

$$v_{p+q} = v_\infty \frac{p+q}{(p+q)+1} = v_p + v_q - v_\infty \frac{p^2 q + 2pq + pq^2}{(p+1)(q+1)(p+q+1)} = v_p + v_q - \frac{2 v_p v_q}{v_\infty} + \frac{v_p v_q v_{p+q}}{v_\infty^2} \tag{19}$$

which produces the new relativity principle

$$v_{p+q} = \frac{v_p + v_q - \frac{2 v_p v_q}{v_\infty}}{1 - \frac{v_p v_q}{v_\infty^2}} \tag{20}$$

Example 3

$$f(p) = \mathrm{th}\, p \tag{21}$$

which still satisfies the condition (12), then

$$v_{p+q} = v_\infty[\mathrm{th}(p+q)] = \frac{v_p + v_q}{1 + \frac{v_p v_q}{v_\infty^2}} \tag{22}$$

This is relativity principle same as Lorenz transformation.

4. The differential equation for v_p

From (15) and (16), we see that the v_p for She-model satisfies the following linear differential equation

$$\frac{dv_p}{dp} = \lg\beta(v_p - v_\infty) \tag{23}$$

For Example 2 of nonGalileo transform, the v_p satisfies the variable coefficient linear differential equation

$$\frac{dv_p}{dp} = v_\infty\left(\frac{1}{p} - \frac{1}{p+1}\right)v_p \tag{24}$$

But, for Example 3 of Lorenz transform, the v_p satisfies the nonlinear differential equation

$$\frac{dv_p}{dp} = v_\infty(1 - v_p^2) \tag{25}$$

5. Conclusion

Fractal space–time physics is important for research dealing with chaotic time series. The most intensive fluctuation of chaotic time series can be denoted by scaling exponent α_∞ and dimension D_∞, which are the result of α_p and D_p when $p \to \infty$. If α_∞ and D_∞ are calculated from the chaotic time series, then the scaling exponent ζ_p of p-order structure function can be obtained based on above analysis. Actually ζ_p is nonlinear function of p, which is corresponding to velocity v_p in different inertial systems. The v_p satisfies a number of different scale relativity principles, in which Lorenz transformation from Einstein theory is indeed interesting.

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