粒子物理

9. 正负电子湮没过程 (Trace Technique)

曹庆宏

北京大学物理学院

Quantum Electrodynamics (QED)

QED-1

便笺标题 2014/11/24

任务：

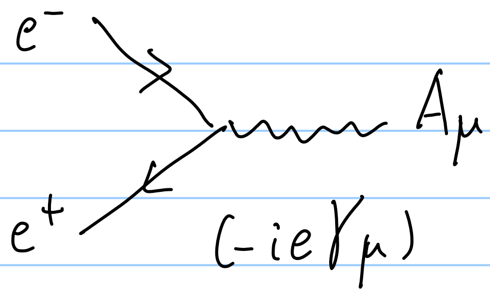
- 我们将讨论 $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to \gamma\gamma$

  和 $R \equiv \dfrac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$ （测定 QCD 色的数目）

- Jet（喷注）— 夸克或胶子的实验观测物理量
- 测量 α_{em}

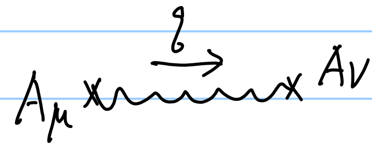
3) Feynman Rules for Feynman diagrams

Vertex:

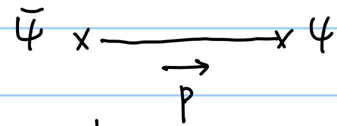


$(-ie\gamma_\mu)$

Propagator



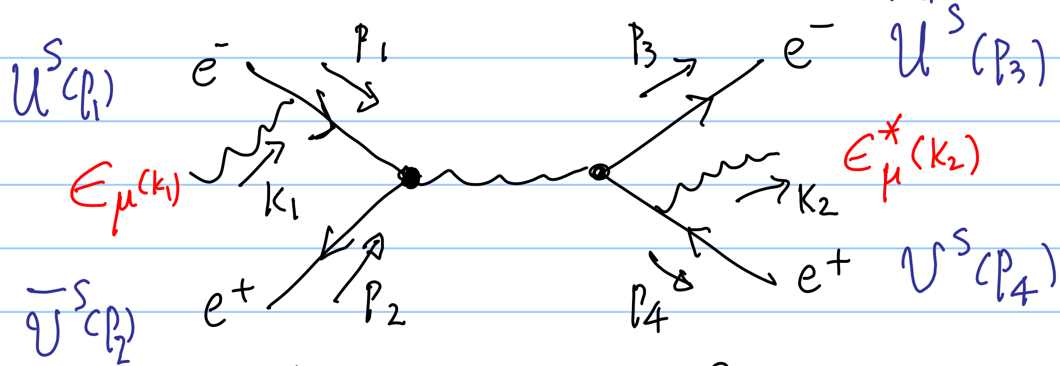
$$\frac{-i g_{\mu\nu}}{q^2 + i\varepsilon} \quad \varepsilon \to 0^+$$



$$\frac{i(\not p + m)}{p^2 - m^2 + i\varepsilon}$$

其中 $p^2 \equiv p_\mu p^\mu$, $\not p \equiv \gamma_\mu p^\mu$

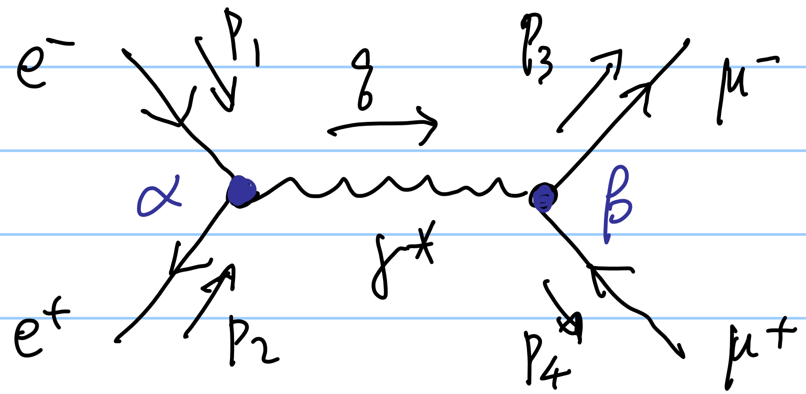
External particle's wave functions



逆着费米子线写振幅

initial $\longrightarrow$ final

5) 示例：$e^+e^- \longrightarrow \mu^+\mu^-$ 散射



$$q = p_1 + p_2$$
$$= p_3 + p_4$$

散射振幅为

$$-iM = \left[\bar{u}(p_3)(ieQ\gamma^\beta)v(p_4)\right]\frac{-ig_{\alpha\beta}}{q^2+i\varepsilon}\left[\bar{v}(p_2)(-ie\gamma^\alpha)u(p_1)\right]$$

$Q \to Q_\mu$　　$Q_e = -1$

$$= \left(\frac{-e^2 Q_e Q_\mu}{q^2}\right)\left[\bar{u}(p_3)\gamma^\alpha v(p_4)\right]\left[\bar{v}(p_2)\gamma_\alpha u(p_1)\right]$$

在QED、QCD和弱相互作用顶点都可以写作为 $\overline{u}(p)\,\Gamma\,u(p')$

$$\overline{u}(p)\,\Gamma\,u(p') = \overline{u}(p)_j\,\Gamma_{ji}\,u(p')_i$$

指标意味着求和。注意：上式仅仅是个复数。

If this is not immediately obvious, consider the 2×2 case of $\mathbf{c}^{\mathrm{T}}\mathbf{B}\mathbf{a}$, where the equivalent product can be written as

$$\begin{aligned}(c_1, c_2)\begin{pmatrix} B_{11} & B_{12} \\ B_{22} & B_{22} \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} &= c_1 B_{11} a_1 + c_1 B_{12} a_2 + c_2 B_{21} a_1 + c_2 B_{22} \\ &= c_j B_{ji} a_i,\end{aligned}$$

which is just the sum over the product of the components of **a**, **c** and **B**.

量子力学原理告诉我们，散射几率依赖于散射振幅模方

$$|\mathcal{M}|^2 = \mathcal{M}^\dagger \mathcal{M}$$

$$= \left|\frac{-e^2 Q_e Q_\mu}{q^2}\right|^2 \left[\bar{u}(p_3)\gamma^\beta v(p_3)\right]^\dagger \left[\bar{u}(p_3)\gamma^\alpha v(p_4)\right]$$

$$\left[\bar{v}(p_2)\gamma_\beta u(p_1)\right]^\dagger \left[\bar{v}(p_2)\gamma_\alpha u(p_1)\right]$$

因为 $\left[\bar{u}(p_3)\gamma^\beta v(p_4)\right]^\dagger = \left[u^\dagger \gamma^0 \gamma^\beta v\right]^\dagger = v^\dagger (\gamma^\beta)^\dagger (\gamma^0)^\dagger (u^\dagger)^\dagger$

$$= v^\dagger \gamma^0 \underbrace{\gamma^0 (\gamma^\beta)^\dagger \gamma^0}_{} u$$

$$(\gamma^0)^\dagger = \gamma^0$$

$$(\gamma^j)^\dagger = -\gamma^j$$

$$= \bar{v}\gamma^\beta u$$

$$(\gamma^0)^2 = 1$$

同理 $\left[\bar{v}(p_2)\gamma_\beta u(p_1)\right]^\dagger = \bar{u}(p_1)\gamma_\beta v(p_2)$

对于没有极化的 e^+ 和 e^- 入射粒子束，我们需要对初态自旋求平均，对末态自旋求和，则有

$$\overline{|m|^2} = \underbrace{\frac{1}{2}\times\frac{1}{2}}_{\text{自旋平均}} \times \sum_{spin} |m|^2$$

其中

$$\sum_{spin} |m|^2 = \sum_{spin} \left(\frac{e^4 Q_\mu^2}{s^2}\right) \left[\bar{v}(p_4)\gamma^\beta u(p_3)\,\bar{u}(p_3)\gamma^\alpha v(p_4)\right]$$

$$\left[\bar{u}(p_1)\gamma_\beta v(p_2)\,\bar{v}(p_2)\gamma_\alpha u(p_1)\right]$$

因为

$$\sum_{spin}\left[\bar{v}_a(p_4)\gamma^{\beta}_{ab}u_b(p_3)\,\bar{u}_c(p_3)\gamma^{\alpha}_{cd}v_d(p_4)\right]$$

$$=\sum_{spin}\left[v_d(p_4)\,\bar{v}_a(p_4)\gamma^{\beta}_{ab}u_b(p_3)\,\bar{u}_c(p_3)\gamma^{\alpha}_{cd}\right]$$

completeness relation

$$\sum_{s=1}^{2} u_s\bar{u}_s = (\gamma^{\mu}p_{\mu} + mI) = \not{p} + m,$$

$$\sum_{r=1}^{2} v_r\bar{v}_r = (\gamma^{\mu}p_{\mu} - mI) = \not{p} - m,$$

我们可得

$$\sum_{spin}\left[\bar{v}_a(p_4)\gamma^{\beta}_{ab}u_b(p_3)\bar{u}_c(p_3)\gamma^{\alpha}_{cd}v_d(p_4)\right]$$

$$=\sum_{spin}\left[v_d(p_4)\bar{v}_a(p_4)\gamma^{\beta}_{ab}u_b(p_3)\bar{u}_c(p_3)\gamma^{\alpha}_{cd}\right]$$

$$=Tr\left[(\not{p}_4-m_\mu)\gamma^{\beta}(\not{p}_3+m_\mu)\gamma^{\alpha}\right]$$

同理，$\sum_{spin}\left[\bar{u}(p_1)\gamma_\beta v(p_2)\bar{v}(p_2)\gamma_\alpha u(p_1)\right]=Tr(\not{p}_1\gamma_\beta\not{p}_2\gamma_\alpha)$

故而

$$\overline{|m|^2}=\left(\frac{1}{2}\times\frac{1}{2}\right)Tr(\not{p}_1\gamma_\beta\not{p}_2\gamma_\alpha)\,Tr\left((\not{p}_4-m_\mu)\gamma^{\beta}(\not{p}_3+m_\mu)\gamma^{\alpha}\right)\left(\frac{e^4Q_\mu^2}{s^2}\right)$$

Trace Theorems

$$\mathrm{Tr}\,(A+B) \equiv \mathrm{Tr}\,(A) + \mathrm{Tr}\,(B) \qquad \mathrm{Tr}\,(AB\ldots YZ) \equiv \mathrm{Tr}\,(ZAB\ldots Y)$$

$$\boxed{\mathrm{Tr}\,(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu}}$$

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu \equiv 2g^{\mu\nu}I,$$

$$\Rightarrow \mathrm{Tr}\,(\gamma^\mu\gamma^\nu) + \mathrm{Tr}\,(\gamma^\nu\gamma^\mu) = 2g^{\mu\nu}\,\mathrm{Tr}\,(I)$$

$$\boxed{\mathrm{Tr}\,(\gamma^\mu\gamma^\nu\gamma^\rho) = 0}$$

$$\mathrm{Tr}\,(\gamma^\mu\gamma^\nu\gamma^\rho) = \mathrm{Tr}\left(\gamma^5\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\right) = \mathrm{Tr}\left(\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^5\right)$$

$$= -\mathrm{Tr}\left(\gamma^5\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\right)$$

Trace Theorems

$$\mathrm{Tr}\left(\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma}\right) = 4g^{\mu\nu}g^{\rho\sigma} - 4g^{\mu\rho}g^{\nu\sigma} + 4g^{\mu\sigma}g^{\nu\rho}.$$

$$\gamma^{a}\gamma^{b} = 2g^{ab} - \gamma^{b}\gamma^{a}$$

$$\begin{aligned}
\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma} &= 2g^{\mu\nu}\gamma^{\rho}\gamma^{\sigma} - \gamma^{\nu}\gamma^{\mu}\gamma^{\rho}\gamma^{\sigma} \\
&= 2g^{\mu\nu}\gamma^{\rho}\gamma^{\sigma} - 2g^{\mu\rho}\gamma^{\nu}\gamma^{\sigma} + \gamma^{\nu}\gamma^{\rho}\gamma^{\mu}\gamma^{\sigma} \\
&= 2g^{\mu\nu}\gamma^{\rho}\gamma^{\sigma} - 2g^{\mu\rho}\gamma^{\nu}\gamma^{\sigma} + 2g^{\mu\sigma}\gamma^{\nu}\gamma^{\rho} - \gamma^{\nu}\gamma^{\rho}\gamma^{\sigma}\gamma^{\mu}
\end{aligned}$$

$$\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma} + \gamma^{\nu}\gamma^{\rho}\gamma^{\sigma}\gamma^{\mu} = 2g^{\mu\nu}\gamma^{\rho}\gamma^{\sigma} - 2g^{\mu\rho}\gamma^{\nu}\gamma^{\sigma} + 2g^{\mu\sigma}\gamma^{\nu}\gamma^{\rho}$$

Trace Theorems

(a) $\mathrm{Tr}(I) = 4$;

(b) the trace of any odd number of γ-matrices is zero;

(c) $\mathrm{Tr}(\gamma^\mu \gamma^\nu) = 4g^{\mu\nu}$;

(d) $\mathrm{Tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma) = 4g^{\mu\nu} g^{\rho\sigma} - 4g^{\mu\rho} g^{\nu\sigma} + 4g^{\mu\sigma} g^{\nu\rho}$;

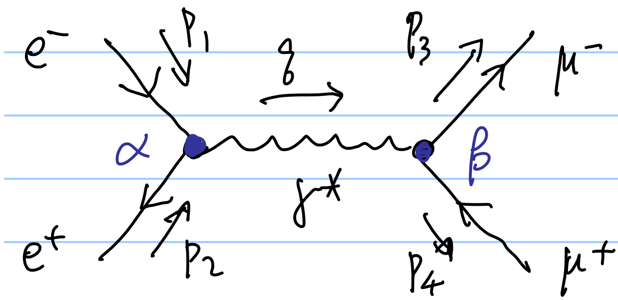
(e) the trace of γ^5 multiplied by an odd number of γ-matrices is zero;

(f) $\mathrm{Tr}\left(\gamma^5\right) = 0$;

(g) $\mathrm{Tr}\left(\gamma^5 \gamma^\mu \gamma^\nu\right) = 0$; and

(h) $\mathrm{Tr}\left(\gamma^5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma\right) = 4i\varepsilon^{\mu\nu\rho\sigma}$, where $\varepsilon^{\mu\nu\rho\sigma}$ is antisymmetric under the interchange of any two indices.

$$\overline{|m|^2} = \underbrace{\frac{1}{2}\times\frac{1}{2}}_{\text{自旋平均}} \times \sum_{\text{spin}} |m|^2$$



$$q = p_1 + p_2$$
$$= p_3 + p_4$$

$$\overline{|m|^2} = \left(\frac{1}{2}\times\frac{1}{2}\right) Tr\left(\not{p}_1 \gamma_\beta \not{p}_2 \gamma_\alpha\right) Tr\left((\not{p}_4 - m_\mu)\gamma^\beta(\not{p}_3 + m_\mu)\gamma^\alpha\right)\left(\frac{e^4 Q_\mu^2}{s^2}\right)$$

其中 $Tr\left(\not{p}_1 \gamma_\beta \not{p}_2 \gamma_\alpha\right) = 4\left(p_{1\beta} p_{2\alpha} - (p_1\cdot p_2) g_{\alpha\beta} + p_{1\alpha} p_{2\beta}\right)$

$$Tr\left((\not{p}_4 - m_\mu)\gamma^\beta(\not{p}_3 + m_\mu)\gamma^\alpha\right) = Tr\left(\not{p}_4 \gamma^\beta \not{p}_3 \gamma^\alpha\right) - m_\mu^2 Tr\left(\gamma^\beta \gamma^\alpha\right)$$

$$= 4\left(p_4^\beta p_3^\alpha - (p_3\cdot p_4) g^{\alpha\beta} + p_4^\alpha p_3^\beta\right) - m_\mu^2\left(4 g^{\alpha\beta}\right)$$

$$\overline{|M|^2} = \left(\frac{e^4 Q_\mu^2}{s^2}\right)\left(\frac{1}{2}\times\frac{1}{2}\right)(4)(4)$$

$$\times \Big\{ (p_1\cdot p_4)(p_2\cdot p_3) - (p_1\cdot p_2)(p_3\cdot p_4) + (p_1\cdot p_3)(p_2\cdot p_4)$$

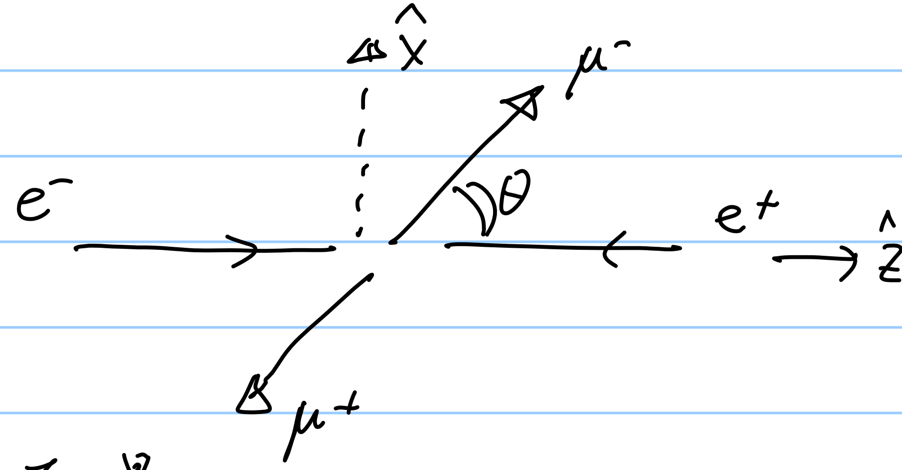
$$+ (p_1\cdot p_4)(p_2\cdot p_3) - (p_3\cdot p_4)(p_1\cdot p_2) + (p_2\cdot p_4)(p_1\cdot p_3)$$

$$+ 4(p_1\cdot p_2)(p_3\cdot p_4) - (p_1\cdot p_2)(p_3\cdot p_4) - (p_1\cdot p_2)(p_3\cdot p_4)$$

$$- m_\mu^2 \left[(p_1\cdot p_2) - 4(p_1\cdot p_2) + (p_1\cdot p_2) \right] \Big\}$$

$$= \left(\frac{e^4 Q_\mu^2}{s^2}\right)\left(\frac{1}{2}\times\frac{1}{2}\times 4\times 4\right)(2) \Big\{ (p_1\cdot p_4)(p_2\cdot p_3) + (p_1\cdot p_3)(p_2\cdot p_4) + m_\mu^2 (p_1\cdot p_2) \Big\}$$

在 e^+e^- 质心系中



$$p^2 = (p_1+p_2)^2 = (p_3+p_4)^2 = s$$

$\sqrt{s} = 2\cdot(\text{beam energy}) = 2E$ (质心系能量)

$$p_1 = (E, 0, 0, E)$$

$$p_2 = (E, 0, 0, -E)$$

$$p_3 = (E, |\vec{p}_3|\sin\theta, 0, |\vec{p}_3|\cos\theta)$$

$$p_4 = (E, -|\vec{p}_3|\sin\theta, 0, -|\vec{p}_3|\cos\theta)$$

$$|\vec{p}_3| = \sqrt{E^2 - m_\mu^2}$$

$$\frac{p_3}{E} = \beta, \quad \beta = \sqrt{1 - \frac{4m_\mu^2}{s}}$$

$$(p_1\cdot p_2) = 2E^2$$

$$(p_1\cdot p_4) = (p_2\cdot p_3) = E^2(1+\beta\cos\theta)$$

$$(p_1\cdot p_3) = (p_2\cdot p_4) = E^2(1-\beta\cos\theta)$$

$$s = 2p_1\cdot p_2$$

$$t = (m_\mu^2 - 2p_1\cdot p_3) = (m_\mu^2 - 2p_2\cdot p_4)$$

$$u = (m_\mu^2 - 2p_1\cdot p_4) = (m_\mu^2 - 2p_2\cdot p_3)$$

微分散射截面.

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\cos\theta\, d\phi} = \frac{1}{64\pi^2 s} \frac{|\vec{P_3}|}{|\vec{P_1}|} \overline{|m|^2}$$

所以,

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\cos\theta\, d\phi} = \frac{\alpha^2 Q_\mu^2}{4s} \beta \left(1+\beta^2\cos^2\theta + \frac{m_\mu^2}{E^2}\right)$$

$$= \frac{\alpha^2 Q_\mu^2}{4s} \beta \left(2-\beta^2 + \beta^2\cos^2\theta\right)$$

$$\alpha = \frac{e^2}{4\pi}$$

极端相对论情况下, $m_\mu \ll E$, $\beta \to 1$

$$\frac{d\sigma}{d\Omega} \xrightarrow[\beta\to 1]{} \frac{Q_\mu^2 \alpha^2 \beta}{4s} (1+\cos^2\theta)$$

总散射截面

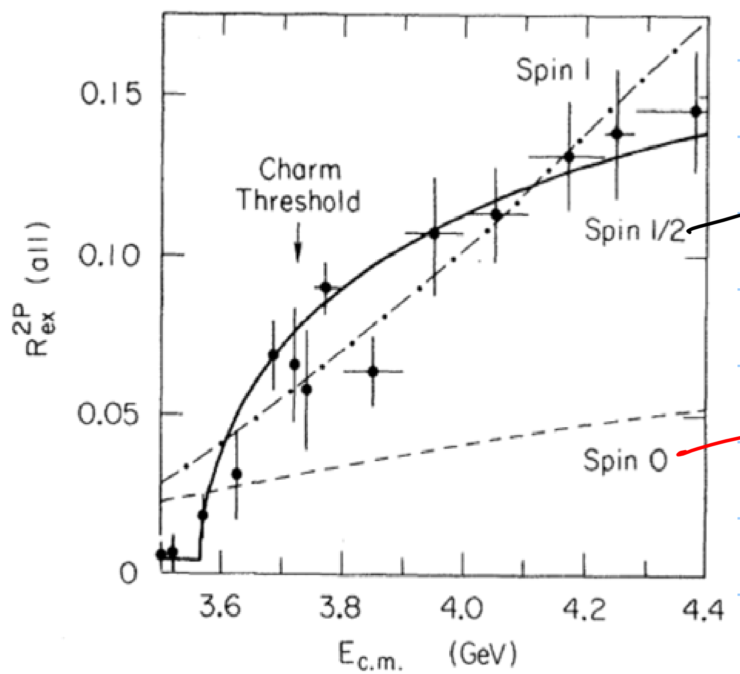
$$\sigma = \int \frac{d\sigma}{d\Omega} = \int_{\cos\theta=-1}^{+1} \int_{\phi=0}^{2\pi} \frac{d\sigma}{d\Omega}\, d\cos\theta\, d\phi = \frac{4\pi\alpha^2 Q_\mu^2 \beta}{3s}\left(\frac{3-\beta^2}{2}\right)$$

threshold附近行为

$\beta(3-\beta^2)$

$$\sigma \xrightarrow[\beta\to 1]{} \frac{4\pi\alpha^2 Q_\mu^2}{3s} \approx \frac{86.6\, Q_\mu^2}{\left(\frac{s}{GeV^2}\right)} \,[nb]$$

$10^{-33} cm^2$



$\beta(3-\beta^2)$

$\beta^?$ Homework

Bacino et al
PRL41,13, 1978

CalcHEP - 高能蒙特卡洛模拟软件

http://theory.sinp.msu.ru/~pukhov/calchep.html

CalcHEP - a package for calculation of Feynman diagrams and integration over multi-particle phase space.

Authors - Alexander Pukhov, Alexander Belyaev, Neil Christensen

The main idea of CalcHEP is to enable one to go directly from the Lagrangian to the cross sections and distributions effectively, with a high level of automation. The package can be compiled on any Unix platform.

General information

- Main features , • Acknowledgments • News&Bugs • Publications&Lectures • Contributions

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Manual

- calchep_man_3.3.6.pdf (manual for version 3.3.6, July 19, 2012)
- HEP computer tools (Lecture by Alexander Belyaev)

See also: Dan Green, High Pt physics at hadron colliders (Cambrige University Press)

Code download.

- Licence • Installation • Current version 3.6.25(23.06.2015) new options
- Archive

Models:

- MSSM_10.14(15.10.2014) • NMSSM_8.15(25.08.2015) • CPVMSSM_10.14(16.10.2014) • SUSY models By A.Semenov • LeptoQuarks • 5DSM • 6DSM

Model database • HEPMDB

Related packages on Web:

Packages for model generation: • LanHEP • FeynRules • SARAH

RGE and spectrum calculation: • SuSpect • Isajet • SoftSUSY • SPheno • CPsuperH • NMSSMTools

Particle widths in MSSM: • SUSY-HIT • HDECAY

Parton showers: • PYTHIA

Contacts

Email: calchep@googlegroups.com *Launchpad service:* • *Ask a question* • *File a bug*

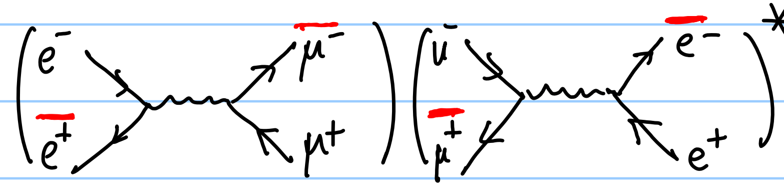
Cut-Diagram方法

在对初末态费米子自旋求和后，我们将 $\sum_{spin} \longrightarrow Tr(\quad)$

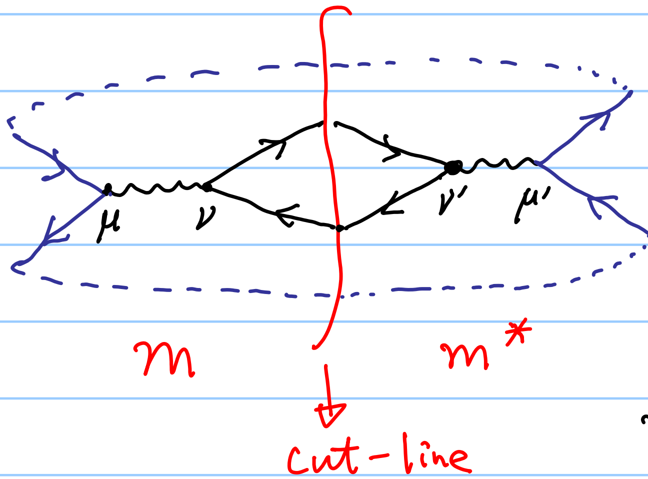
采用 Cut-diagram notation 可以帮助我们更快地计算 $\overline{|M|^2}$

$$\sum_{spin} |M|^2 = \sum_{spin} M M^{\dagger}$$

$$\sum_{spin} \left| \right|^2 = \sum_{spin} \left(\right) \left(\right)^*$$



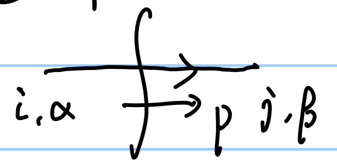
$= \sum_{spin}$



m^*中动量和费米子线都反向

Feynman Rule for cut-diagrams

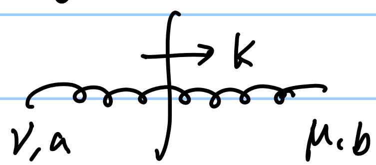
① fermion line



$$(2\pi)\,\delta^+(p^2-m^2)\,(\not p+m)_{\beta\alpha}\,\delta_{ij}$$

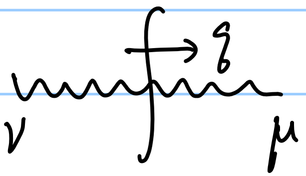
$$\hookrightarrow \delta(p^2-m^2)\,\theta(p_0)$$

② gluon line



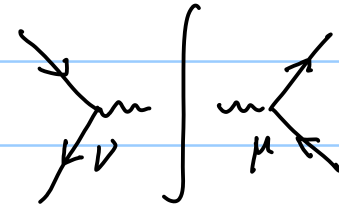
$$(2\pi)\,\delta^+(k^2)\,(-g_{\mu\nu})\,\delta_{ab}$$

③ W-boson line



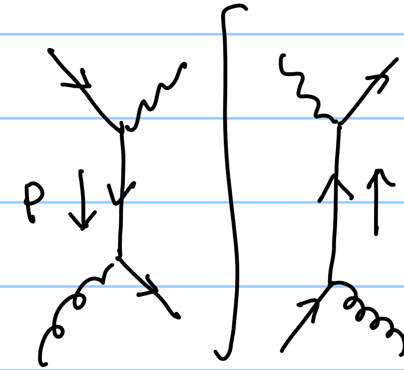
$$(2\pi)\,\delta^+(q^2-M^2)\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{M^2}\right)$$

④



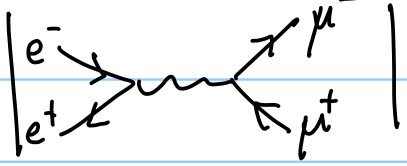
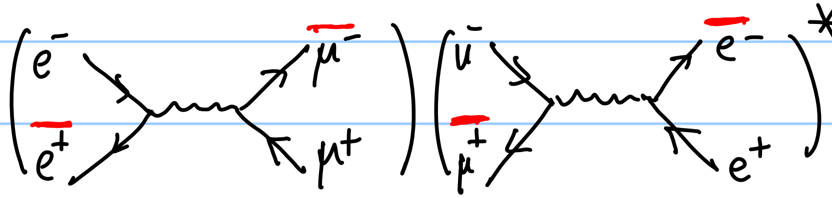
$$i\frac{g}{\sqrt{2}}\gamma_\nu\frac{1}{2}(1-\gamma_5) \qquad -i\frac{g}{\sqrt{2}}\gamma_\mu\frac{1}{2}(1-\gamma_5)$$

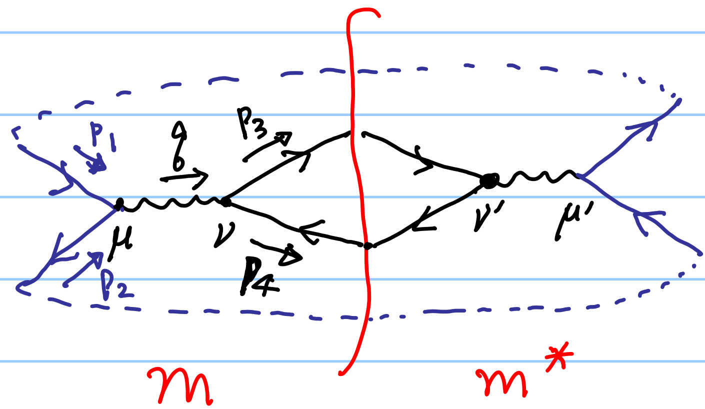
⑤



$$\frac{i(\not p+m)}{p^2-m^2+i\varepsilon} \qquad \frac{-i(\not p+m)}{p^2-m^2+i\varepsilon}$$

$e^+e^- \to \mu^+\mu^-$

$\sum_{\text{spin}} \Big|$  $\Big|^2 = \sum_{\text{spin}}$ 

$= \sum_{\text{spin}}$ 

每个封闭的费米子圈都意味着求迹

$$= \mathrm{Tr}\left[\not{p}_1\,\gamma_{\mu'}\,(-\not{p}_2)\,\gamma_\mu\right]\mathrm{Tr}\left[(\not{p}_3+m_\mu)\gamma_\nu(-\not{p}_4+m_\mu)\gamma_{\nu'}\right]$$

$$\times\ \frac{-g^{\mu\nu}}{q^2}\times\frac{-g^{\mu'\nu'}}{q^2}$$

（相空间因子另外计算）

$$= \mathrm{Tr}\left[\not{p}_1 \gamma_{\mu'} (-\not{p}_2) \gamma_\mu \right] \mathrm{Tr}\left[(\not{p}_3 + m_\mu) \gamma_\nu (-\not{p}_4 + m_\mu) \gamma_{\nu'} \right]$$

$$\times \frac{-g^{\mu\nu}}{q^2} \times \frac{-g^{\mu'\nu'}}{q^2}$$

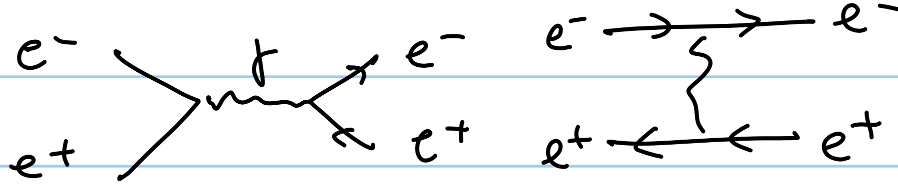
(相空间因子另外计算)

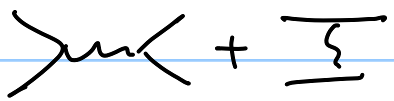
$$= \mathrm{Tr}\left[\not{p}_1 \gamma_{\mu'} \not{p}_2 \gamma_\mu \right] \mathrm{Tr}\left[(\not{p}_3 + m_\mu) \gamma_\mu (\not{p}_4 - m_\mu) \gamma_{\mu'} \right] \frac{1}{q^4}$$

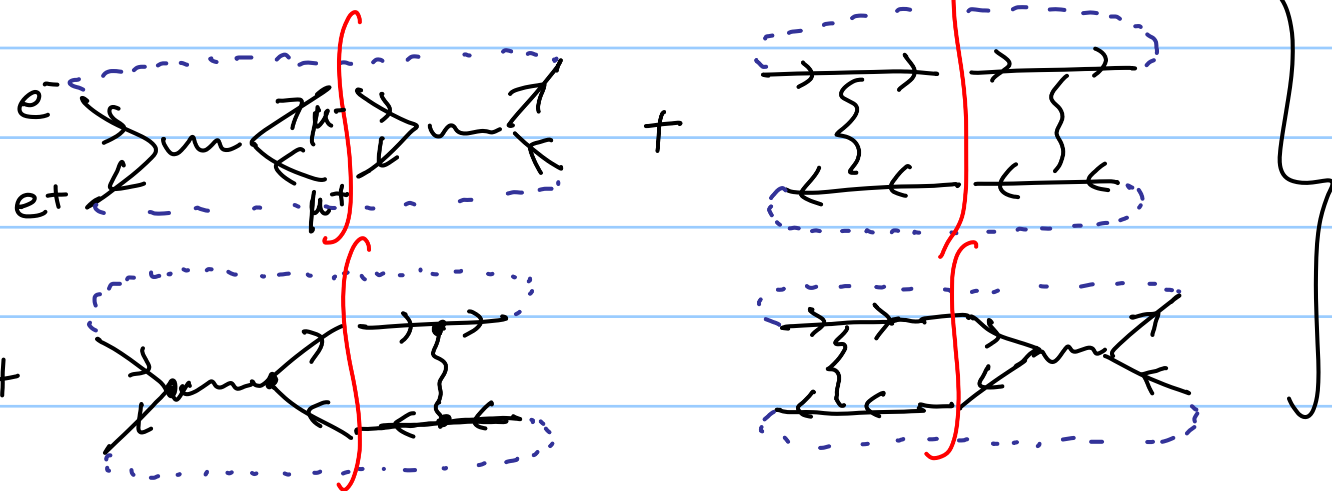
$$= \mathrm{Tr}\left[\not{p}_1 \gamma_{\mu'} \not{p}_2 \gamma_\mu \right] \mathrm{Tr}\left[(\not{p}_4 - m_\mu) \gamma^{\mu'} (\not{p}_3 + m_\mu) \gamma^{\mu} \right] \frac{1}{q^4}$$

$$= \frac{1}{q^4} \mathrm{Tr}\left[\not{p}_1 \gamma_\beta \not{p}_2 \gamma_\alpha \right] \mathrm{Tr}\left[(\not{p}_4 - m_\mu) \gamma^\beta (\not{p}_3 + m_\mu) \gamma^\alpha \right]$$

$e^- e^+ \to e^- e^+$



$\sum_{\text{spin}} \Big|$  $\Big|^2$

$=$ 

$\mathrm{Tr}[\quad] \times \mathrm{Tr}[\quad]$

FeynCalc - 高能物理符号计算软件

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About

FeynCalc is a Mathematica package for algebraic calculations in elementary particle physics.

Some of the features of FeynCalc are:

- Passarino-Veltman reduction of one-loop amplitudes to standard scalar integrals
- Tools for frequently occuring tasks like Lorentz index contraction, color factor calculation, Dirac matrix manipulation and traces, etc.
- Tensor and Dirac algebra manipulations (including traces) in 4 or D dimensions
- Generation of Feynman rules from a lagrangian
- Tools for non-commutative algebra
- SU(N) algebra

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Loading the package FeynCalc

```
<< HighEnergyPhysics`fc`
```

Loading FeynCalc from /Users/cao/Library/Mathematica/Applications/HighEnergyPhysics
FeynCalc 8.2.0 For help, type ?FeynCalc, open `FeynCalcRef8.nb` or visit www.feyncalc.org
Loading FeynArts, see www.feynarts.de for documentation

输入散射振幅模方

```
term1 = Tr[GS[p1].GA[β].GS[p2].GA[α]]
```

$$4\left(-g^{\alpha\beta}\,\mathrm{p1}\cdot\mathrm{p2}+\mathrm{p1}^{\beta}\,\mathrm{p2}^{\alpha}+\mathrm{p1}^{\alpha}\,\mathrm{p2}^{\beta}\right)$$

```
term2 = Tr[(GS[p3] - mf).GA[β].(GS[p4] + mf).GA[α]]
```

$$4\left(\mathrm{mf}^2\left(-g^{\alpha\beta}\right)-g^{\alpha\beta}\,\mathrm{p3}\cdot\mathrm{p4}+\mathrm{p3}^{\beta}\,\mathrm{p4}^{\alpha}+\mathrm{p3}^{\alpha}\,\mathrm{p4}^{\beta}\right)$$

```
numsq = Calc[term1 term2]
```

$$32\,\mathrm{mf}^2\,\mathrm{p1}\cdot\mathrm{p2}+32\,\mathrm{p1}\cdot\mathrm{p4}\,\mathrm{p2}\cdot\mathrm{p3}+32\,\mathrm{p1}\cdot\mathrm{p3}\,\mathrm{p2}\cdot\mathrm{p4}$$

```
spin = (1 / 2) * (1 / 2);
prefactor = (e^4 Q^2 / s^2);
matsq = spin * prefactor * numsq // Simplify
```

$$\frac{8\,e^4\,Q^2\left(\mathrm{mf}^2\,\mathrm{p1}\cdot\mathrm{p2}+\mathrm{p1}\cdot\mathrm{p4}\,\mathrm{p2}\cdot\mathrm{p3}+\mathrm{p1}\cdot\mathrm{p3}\,\mathrm{p2}\cdot\mathrm{p4}\right)}{s^2}$$

定义洛伦兹不变量：

replace rule #1

```
rr1 = {
   Pair[Momentum[p1], Momentum[p1]] → 0,
   Pair[Momentum[p2], Momentum[p2]] → 0,
   Pair[Momentum[p3], Momentum[p3]] → mf,
   Pair[Momentum[p4], Momentum[p4]] → mf,
   Pair[Momentum[p1], Momentum[p2]] → s / 2,
   Pair[Momentum[p1], Momentum[p3]] → (mf^2 - t) / 2,
   Pair[Momentum[p1], Momentum[p4]] → (mf^2 - u) / 2,
   Pair[Momentum[p2], Momentum[p3]] → (mf^2 - u) / 2,
   Pair[Momentum[p2], Momentum[p4]] → (mf^2 - t) / 2,
   Pair[Momentum[p3], Momentum[p4]] → (s - 2 mf^2) / 2
  }
```

$$\left\{\mathrm{p1}^2\to 0,\ \mathrm{p2}^2\to 0,\ \mathrm{p3}^2\to\mathrm{mf},\ \mathrm{p4}^2\to\mathrm{mf},\ \mathrm{p1}\cdot\mathrm{p2}\to\frac{s}{2},\ \mathrm{p1}\cdot\mathrm{p3}\to\frac{1}{2}\left(\mathrm{mf}^2-t\right),\right.$$
$$\left.\mathrm{p1}\cdot\mathrm{p4}\to\frac{1}{2}\left(\mathrm{mf}^2-u\right),\ \mathrm{p2}\cdot\mathrm{p3}\to\frac{1}{2}\left(\mathrm{mf}^2-u\right),\ \mathrm{p2}\cdot\mathrm{p4}\to\frac{1}{2}\left(\mathrm{mf}^2-t\right),\ \mathrm{p3}\cdot\mathrm{p4}\to\frac{1}{2}\left(s-2\,\mathrm{mf}^2\right)\right\}$$

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选取质心系：

In the frame of center of mass of e + and e -

```
metric = {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}};
```

$$p1 = \left\{\frac{\sqrt{s}}{2}, 0, 0, \frac{\sqrt{s}}{2}\right\};$$

$$p2 = \left\{\frac{\sqrt{s}}{2}, 0, 0, -\frac{\sqrt{s}}{2}\right\};$$

$$p3 = \frac{\sqrt{s}}{2}\{1, \beta\,\mathrm{Sin}[\theta], 0, \beta\,\mathrm{Cos}[\theta]\};$$

$$p4 = \frac{\sqrt{s}}{2}\{1, -\beta\,\mathrm{Sin}[\theta], 0, -\beta\,\mathrm{Cos}[\theta]\};$$

```
rr2 = {t → (p1 - p3).metric.(p1 - p3) // Simplify,
  u → (p1 - p4).metric.(p1 - p4) // Simplify}
```

$$rr3 = \left\{mf \to \sqrt{1-\beta\hat{}2} * \sqrt{s} \Big/ 2\right\}$$

微分散射截面

$$\mathrm{dsd}\Omega = \frac{1}{64\,\pi^2\,s} * \beta * \mathrm{matsq} \;/.\; \mathrm{rr1} \;/.\; \{e \to \mathrm{Sqrt}[4\,\pi\,\alpha]\}$$

$$\frac{2\,\alpha^2\,\beta\,Q^2\left(\frac{\mathrm{mf}^2\,s}{2} + \frac{1}{4}\left(\mathrm{mf}^2 - t\right)^2 + \frac{1}{4}\left(\mathrm{mf}^2 - u\right)^2\right)}{s^3}$$

```
dsdΩ = dsdΩ /. rr2 /. rr3 // Expand
```

$$\frac{\alpha^2\,\beta^3\,Q^2\cos^2(\theta)}{4\,s} - \frac{\alpha^2\,\beta^3\,Q^2}{4\,s} + \frac{\alpha^2\,\beta\,Q^2}{2\,s}$$

```
dsdz = dsdΩ * (2 π);
```

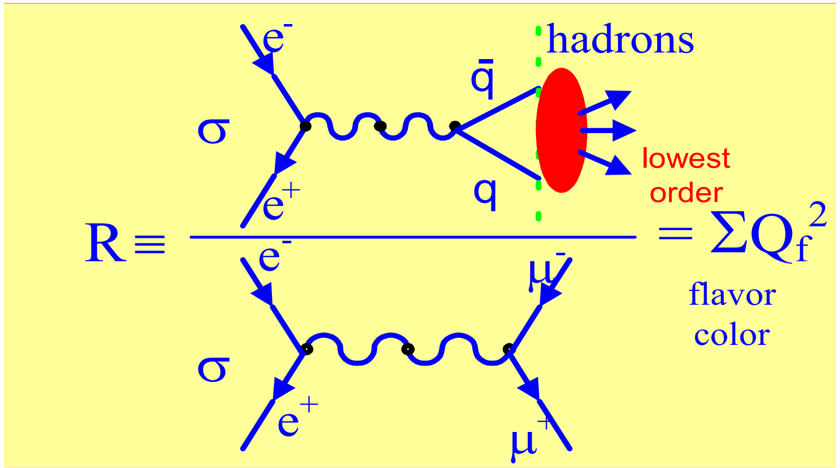
总散射截面

```
XSEC[dsdzfunc_] := Module[{z, tmp, integral, result},
   tmp = dsdzfunc /. Cos[θ] → z;
   integral = ∫tmp dz;
   result = (integral /. z → 1) - (integral /. z → -1);
   Return[result]
  ]

XSEC[dsdz] // Simplify
```

$$-\frac{2\,\pi\,\alpha^2\,\beta\left(\beta^2 - 3\right)Q^2}{3\,s}$$

R值测量



测量夸克味道和颜色数目

$$R \equiv \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$\sigma(e^+e^- \to \mu^+\mu^-) \xrightarrow{s \gg m_\mu^2} \frac{4\pi\alpha^2 Q_\mu^2}{3s}$$

$$\Rightarrow \quad R = \sum_q Q_q^2 \qquad (s \gg m_q^2)$$

还要考虑其他内部动力学自由度，例如颜色等

$$R = \sum_q Q_q^2 \quad (s \geq m_q^2)$$

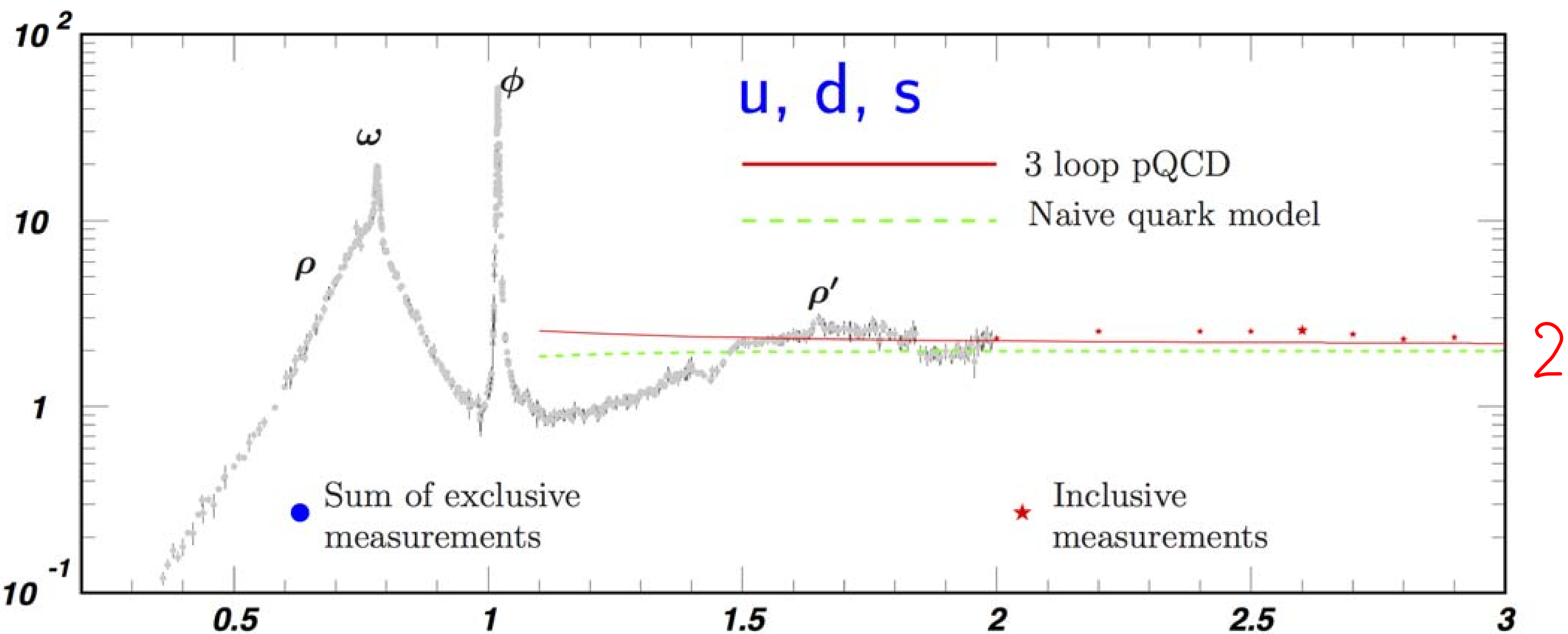
当 $1.5\,\text{GeV} < Q < 3.6\,\text{GeV}$ 时，$u\bar{u}$, $d\bar{d}$ 和 $s\bar{s}$ 可以被成对产生

如果夸克没有颜色，

$$R = Q_u^2 + Q_d^2 + Q_s^2 = \frac{2}{3}$$

如果夸克有3种颜色

$$R = 3 \times (Q_u^2 + Q_d^2 + Q_s^2) = \frac{2}{3} \times 3 = 2 \quad (\checkmark)$$

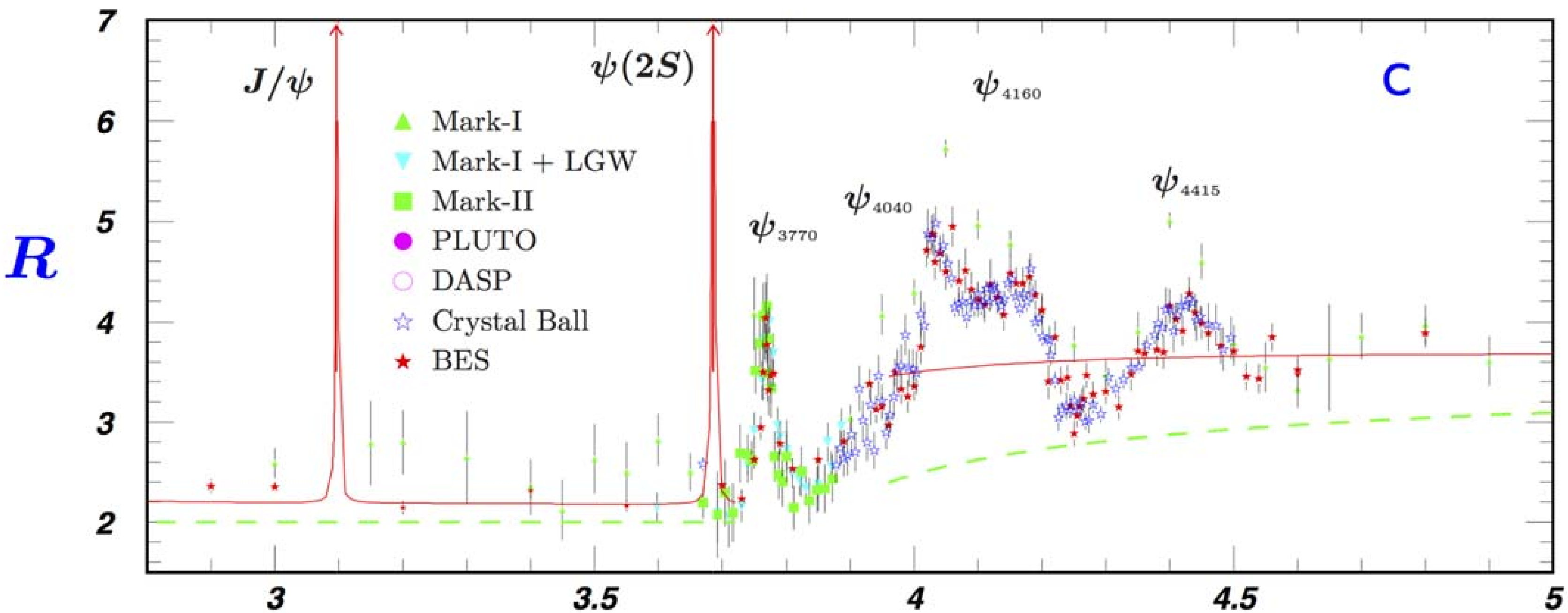


$$R = \sum_q Q_q^2 \qquad (s \gg m_q^2)$$

当能量提高时 $c\bar{c}$ 和 $b\bar{b}$ 都可能成对产生

$$R = 3 \times (Q_u^2 + Q_d^2 + Q_s^2 + Q_c^2) = \frac{10}{3}$$

$$R = 3 \times (Q_u^2 + Q_d^2 + Q_s^2 + Q_c^2 + Q_b^2) = \frac{11}{3}$$



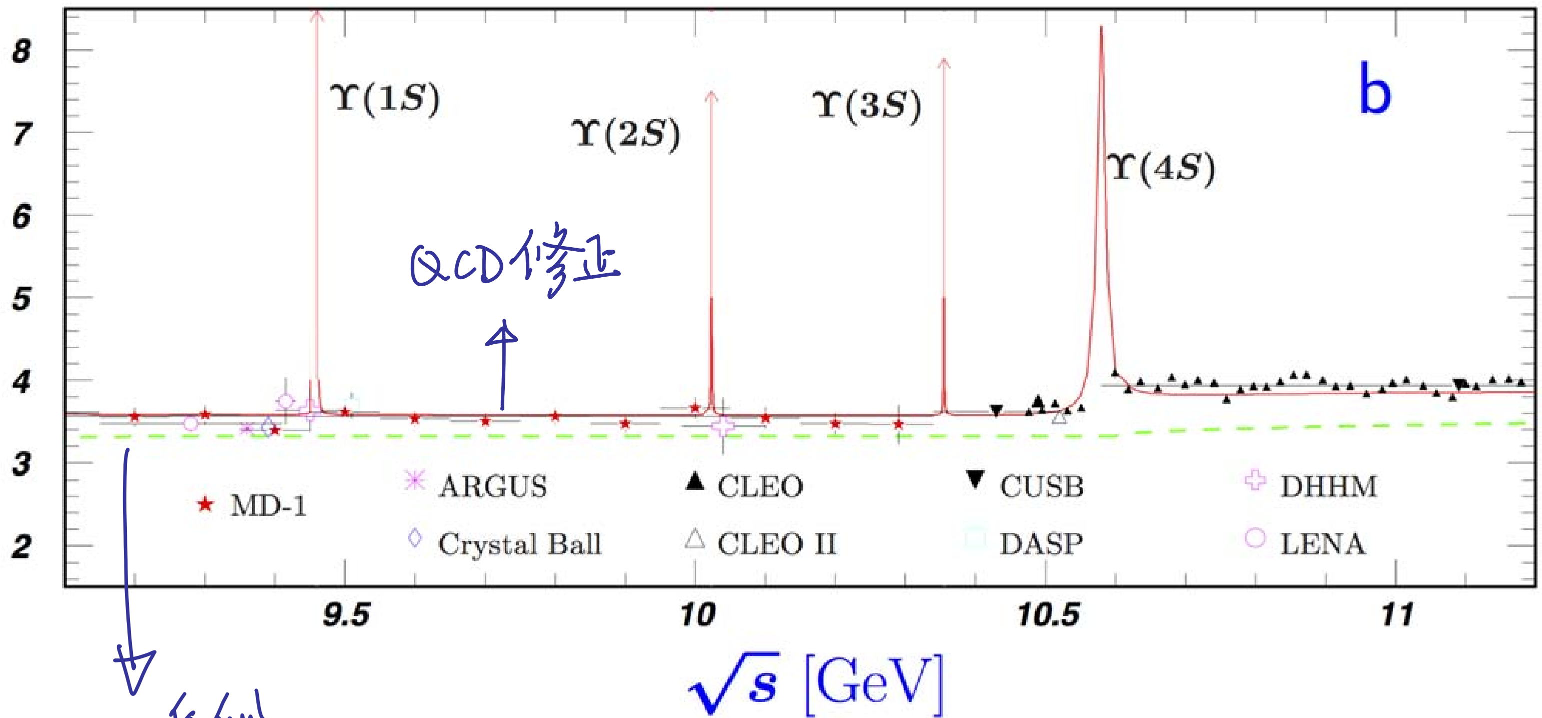
$\sim \frac{10}{3}$

$$R = \sum_q Q_q^2 \quad (s \geq m_q^2)$$

当能量提高时 $c\bar{c}$ 和 $b\bar{b}$ 都可能成对产生

$$R = 3 \times (Q_u^2 + Q_d^2 + Q_s^2 + Q_c^2) = \frac{10}{3}$$

$$R = 3 \times (Q_u^2 + Q_d^2 + Q_s^2 + Q_c^2 + Q_b^2) = \frac{11}{3}$$



QCD修正

$\sim \frac{11}{3}$

LO近似

$$R = 3\times\left(Q_u^2 + Q_d^2 + Q_s^2 + Q_c^2\right) = \frac{10}{3}$$

$$R = 3\times\left(Q_u^2 + Q_d^2 + Q_s^2 + Q_c^2 + Q_b^2\right) = \frac{11}{3}$$

好的近似

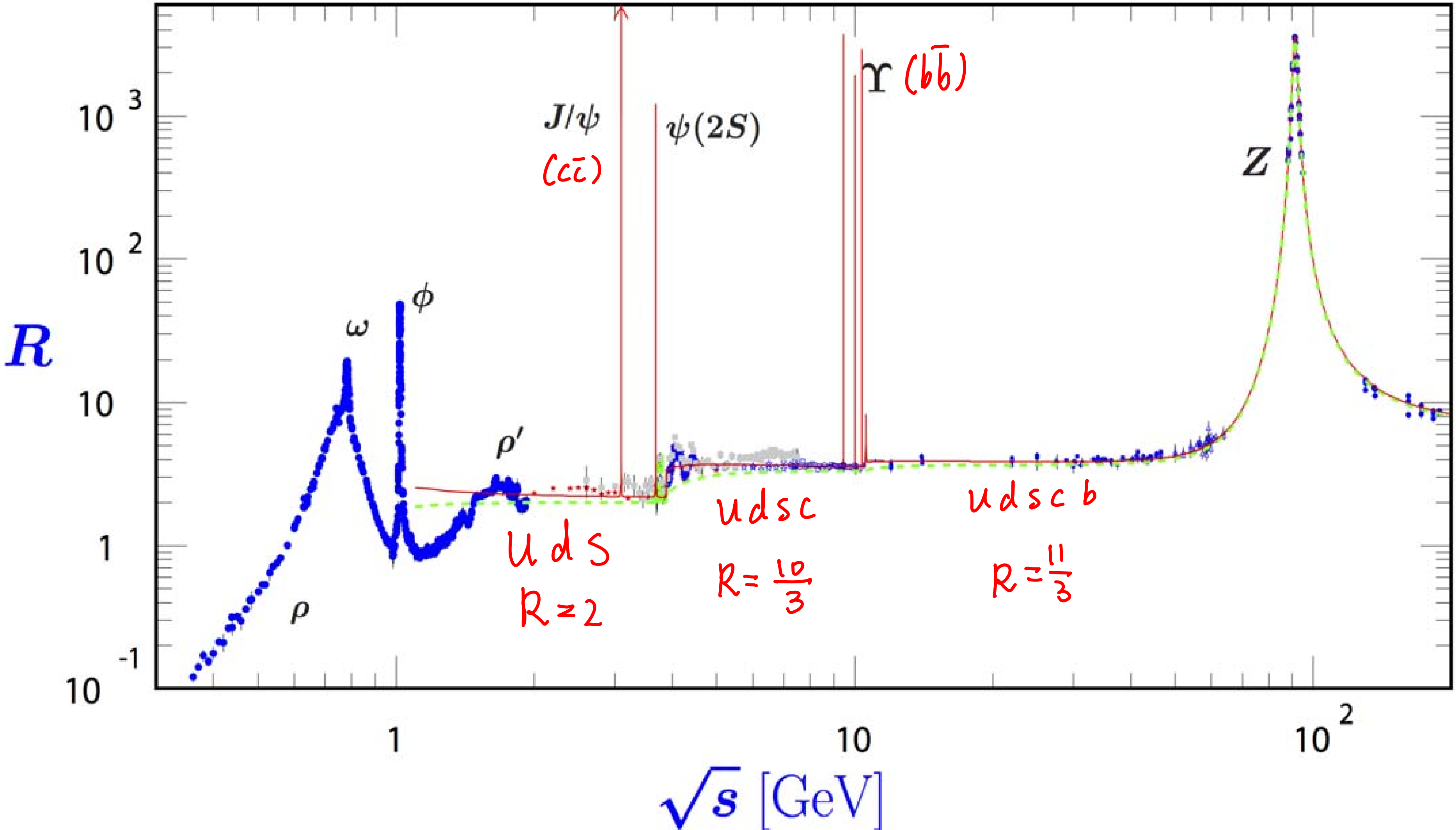
注意：考虑QCD修正后（$O(\alpha_s)$）

$$R = 3\sum_q Q_q^2\left(1 + \frac{\alpha_s(Q^2)}{\pi}\right)$$

R值的Q^2无关性被QCD的效应 $\ln(Q^2)$ 破坏

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2n_f)\ln\left(\frac{Q^2}{\Lambda^2}\right)}$$

Q^2依赖性很难测量$\ln Q^2$行为



R
$\sqrt{s}$ [GeV]
10^3
10^2
10
1
10^{-1}
1
10
10^2
ρ
ω
ϕ
ρ'
J/ψ
$(c\bar{c})$
$\psi(2S)$
Υ $(b\bar{b})$
Z
u d s
R = 2
u d s c
R = $\frac{10}{3}$
u d s c b
R = $\frac{11}{3}$

总结

1）QED散射截面：自旋求和

1.1) Completion Relations

$$\sum_{s=1}^{2} u_s \bar{u}_s = (\gamma^\mu p_\mu + mI) = \not{p} + m, \qquad \sum_{r=1}^{2} v_r \bar{v}_r = (\gamma^\mu p_\mu - mI) = \not{p} - m,$$

1.2) Trace Theorems

(a) $\mathrm{Tr}\,(I) = 4$;

(b) the trace of any odd number of γ-matrices is zero;

(c) $\mathrm{Tr}\,(\gamma^\mu \gamma^\nu) = 4g^{\mu\nu}$;

(d) $\mathrm{Tr}\,(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma) = 4g^{\mu\nu} g^{\rho\sigma} - 4g^{\mu\rho} g^{\nu\sigma} + 4g^{\mu\sigma} g^{\nu\rho}$;

(e) the trace of γ^5 multiplied by an odd number of γ-matrices is zero;

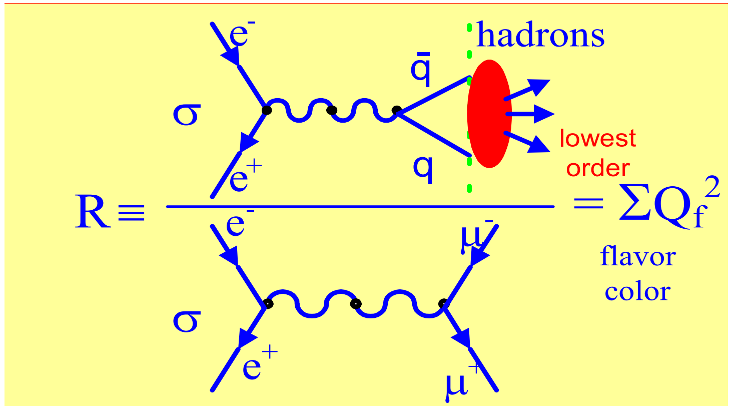
(f) $\mathrm{Tr}\left(\gamma^5\right) = 0$;

(g) $\mathrm{Tr}\left(\gamma^5 \gamma^\mu \gamma^\nu\right) = 0$; and

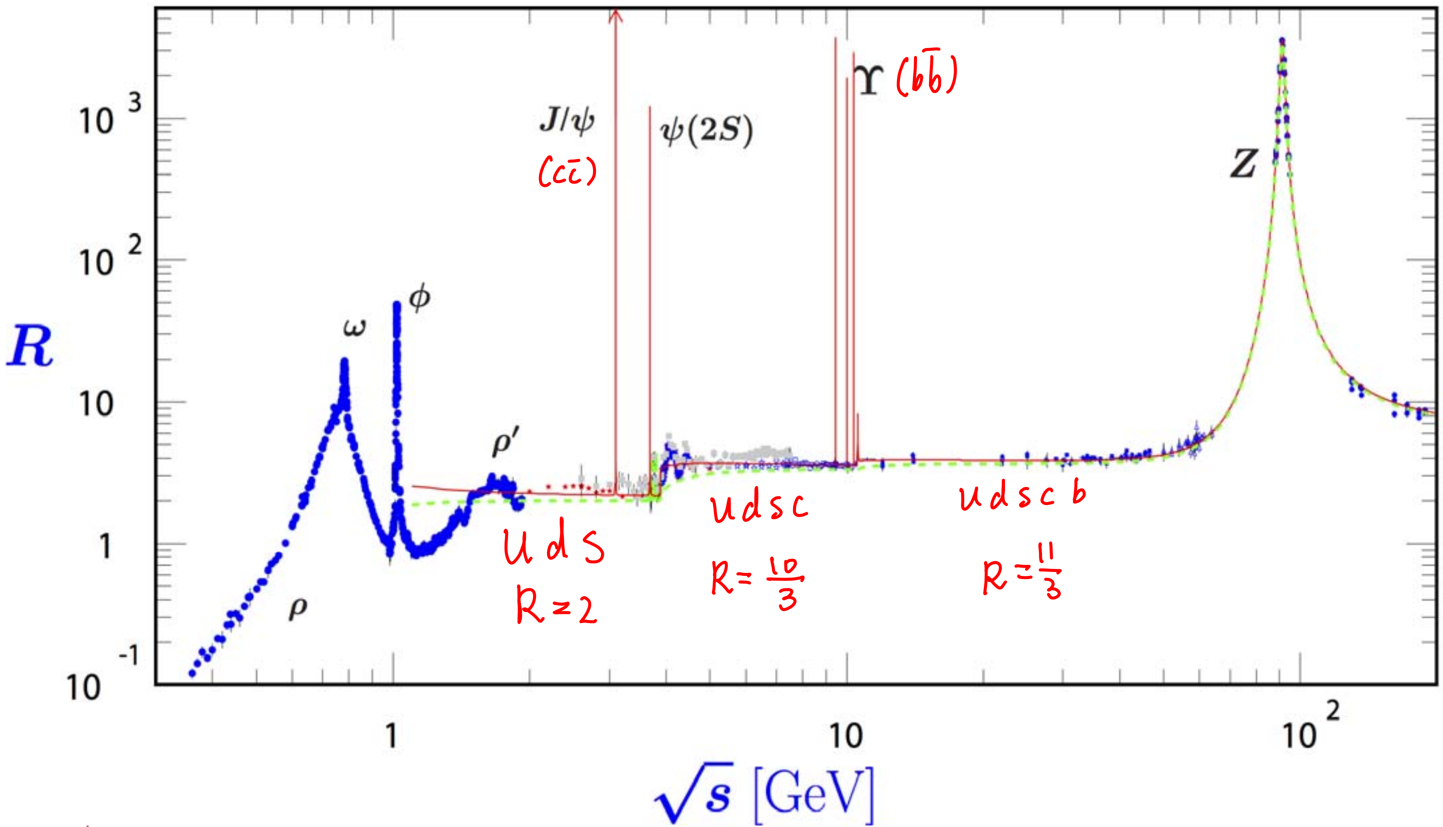
(h) $\mathrm{Tr}\left(\gamma^5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma\right) = 4i\varepsilon^{\mu\nu\rho\sigma}$, where $\varepsilon^{\mu\nu\rho\sigma}$ is antisymmetric under the interchange of any two indices.

总结

2）R值测量



hadrons
lowest
order
flavor
color
$R \equiv \frac{\sigma(e^- e^+ \to q\bar{q})}{\sigma(e^- e^+ \to \mu^- \mu^+)} = \Sigma Q_f^2$



R
$\sqrt{s}$ [GeV]
J/ψ
$(c\bar{c})$
$\psi(2S)$
Υ $(b\bar{b})$
Z
ρ
ω
ϕ
ρ'
u d s
R ≈ 2
u d s c
$R = \frac{10}{3}$
u d s c b
$R = \frac{11}{3}$