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A universal model to characterize different multi-fractal behaviors of daily temperature records over China

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Abstract

The multi-fractal properties of surface air temperature over China are studied through Multi-fractal detrended fluctuation analysis (MF-DFA). It is shown that there indeed exists the multi-fractal phenomenon in daily surface air temperature time series, reflecting a great deal of fluctuations at various time scales. The multi-fractal properties can be characterized very well by a universal generalized binomial multiplicative cascade model with only two parameters a and b. For different stations, the width of singularity spectrum $f(\alpha)$ is different, indicating different strengths of temperature multi-fractal behavior from station to station.

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Keywords: Multi-fractal behavior; Multi-fractal detrended fluctuation analysis; Singularity spectrum

1. Introduction

The climate system is governed by a variety of physical processes and exhibits a great deal of fluctuations at various spatial and temporal scales, indicating a hierarchy of structures. Similarly, the surface air temperature, one of the most fundamental indicators of fluctuations or changes in climate system, shows the fractal phenomenon by taking the self-similar structure over a wide range of time scales from one day to ten years, one hundred years and even longer time. To understand the dynamical variability of these phenomena, many studies have dealt with the fluctuations and correlations of air temperature records, using various methods such as power density spectra, autocorrelation functions, Hurst re-scaled ranges, detrended fluctuations analysis (DFA) and so on, and have identified asymptotic power-law scaling for several long records [1–5]. The existence of power-laws (scaling) in the statistics used to describe the patterns of temporally fluctuating systems indicates the presence of a fractal behavior.

The above scaling exponent approaches, however, are not sufficient to fully characterize the complex dynamics of air temperatures, since they exclusively focus on the variance which can be regarded as the second moment of the full distribution of the fluctuations [6] and a single scaling exponent which describes that a mono-fractals cannot capture

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the characteristics of the multi-fractals completely [7]. To further characterize the air temperature record, we extend the study to include all moments of the full distribution of the fluctuations and adopt different scaling exponents, using a fairly robust and powerful technique called multi-fractal detrended fluctuation analysis (MF-DFA) [8]. This method provides a systematic means to identify and more importantly quantify the multiple scaling exponents in the data [8], and has been successfully utilized in different fields to study multi-fractals [6,9–12].

The paper is organized as follows. In Section 2, we briefly describe our data and the method of MF-DFA. In Section 3, the results of MF-DFA and the calculation of the multi-fractal spectrum are provided. And we will summarize and discuss the conclusions in the last section.

2. Methodology and data

2.1. Methodology outline

Before applying the method of MF-DFA, the annual cycle is removed from the raw data T_i by computing the temperature anomaly series $x_i = T_i - \langle T_i \rangle_d$, where $i = 1, \ldots, N$, and $\langle . \rangle_d$ denotes the long-time average for the given calendar day.

In the MF-DFA procedure [8], the anomaly time series are firstly integrated to the so-called profile. To determine the moments $F_q(s)$, the profile time series is partitioned into segments of s, and polynomial fits of order N are calculated separately for each segment ν. Then calculate the mean-square deviations $F^2(\nu, s)$ from polynomial fits and take the qth root of the average $(F^2(\nu, s))^{q/2}$ over all segments. In this paper, we have used fourth-order polynomials to eliminate cubic trends in the data. In general, the index variable q can take any real value except zero. If the time series is long-range power-law correlated, $F_q(s)$ increases for large values of s as a power-law: $F_q(s) \approx s^{h(q)}$, where the exponent $h(q)$ describes the scaling behavior of the qth-order fluctuation function. For positive values of q, $h(q)$ describes the scaling behavior of segments with large fluctuations while for negative values of q, it describes scaling behavior of segments with small fluctuations [8]. For stationary series, $h(2)$ is the well-defined Hurst exponent. Thus we call $h(q)$ the generalized Hurst exponent [8]. For mono-fractal time series which are characterized by a single exponent over all scales, $h(q)$ is independent of q, whereas for a multi-fractal time series, $h(q)$ varies with q. This dependence is considered to be a characteristics of multi-fractal process [8].

It is well-known that the traditional way to characterize a multi-fractal series is to calculate the singularity spectrum $f(\alpha)$ [13,14]. This singularity spectrum can be related to $h(q)$ via a Legendre transform [8,13],

$$\alpha = h(q) + q\frac{\mathrm{d}h(q)}{\mathrm{d}q} \qquad f(\alpha) = q(\alpha - h(q)) + 1, \tag{1}$$

where α is the singularity strength or Hölder exponent, while $f(\alpha)$ denotes the dimension of the subset of the series that is characterized by α.

2.2. Data sets

In this paper, we study the fractal behavior of daily mean temperature records at 191 weather stations over China mainly using MF-DFA. The records were obtained from a high-quality daily surface climatic data set, processed by Chinese National Meteorological Information Center (NMIC), of 194 Chinese meteorological stations taking part in international exchange. The same collection was utilized in many studies to analyze climate change over China in the recent 50 years [15–18]. Data for three stations, station 54618, 52203 and 54909, were kicked out because of their short time span about only 10 years, while records of the other stations last about 50 years, from 1951 to 2000. According to Ref. [19], scaling of correlated data series is not affected by randomly cutting out segments and stitching together the remaining parts, even when 50% of the points are removed. Therefore we remove the missing values from the raw data, since they are only a small part of the series.

3. Results

As a representative example, Fig. 1 shows three years of the anomaly daily temperature record of the station Shantou in the South China (a) and the station Tengchong in the Southwest of China (b). It can be seen that the

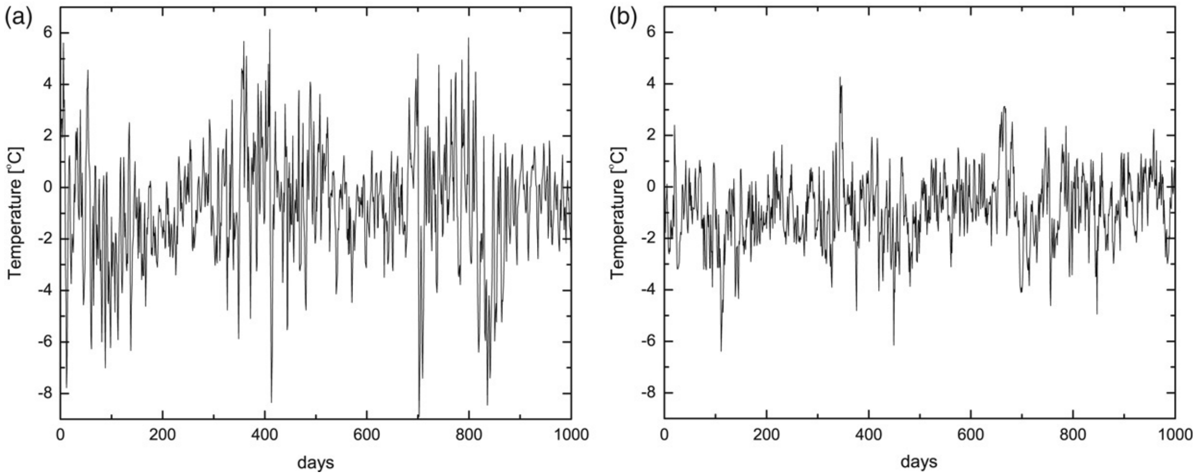

Fig. 1. Three years of the anomaly daily temperature record for the station Shantou (a) and the station Tengchong in the Southwest of China (b).

fluctuation of Shantou record appears more furious than that of the Tengchong record. But both time series show a clear irregular dynamics, characterized by sharp bursts of high-frequency fluctuations.

We then applied the method of MF-DFA4 (actually, MF-DFA1, MF-DFA2, MF-DFA3, MF-DFA4 and even the still higher-order MF-DFA will give the similar results, in order to eliminate the possible nonlinear trends imposing on the original time series, here we select MF-DFA4 to analyze the data in this paper) to the records and calculated the fluctuation functions $F_q(s)$ for scales ranging from 32 days to $N/4$ days, where N is the total length of the series. The log–log plots of the fluctuation function ($F_q(s)$ vs s) with varying moments ($q = -5, -4, -3, -2, -1, 1, 2, 3, 4, 5$) by using fourth-order polynomial detrending for the representative records are shown in Fig. 2(a) and (b). For station Shantou, it is observed that while all the curves take the similar shape, the curve becomes steeper and steeper from the top to the bottom. But the case is not the same for station Tengchong: the slope of the curve changes little.

To compute the q dependence of the scaling exponent $h(q)$ quantitatively, we selected the time scale in the range [2.0–3.0], where the scaling is more or less constant for a given q. For the sake of comparison, all the slopes of the fluctuation curves $h(q)$ in this paper are calculated for every q in this range, corresponding to the time span of [100–1000] days. By this method, the generalized Hurst exponents $h(-5)$, $h(-4)$, $h(-3)$, $h(-2)$, $h(-1)$, $h(1)$, $h(2)$, $h(3)$, $h(4)$ and $h(5)$ of the daily mean temperature record for the station Shantou and Tengchong are shown in the Fig. 2(c) and (d) (filled squares), respectively. For station Shantou, the behavior that the scaling exponent $h(q)$ depends on the value of q represents the presence of multi-fractal behavior; while the value of $h(q)$ seems to be constant for all q in station Tengchong, indicating the phenomena of the mono-fractals. From the Ref. [8], it is known that the origin of multi-fractality is twofold: broad probability density function and different long-range correlation for small and large fluctuations. By comparing the multi-fractal DFA results for original series with those for shuffled series we can distinguish multi-fractality due to long-range correlations from multi-fractality due to a broad probability density function. It is obvious that the multi-fractality of temperature time series over station Shantou is not from broad probability density function since the slopes of multi-fractal DFA with different orders for shuffled data over station Shantou are the same and taking the same results as random series, see the Fig. 2(e).

Besides, in the Fig. 2(c) and (d) we showed least-squares fits (open circles) according to the formula

$$h(q) = \frac{1}{q} - \frac{\ln(a^q + b^q)}{q \ln 2}, \tag{2}$$

assuming a generalized binomial multiplicative cascade model, which has served as one of the standard paradigms to describe multi-fractal scaling (see Refs. [6,8] for details). The values of the two parameters a and b are also reported in the Fig. 2(c) and (d). We have fitted the $h(q)$ in the range $-5 \le q \le 5$, with a 0.5 step for all 191 stations over China. Surprisingly, for all of these stations the generalized binomial multiplicative cascade model fits the $h(q)$ curves very well (limited by the space, only representative examples are shown in Fig. 2(c) and (d)). It is worthy of strengthening that not all the scaling exponents are universal which has been given in the analysis of the atmospheric variability [2],

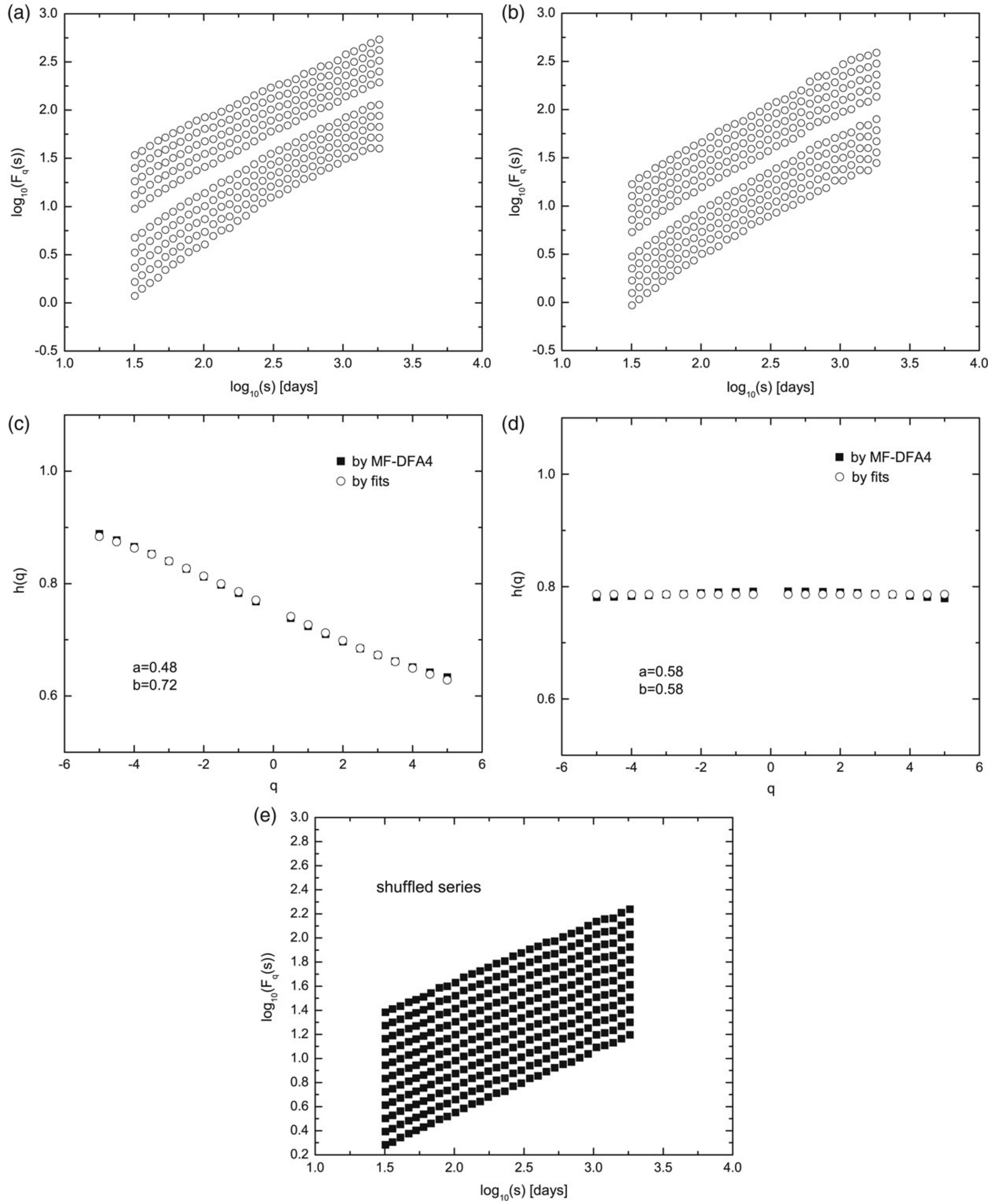

Fig. 2. (a) Log–Log plots of the MF-DFA4 curves of temperature for Shantou. From the top to the bottom curves correspond to different q (from $q = -5$ to $q = 5$) and are shifted vertically for clarity. (b) The analog to (a) but for Tengchong. (c) $h(q)$ vs q plots for Shantou. Solid squares: obtained from MF-DFA4 results in (a); open circles: obtained by fits of the two-parameters binomial model. (d) the analog to (c) but for Tengchong. (e) the analog to (a) but for the shuffled data of Shantou.

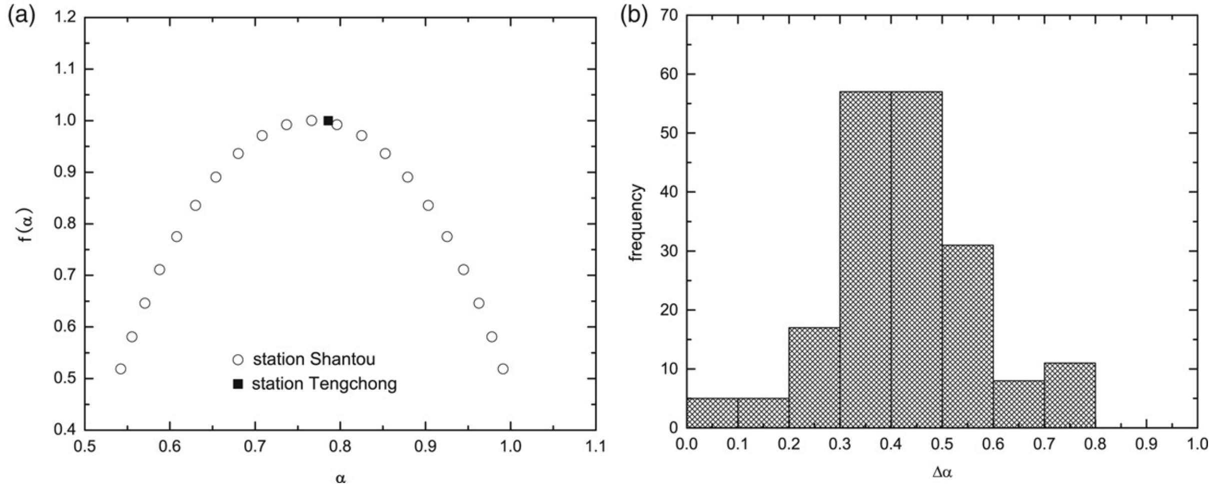

Fig. 3. (a) The multi-fractal spectra $f(\alpha)$ for two representative temperature records of station Shantou (open circles) and station Tengchong (solid squares). (b) the histogram of $\Delta\alpha$ of temperature records for 191 stations over China.

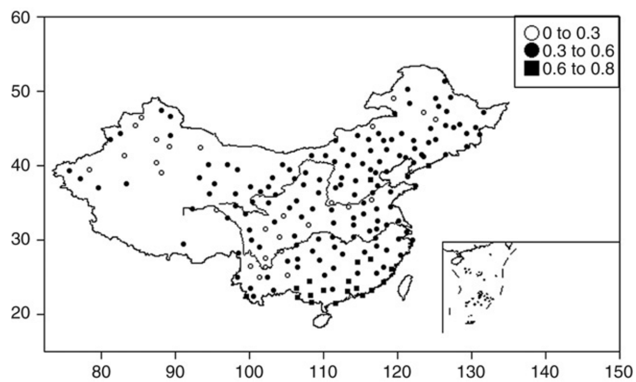

Fig. 4. Geographical distribution of $\Delta\alpha$ of temperature records for 191 stations over China, where open circles for $0 \le \Delta\alpha \le 03$, solid circles for $0.3 < \Delta\alpha < 0.6$, solid squares for $0.6 \le \Delta\alpha \le 0.8$.

but there is a universal generalized binomial multiplicative cascade model to describe the multi-fractal behavior of the surface air temperature anomalies.

From $h(q)$ we obtained the Hölder exponent α and the singularity spectrum $f(\alpha)$ (Eq. (1)). Fig. 3(a) shows the multi-fractal spectra of air temperature for the station Shantou and the station Tengchong. Apparently, the spectra for the two stations exhibit completely different shapes: the spectra for station Shantou takes bell-like shape with wide distribution of $f(\alpha)$ values, while the one for station Tengchong is only one dot with single value. In fact, the width of $f(\alpha)$ taken at $f = 0$ characterizes the strength of the multi-fractal behavior. The bell-like spectrum represents the strong multi-fractal behavior. In contrast, the spectrum for station Tengchong reflects mono-fractals. In the generalized binomial multi-fractal model, the strength of the multi-fractal behavior is identical to the difference of the asymptotical values of $h(q)$, $\Delta\alpha = h(-\infty) - h(+\infty) = \frac{\ln(b)-\ln(a)}{\ln(2)}$.

In Fig. 3(a), it is shown that both the widths of $f(\alpha)$ are very different, which indicates that the strength of the multi-fractal behavior of air temperature appears to be not universal. Therefore, next we focus on the difference of the multi-fractal behavior between stations. Fig. 3(b) summarizes the results for $\Delta\alpha$ for temperature records. One can see clearly that for 191 stations the average $\Delta\alpha$ is close to 0.42 with quite a large variance 0.15.

The geographical distribution of $\Delta\alpha$ is exhibited in Fig. 4, where the intervals for $\Delta\alpha$ are chosen as the multiples of the variance calculated from 191 stations' $\Delta\alpha$. The figure shows that there are stations with quite strong multi-fractal fluctuations, i.e. large $\Delta\alpha$ above the averaged value, for the areas of South China and North China and ones with weak multi-fractal behavior, i.e. $\Delta\alpha \approx 0$, for the areas of Northeast of China, Southwest of China, Northwest of China, and the areas between Yangtze River and Huanghe River. This means that the air temperature records over the South China and North China are more nonlinear and singular than that over the rest of China. This difference of temperature multi-fractal behavior is probably caused by the different climatic conditions around the different weather stations. Similar result that there is transitional behavior over the areas of Northeast of China, Northwest of China and the areas between Yangtze River and Huanghe River, has been reported in the Ref. [18], where a new index constructed with the product of the scaling exponent and the standard deviation of the same relative humidity record is used to classify the climate regions. This transitional behavior can be still found in this letter but from the multi-fractal behaviors, so it can be considered that this transitional behavior is robust and deserves to be studied deeply.

4. Conclusion and discussion

In this letter, we have analyzed daily air temperature records over China by using the MF-DFA method. We indeed found that the temperature records demonstrate the presence of multi-fractal behavior. But the strength of the multi-fractal behavior seems not to be universal over China. For stations over South China and North China, the temperature records exhibit very strong multi-fractal behavior; while the multi-fractal behavior of temperature is week, even almost vanishing for stations over Northwest of China, Southwest of China, Northeast of China, and the areas between Yangtze River and Huanghe River. This difference reflects the different climate dynamic processes controlling the area around different weather stations.

Surprisingly, the type of multi-fractal behavior occurring in temperature records over China is consistent with a modified version of the binomial multi-fractal model [6,20], with only two parameters a and b between 0 and 1, and $a + b > 1$. This model provides one ideal of a universal multi-fractal behavior of temperatures for stations over China, and the multi-fractal exponents can be regarded as 'fingerprints' for each station.

We found the different multi-fractal behaviors over the different areas of China, these different multi-fractal behaviors can be modeled with a universal extended binomial multi-fractal model. There is still a lot of work needed to be done to deeply understand the difference over different climate regions, such as what is the origin of multi-fractals and what causes the difference of fractal strength over different areas, why we can find the similar transitional behavior from different ways and what is the underlying mechanism? All these will be answered in the future reports.

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