粒子物理

18. 量子色动力学（II）
(Quantum Chromodynamics)

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From QED to QCD

★ Suppose there is another fundamental symmetry of the universe, say

"invariance under SU(3) local phase transformations"

- i.e. require invariance under $\psi \to \psi' = \psi e^{ig\vec{\lambda}.\vec{\theta}(x)}$ where

  $\vec{\lambda}$ are the eight 3x3 Gell-Mann matrices introduced in handout 7

  $\vec{\theta}(x)$ are 8 functions taking different values at each point in space-time

  ⇒ 8 spin-1 gauge bosons

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$ wave function is now a vector in **COLOUR SPACE**

⇒ **QCD !**

★ QCD is fully specified by require invariance under SU(3) **local** phase transformations

Corresponds to rotating states in colour space about an axis whose direction is different at every space-time point

⇒ interaction vertex: $-\frac{1}{2} i g_s \lambda^a \gamma^\mu$

★ Predicts 8 massless gauge bosons – the gluons (one for each λ)

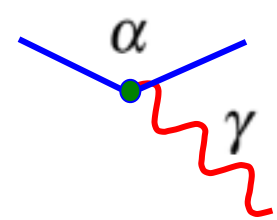
★ Also predicts exact form for interactions between gluons, i.e. the 3 and 4 gluon vertices – the details are beyond the level of this course

Colour in QCD

★ The theory of the strong interaction, Quantum Chromodynamics (QCD), is very similar to QED but with 3 conserved "colour" charges

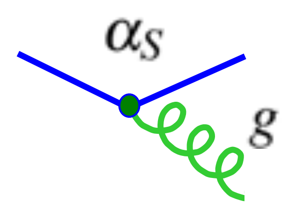
In QED:

- the electron carries one unit of charge $-e$
- the anti-electron carries one unit of anti-charge $+e$
- the force is mediated by a massless "gauge boson" – the photon



In QCD:

- quarks carry colour charge: r, g, b
- anti-quarks carry anti-charge: $\bar{r}, \bar{g}, \bar{b}$
- The force is mediated by massless gluons



★ In QCD, the strong interaction is invariant under rotations in colour space

$$r \leftrightarrow b;\ r \leftrightarrow g;\ b \leftrightarrow g$$

i.e. the same for all three colours

⇒ **SU(3) colour symmetry** ⇒ Color is conserved!

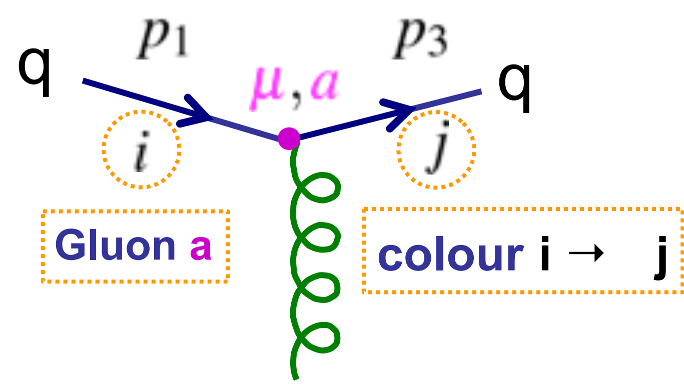
The Quark – Gluon Interaction

- **Representing the colour part of the fermion wave-functions by:**

$$r = c_1 = \begin{pmatrix}1\\0\\0\end{pmatrix} \quad g = c_2 = \begin{pmatrix}0\\1\\0\end{pmatrix} \quad b = c_3 = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

- **Particle wave-functions** $u(p) \longrightarrow c_i u(p)$
- **The QCD** qqg **vertex is written:**

$$\overline{u}(p_3)c_j^\dagger\{-\tfrac{1}{2}ig_s\lambda^a\gamma^\mu\}c_i u(p_1)$$



- **Only difference w.r.t. QED is the insertion of the 3x3 SU(3) Gell-Mann matrices**

- **Isolating the colour part:**

$$c_j^\dagger \lambda^a c_i = c_j^\dagger \begin{pmatrix}\lambda^a_{1i}\\ \lambda^a_{2i}\\ \lambda^a_{3i}\end{pmatrix} = \lambda^a_{ji}$$

Hence the fundamental quark - gluon QCD interaction can be written

$$\overline{u}(p_3)c_j^\dagger\{-\tfrac{1}{2}ig_s\lambda^a\gamma^\mu\}c_i u(p_1) \equiv \overline{u}(p_3)\{-\tfrac{1}{2}ig_s\lambda^a_{ji}\gamma^\mu\}u(p_1)$$

Feynman Rules for QCD

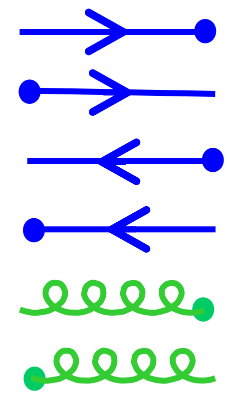
- **External Lines**

spin 1/2

incoming quark	$u(p)$	
outgoing quark	$\bar{u}(p)$	
incoming anti-quark	$\bar{v}(p)$	
outgoing anti-quark	$v(p)$	

spin 1

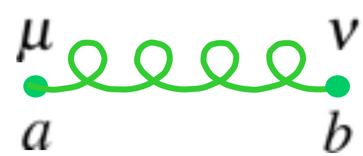
incoming gluon	$\varepsilon^{\mu}(p)$	
outgoing gluon	$\varepsilon^{\mu}(p)^{*}$	



- **Internal Lines (propagators)**

spin 1 gluon

$$\frac{-ig_{\mu\nu}}{q^2}\delta^{ab}$$

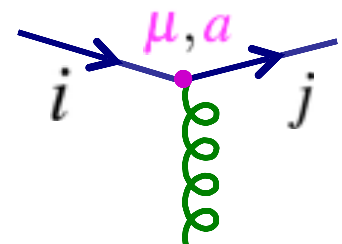


a, b = 1,2,…,8 are gluon colour indices

- **Vertex Factors**

spin 1/2 quark

$$-ig_s\tfrac{1}{2}\lambda^{a}_{ji}\gamma^{\mu}$$



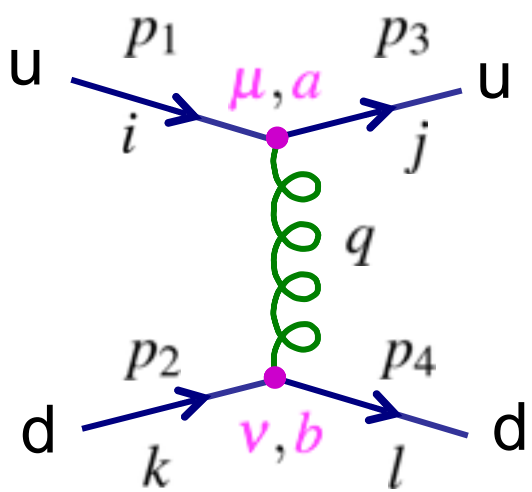
i, j = 1,2,3 are quark colours,

λ^a a = 1,2,..8 are the Gell-Mann SU(3) matrices

- **+ 3 gluon and 4 gluon interaction vertices**
- **Matrix Element** $-iM$ = product of all factors

Matrix Element for quark-quark scattering

★ Consider QCD scattering of an up and a down quark



- The incoming and out-going quark colours are labelled by $i, j, k, l = \{1,2,3\}$ (or $\{r,g,b\}$)
- In terms of colour this scattering is $$ik \rightarrow jl$$
- The 8 different gluons are accounted for by the colour indices $a, b = 1, 2, \ldots, 8$
- NOTE: the δ-function in the propagator ensures $a = b$, i.e. the gluon "emitted" at a is the same as that "absorbed" at b

★ Applying the Feynman rules:

$$-iM = \left[\bar{u}_u(p_3)\{-\tfrac{1}{2}ig_s\lambda^a_{ji}\gamma^\mu\}u_u(p_1)\right]\frac{-ig_{\mu\nu}}{q^2}\delta^{ab}\left[\bar{u}_d(p_4)\{-\tfrac{1}{2}ig_s\lambda^b_{lk}\gamma^\nu\}u_d(p_2)\right]$$

where summation over a and b (and μ and ν) is implied.

★ Summing over a and b using the δ-function gives:

$$M = -\frac{g_s^2}{4}\lambda^a_{ji}\lambda^a_{lk}\frac{1}{q^2}g_{\mu\nu}[\bar{u}_u(p_3)\gamma^\mu u_u(p_1)][\bar{u}_d(p_4)\gamma^\nu u_d(p_2)]$$

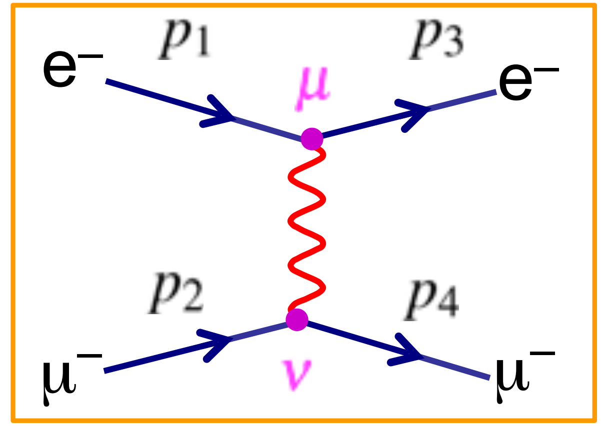
Sum over all 8 gluons (repeated indices)

QCD vs QED

QED

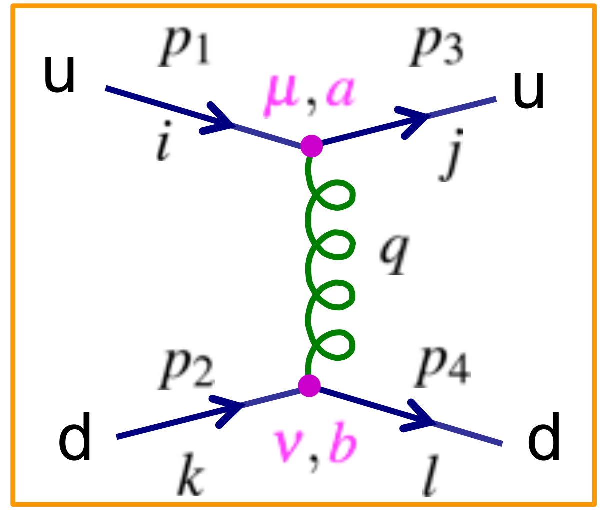
$$-iM = [\overline{u}(p_3)ie\gamma^{\mu}u(p_1)]\frac{-ig_{\mu\nu}}{q^2}[\overline{u}(p_4)ie\gamma^{\nu}u(p_2)]$$

$$M = -e^2\frac{1}{q^2}g_{\mu\nu}[\overline{u}(p_3)\gamma^{\mu}u(p_1)][\overline{u}(p_4)\gamma^{\nu}u(p_2)]$$



QCD

$$M = -\frac{g_s^2}{4}\lambda^a_{ji}\lambda^a_{lk}\frac{1}{q^2}g_{\mu\nu}[\overline{u}_u(p_3)\gamma^{\mu}u_u(p_1)][\overline{u}_d(p_4)\gamma^{\nu}u_d(p_2)]$$



★ **QCD Matrix Element = QED Matrix Element with:**

- $e^2 \to g_s^2$ **or equivalently** $\alpha = \frac{e^2}{4\pi} \to \alpha_s = \frac{g_s^2}{4\pi}$

+ QCD Matrix Element includes an additional "colour factor"

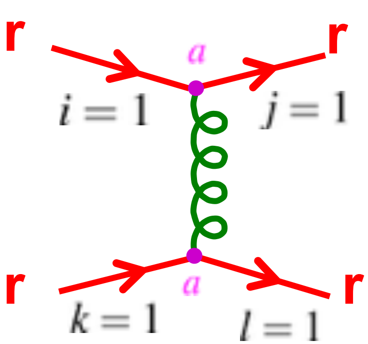
$$C(ik \to jl) \equiv \frac{1}{4}\sum_{a=1}^{8}\lambda^a_{ji}\lambda^a_{lk}$$

Evaluation of QCD Colour Factors

- QCD colour factors reflect the gluon states that are involved

$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ $\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$	$\lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ $\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$	$\lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$
Gluons: $r\bar{g},\, g\bar{r}$	$r\bar{b},\, b\bar{r}$	$g\bar{b},\, b\bar{g}$	$\frac{1}{\sqrt{2}}(r\bar{r} - g\bar{g})$ $\frac{1}{\sqrt{6}}(r\bar{r} + g\bar{g} - 2b\bar{b})$

❶ Configurations involving a single colour



Only matrices with non-zero entries in **11** position are involved

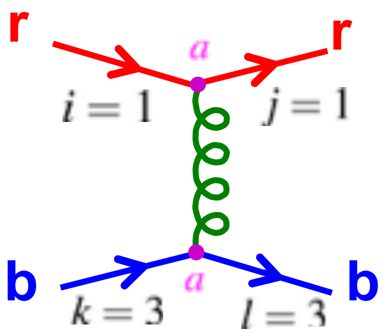
$$C(rr \to rr) = \frac{1}{4}\sum_{a=1}^{8} \lambda^a_{11}\lambda^a_{11} = \frac{1}{4}(\lambda^3_{11}\lambda^3_{11} + \lambda^8_{11}\lambda^8_{11})$$

$$= \frac{1}{4}\left(1 + \frac{1}{3}\right) = \frac{1}{3}$$

Similarly find

$$C(rr \to rr) = C(gg \to gg) = C(bb \to bb) = \frac{1}{3}$$

❷ Other configurations where quarks don't change colour e.g. $rb \rightarrow rb$



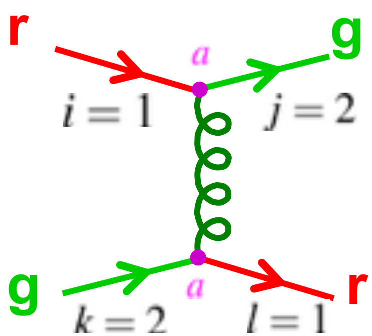
Only matrices with non-zero entries in 11 and 33 position are involved

$$C(rb \rightarrow rb) = \frac{1}{4}\sum_{a=1}^{8} \lambda_{11}^{a}\lambda_{33}^{a} = \frac{1}{4}(\lambda_{11}^{8}\lambda_{33}^{8})$$

$$= \frac{1}{4}\left(\frac{1}{\sqrt{3}} \cdot \frac{-2}{\sqrt{3}}\right) = -\frac{1}{6}$$

Similarly $C(rb \rightarrow rb) = C(rg \rightarrow rg) = C(gr \rightarrow gr) = C(gb \rightarrow gb) = C(br \rightarrow br) = C(bg \rightarrow bg) = -\frac{1}{6}$

❸ Configurations where quarks swap colours e.g. $rg \rightarrow gr$



Only matrices with non-zero entries in 12 and 21 position are involved

Gluons $r\bar{g}, g\bar{r}$

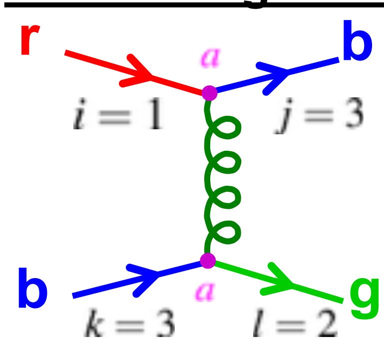
$$C(rg \rightarrow gr) = \frac{1}{4}\sum_{a=1}^{8} \lambda_{21}^{a}\lambda_{12}^{a} = \frac{1}{4}(\lambda_{21}^{1}\lambda_{12}^{1} + \lambda_{21}^{2}\lambda_{12}^{2})$$

$$= \frac{1}{4}(i(-i)+1) = \frac{1}{2}$$

$\hat{T}_{+}^{(ij)}\hat{T}_{-}^{(kl)}$

$$C(rb \rightarrow br) = C(rg \rightarrow gr) = C(gr \rightarrow rg) = C(gb \rightarrow bg) = C(br \rightarrow rb) = C(bg \rightarrow gb) = \frac{1}{2}$$

❹ Configurations involving 3 colours e.g. $rb \rightarrow bg$



Only matrices with non-zero entries in the 13 and 32 position

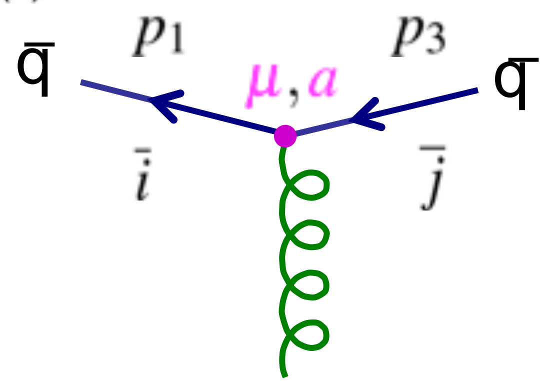
But none of the λ matrices have non-zero entries in the 13 and 32 positions. Hence the colour factor is zero

★ colour is conserved

Colour Factors : Quarks vs Anti-Quarks

- Recall the colour part of wave-function: $r = c_1 = \begin{pmatrix}1\\0\\0\end{pmatrix} \quad g = c_2 = \begin{pmatrix}0\\1\\0\end{pmatrix} \quad b = c_3 = \begin{pmatrix}0\\0\\1\end{pmatrix}$
- The QCD qqg vertex was written:

$$\overline{u}(p_3)c_j^\dagger\{-\tfrac{1}{2}ig_s\lambda^a\gamma^\mu\}c_i u(p_1)$$



★Now consider the anti-quark vertex

The QCD q̄q̄g vertex is:

$$\overline{v}(p_1)c_i^\dagger\{-\tfrac{1}{2}ig_s\lambda^a\gamma^\mu\}c_j v(p_3)$$

Note that the incoming anti-particle now enters on the LHS of the expression

- For which the colour part is

$$c_i^\dagger\lambda^a c_j = c_i^\dagger\begin{pmatrix}\lambda^a_{1j}\\ \lambda^a_{2j}\\ \lambda^a_{3j}\end{pmatrix} = \lambda^a_{ij}$$

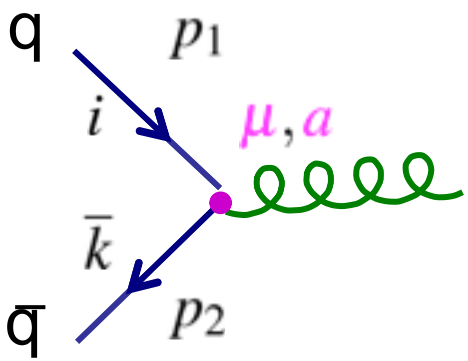
i.e indices *ij* are swapped with respect to the quark case

Hence $\overline{v}(p_1)c_i^\dagger\{-\tfrac{1}{2}ig_s\lambda^a\gamma^\mu\}c_j v(p_3) \equiv \overline{v}(p_1)\{-\tfrac{1}{2}ig_s\lambda^a_{ij}\gamma^\mu\}v(p_3)$

c.f. the quark - gluon QCD interaction

$$\overline{u}(p_3)c_j^\dagger\{-\tfrac{1}{2}ig_s\lambda^a\gamma^\mu\}c_i u(p_1) \equiv \overline{u}(p_3)\{-\tfrac{1}{2}ig_s\lambda^a_{ji}\gamma^\mu\}u(p_1)$$

★ **Finally we can consider the quark – anti-quark annihilation**

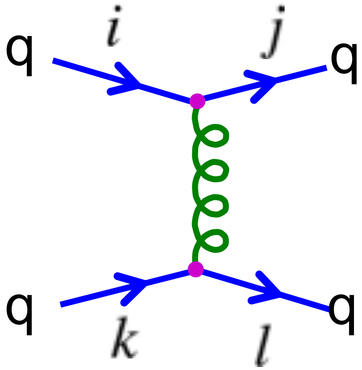


QCD vertex: $\quad \overline{v}(p_2)c_k^{\dagger}\{-\frac{1}{2}ig_s\lambda^a\gamma^{\mu}\}c_i u(p_1)$

with $\quad c_k^{\dagger}\lambda^a c_i = \lambda^a_{ki}$

$$\boxed{\overline{v}(p_2)c_k^{\dagger}\{-\tfrac{1}{2}ig_s\lambda^a\gamma^{\mu}\}c_i u(p_1) \equiv \overline{v}(p_2)\{-\tfrac{1}{2}ig_s\lambda^a_{ki}\gamma^{\mu}\}u(p_1)}$$

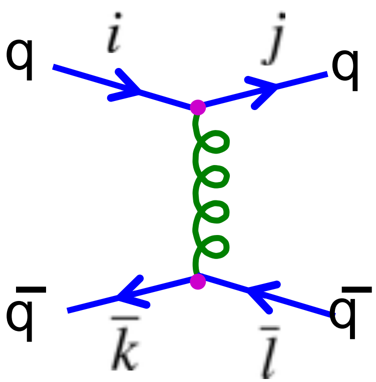
- Consequently the colour factors for the different diagrams are:



$$C(ik \to jl) \equiv \frac{1}{4}\sum_{a=1}^{8} \lambda^a_{ji}\lambda^a_{lk}$$

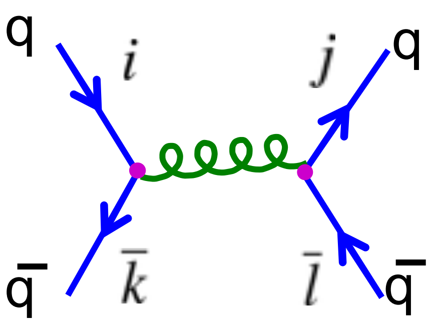
e.g.

$$C(rr \to rr) = \frac{1}{3}$$
$$C(rg \to rg) = -\frac{1}{6}$$
$$C(rg \to gr) = \frac{1}{2}$$



$$C(i\bar{k} \to j\bar{l}) \equiv \frac{1}{4}\sum_{a=1}^{8} \lambda^a_{ji}\lambda^a_{kl}$$

$$C(r\bar{r} \to r\bar{r}) = \frac{1}{3}$$
$$C(r\bar{g} \to r\bar{g}) = -\frac{1}{6}$$
$$C(r\bar{r} \to g\bar{g}) = \frac{1}{2}$$

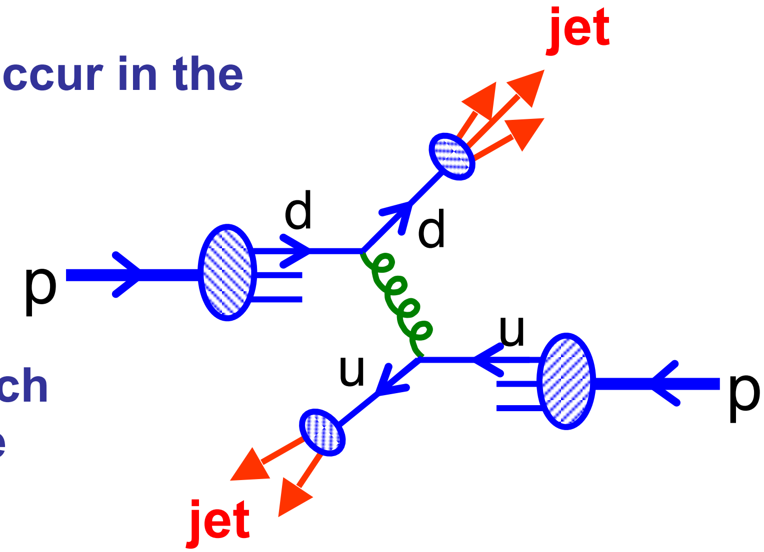


$$C(i\bar{k} \to j\bar{l}) \equiv \frac{1}{4}\sum_{a=1}^{8} \lambda^a_{ki}\lambda^a_{jl}$$

$$C(r\bar{r} \to r\bar{r}) = \frac{1}{3}$$
$$C(r\bar{g} \to r\bar{g}) = \frac{1}{2}$$
$$C(r\bar{r} \to g\bar{g}) = -\frac{1}{6}$$

Colour index of adjoint spinor comes first

Quark-Quark Scattering



- Consider the process $u+d \to u+d$ which can occur in the high energy proton-proton scattering
- There are nine possible colour configurations of the colliding quarks which are all equally likely.
- Need to determine the average matrix element which is the sum over all possible colours divided by the number of possible initial colour states

$$\langle |M_{fi}|^2 \rangle = \frac{1}{3} \cdot \frac{1}{3} \sum_{i,j,k,l=1}^{3} |M_{fi}(ij \to kl)|^2$$

- The colour average matrix element contains the average colour factor

$$\langle |C|^2 \rangle = \frac{1}{9} \sum_{i,j,k,l=1}^{3} |C(ij \to kl)|^2$$

For $qq \to qq$ $\quad$ $rr \to rr,..$ $\quad$ $rb \to rb,..$ $\quad$ $rb \to br,..$

$$\langle |C|^2 \rangle = \frac{1}{9} \left[3 \times \left(\frac{1}{3}\right)^2 + 6 \times \left(-\frac{1}{6}\right)^2 + 6 \times \left(\frac{1}{2}\right)^2 \right] = \frac{2}{9}$$

Previously derived the Lorentz Invariant cross section for $e^-\mu^- \to e^-\mu^-$ elastic scattering in the ultra-relativistic limit

QED

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2}{q^4}\left[1+\left(1+\frac{q^2}{s}\right)^2\right]$$

- For $ud \to ud$ in QCD replace $\alpha \to \alpha_s$ and multiply by $\langle |C|^2 \rangle$

QCD

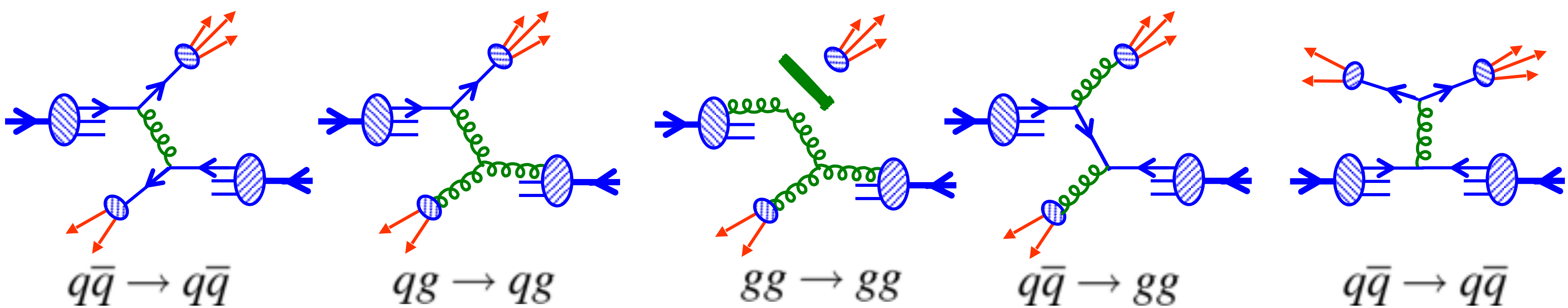
$$\frac{d\sigma}{dq^2} = \frac{2}{9}\frac{2\pi\alpha_S^2}{q^4}\left[1+\left(1+\frac{q^2}{\hat{s}}\right)^2\right]$$

Never see colour, but enters through colour factors. Can tell QCD is SU(3)

Here $\hat{s}$ is the centre-of-mass energy of the quark-quark collision

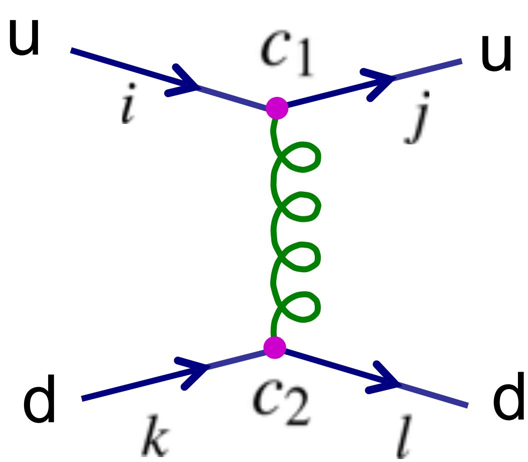
- The calculation of hadron-hadron scattering is very involved, need to include parton structure functions and include all possible interactions

e.g. two jet production in proton-antiproton collisions



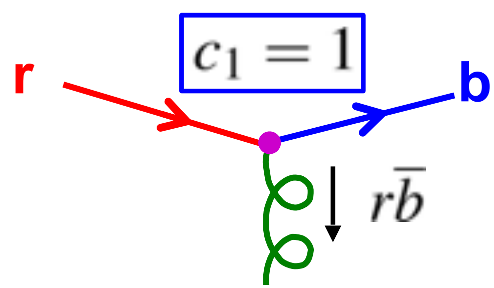
Alternative evaluation of colour factors

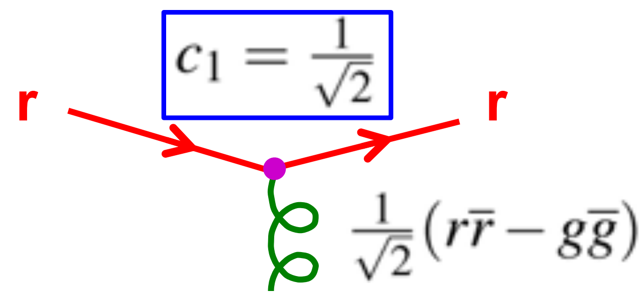
★ The colour factors can be obtained (more intuitively) as follows :



Write $C(ik \to jl) = \frac{1}{2} c_1 c_2$

Where the colour coefficients at the two vertices depend on the quark and gluon colours

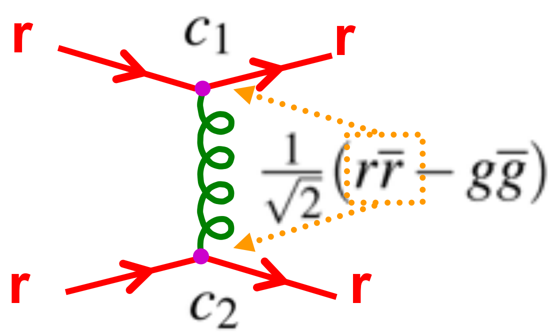


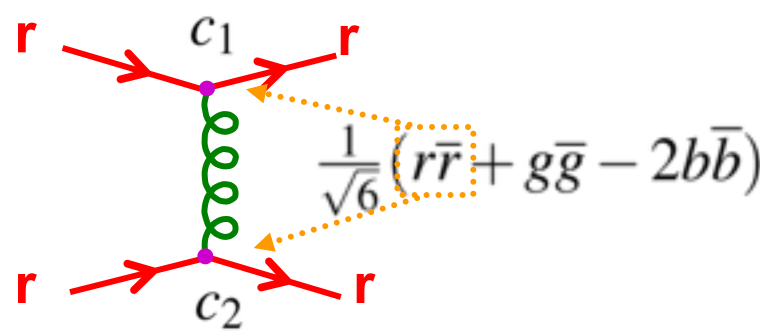


Sum over all possible exchanged gluons conserving colour at both vertices

① Configurations involving a single colour

e.g. $rr \to rr$: two possible exchanged gluons



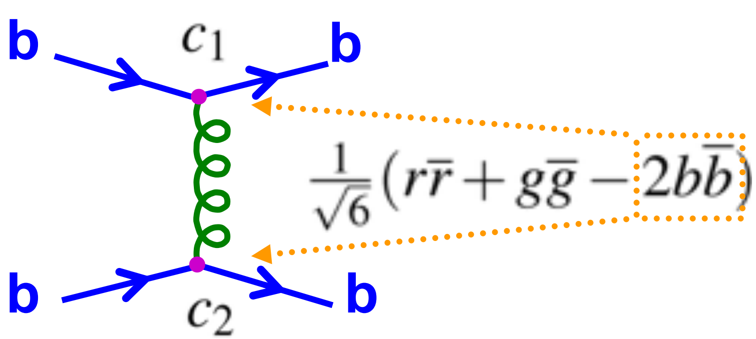


$$c_1 = c_2 = \frac{1}{\sqrt{2}}$$

$$c_1 = c_2 = \frac{1}{\sqrt{6}}$$

$$C(rr \to rr) = \tfrac{1}{2}\left(\tfrac{1}{2} + \tfrac{1}{6}\right) = \tfrac{1}{3}$$

e.g. $bb \to bb$: only one possible exchanged gluon

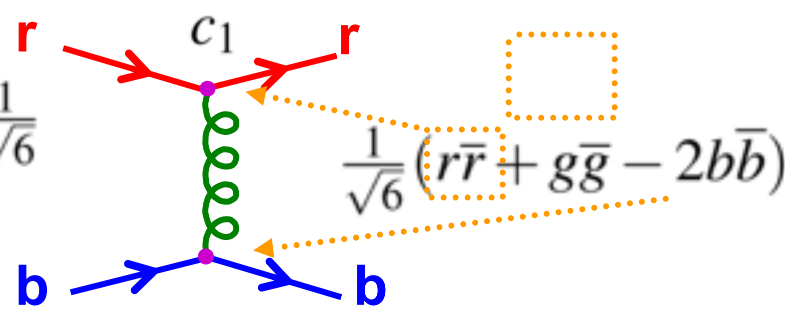


$$c_1 = c_2 = -\frac{2}{\sqrt{6}}$$

$$\Rightarrow \quad C(bb \to bb) = \tfrac{1}{2}\left(\tfrac{2}{\sqrt{6}}\tfrac{2}{\sqrt{6}}\right) = \tfrac{1}{3}$$

② Other configurations where quarks don't change colour

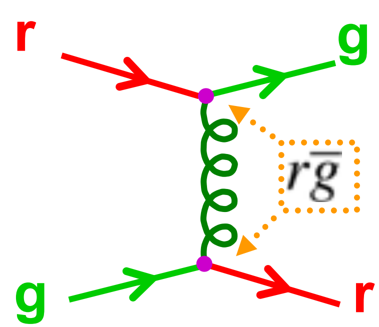
$$c_1 = \frac{1}{\sqrt{6}}$$



$$c_2 = -\frac{2}{\sqrt{6}}$$

$$C(rb \to rb) = \tfrac{1}{2}\left(-\frac{1}{\sqrt{6}}\frac{2}{\sqrt{6}}\right) = -\tfrac{1}{6}$$

③ Configurations where quarks swap colours



$$c_1 = c_2 = 1$$

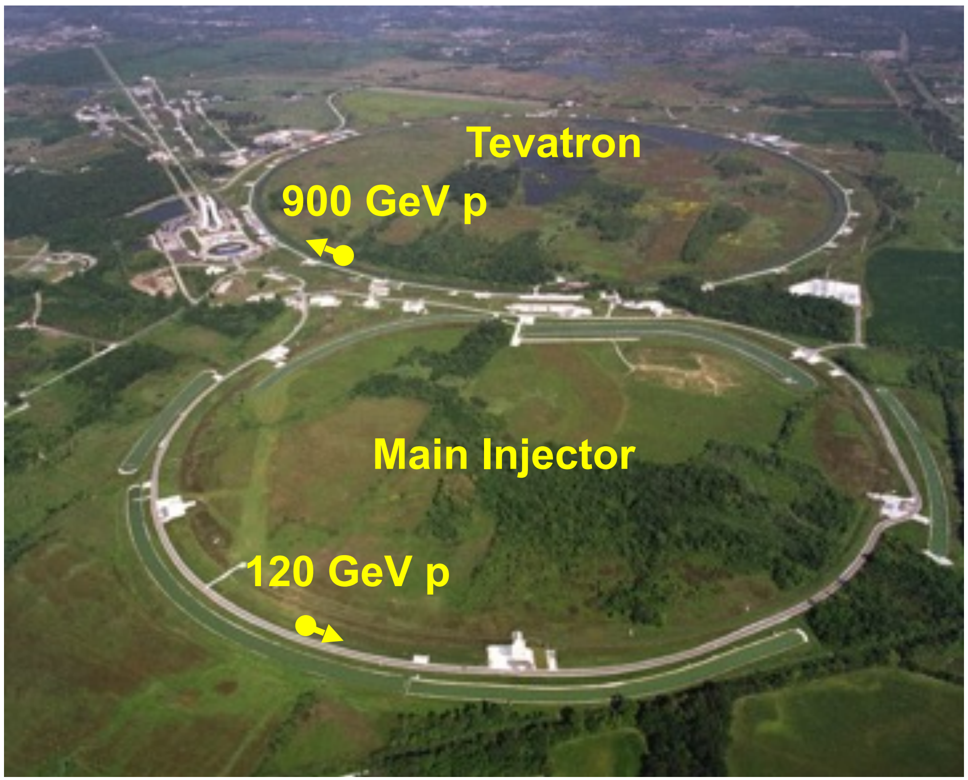
$$C(rg \to gr) = \tfrac{1}{2}$$

e.g. $p\bar{p}$ collisions at the Tevatron

★ Tevatron collider at Fermi National Laboratory (FNAL)
- located ~40 miles from Chigaco, US
- started operation in 1987 (will run until 2009/2010)

★ $p\bar{p}$ collisions at $\sqrt{s}$ = 1.8 TeV

c.f. 14 TeV at the LHC



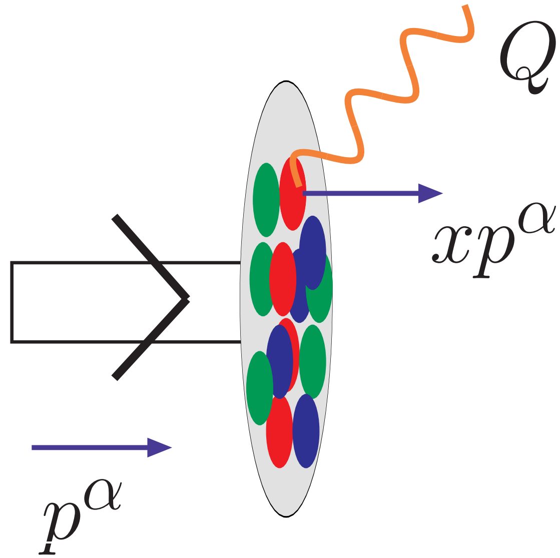
Two main accelerators:

★ Main Injector
- Accelerates 8 GeV p to 120 GeV
- also $\bar{p}$ to 120 GeV
- Protons sent to Tevatron & MINOS
- $\bar{p}$ all go to Tevatron

★ Tevatron
- 4 mile circumference
- accelerates $p/\bar{p}$ from 120 GeV to 900 GeV

Hadron Collider

In the simplest (leading-order) interpretation, the PDF $f_{a/p}(x, Q)$ is a probability for finding a parton a with 4-momentum xp^α in a proton with 4-momentum p^α



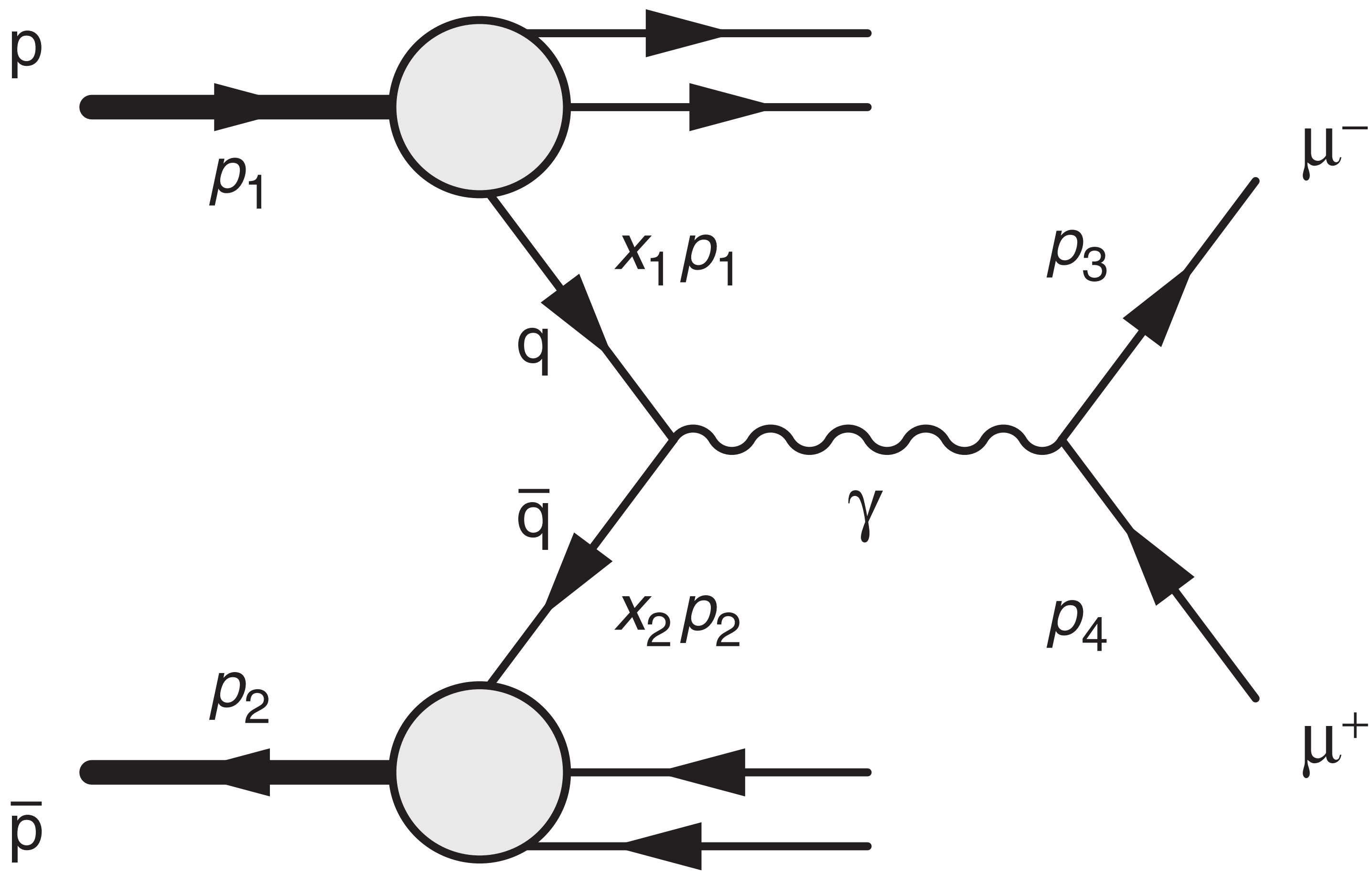
$f_{a/p}(x, Q)$ depends on **nonperturbative** QCD interactions

Drell-Yan Process at Hadron Colliders

Suggested by Sidney Drell and Tung-Mow Yan in 1970

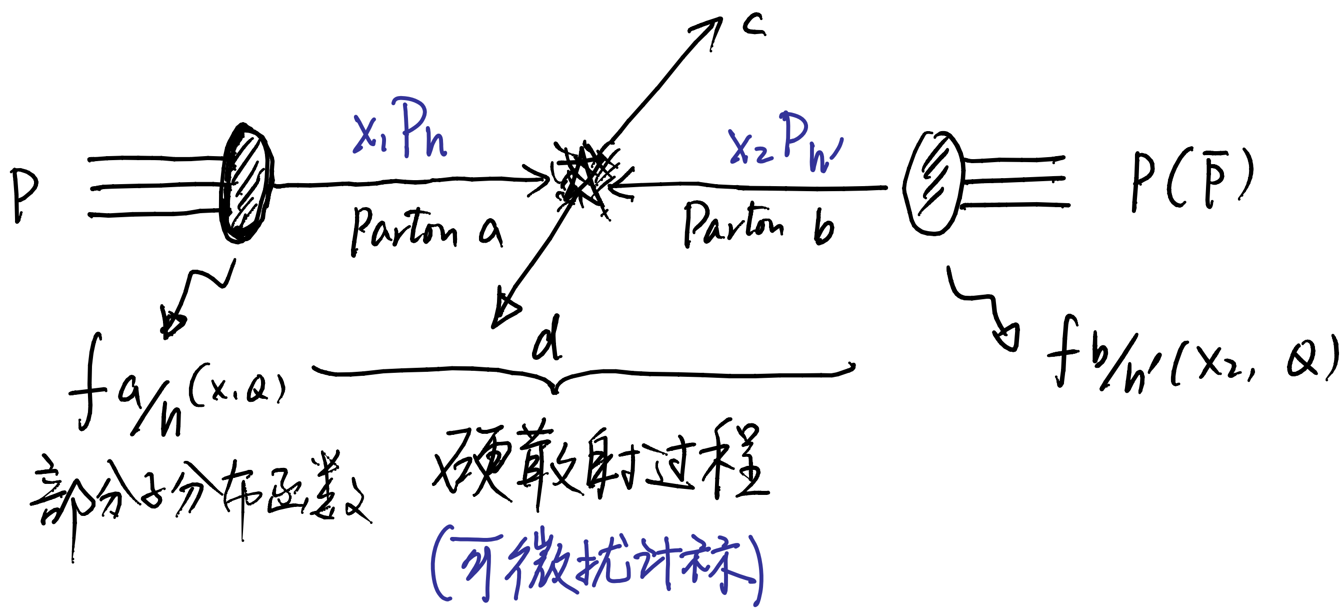
颜东茂

Observed by Christenson et al in 1970. PRL 25 (21), 1523



强子对撞机上的散射过程

$$p + p \longrightarrow c + d$$



$f_{a/h}(x,Q)$：在强子h中找到动量为 xP_h 的部分子a的几率

$$\sigma(PP \to cd) = \int_0^1 dx_1 \int_0^{x_2} dx_2 \; f_{a/h}(x_1, Q^2)\, f_{b/h'}(x_2, Q^2)\, \hat{\sigma}(a+b \to c+d)$$

$P_a = x_1 P_h$, $P_b = x_2 P_{h'}$

$$+ \; (x_1 \leftrightarrow x_2)$$

a) x_1 和 x_2 独立 $(x_1 \neq x_2)$ $\Longrightarrow$ $P_a + P_b = x_1 P_n + x_2 P_{n'} \neq 0$

实验室系并不是硬散射过程的质心系

b) 质心系能量是连续变化的

$$\hat{s} = (P_a + P_b)^2 = (x_1 P_n + x_2 P_{n'})^2 = x_1^2 P_n^2 + x_2^2 P_{n'}^2 + 2x_1 x_2 P_n P_{n'}$$

入射质子对的不变质量

$$S = (P_n + P_{n'})^2 = P_n^2 + P_{n'}^2 + 2 P_n P_{n'}$$

$$\Longrightarrow \hat{S} = x_1 x_2 S$$

好处：可同时扫描不同的质心系能量

坏处：无法确认质心系，特别是末态中有不可见粒子

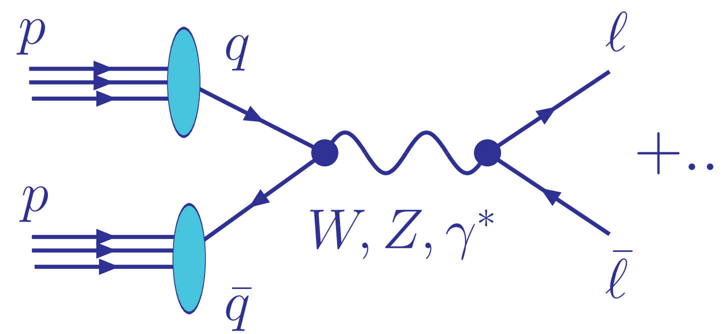
c) 为计及不同部分子的贡献，我们需要对所有可能的部分子求和

PDFs and QCD factorization

According to QCD factorization theorems, typical cross sections (e.g., for vector boson production $p(k_1)p(k_2) \to [V(q) \to \ell(k_3)\bar{\ell}(k_4)]\, X$) take the form

$$\sigma_{pp\to\ell\bar{\ell}X} = \sum_{a,b=q,\bar{q},g} \int_0^1 d\xi_1 \int_0^1 d\xi_2 \widehat{\sigma}_{ab\to V\to\ell\bar{\ell}} \left(\frac{x_1}{\xi_1}, \frac{x_2}{\xi_2}; \frac{Q}{\mu}\right) f_{a/p}(\xi_1,\mu) f_{b/p}(\xi_2,\mu)$$
$$+ \mathcal{O}\left(\Lambda^2_{QCD}/Q^2\right)$$

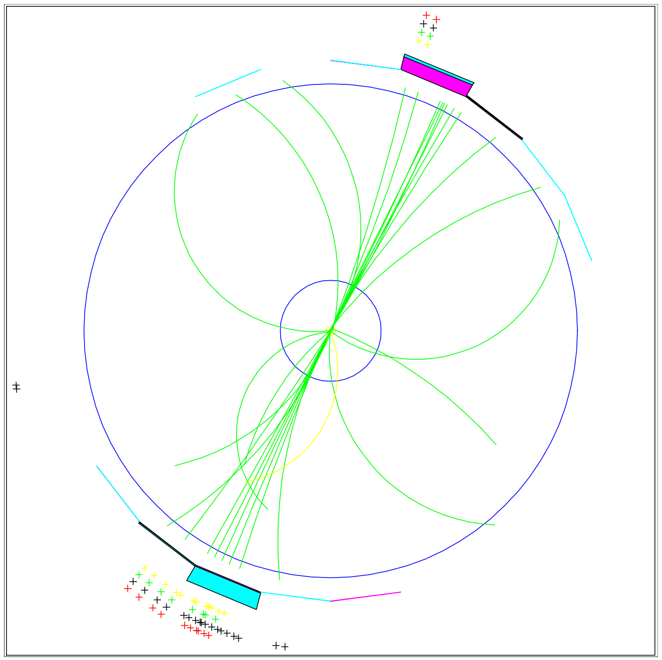
- $\widehat{\sigma}_{ab\to V\to\ell\bar{\ell}}$ is the **hard-scattering cross section**
- $f_{a/p}(\xi,\mu)$ are the **PDFs**
- $Q^2 = (k_3+k_4)^2$, $x_{1,2} = (Q/\sqrt{s})\, e^{\pm y_V}$ — measurable quantities

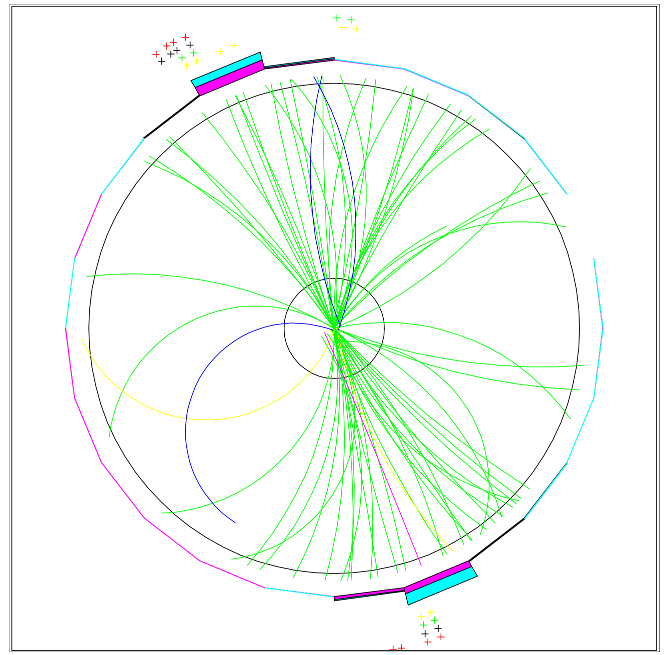


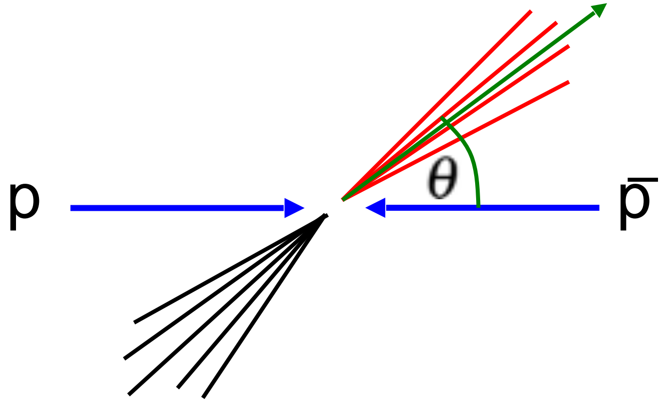
- ξ_1, ξ_2 are partonic momentum fractions (integrated over)
- μ is a factorization scale (=renormalization scale from now on)
- Factorization holds up to terms of order Λ^2_{QCD}/Q^2

★ **Test QCD predictions by looking at production of pairs of high energy jets**

$$p\bar{p} \rightarrow \text{jet jet} + X$$



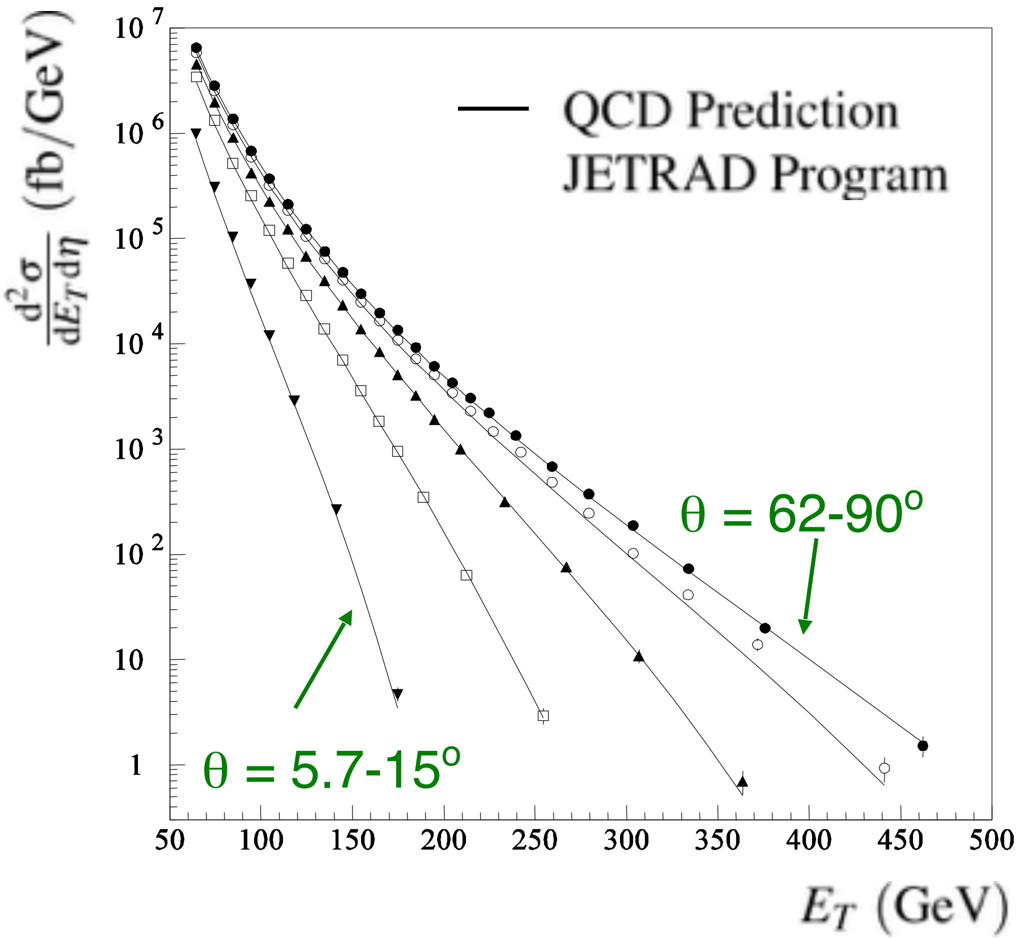




★ **Measure cross-section in terms of**

- **"transverse energy"** $E_T = E_{\text{jet}} \sin\theta$
- **"pseudorapidity"** $\eta = \ln\left[\cot\left(\frac{\theta}{2}\right)\right]$

…don't worry too much about the details here, what matters is that…



D0 Collaboration, Phys. Rev. Lett. 86 (2001)

★ **QCD predictions provide an excellent description of the data**

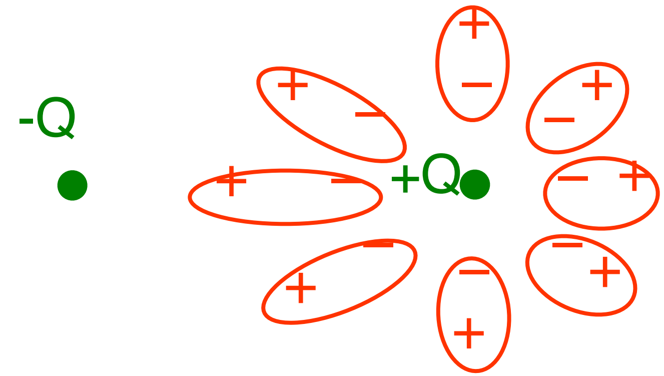
★ **NOTE:**

- **at low E_T cross-section is dominated by low x partons i.e. gluon-gluon scattering**
- **at high E_T cross-section is dominated by high x partons i.e. quark-antiquark scattering**

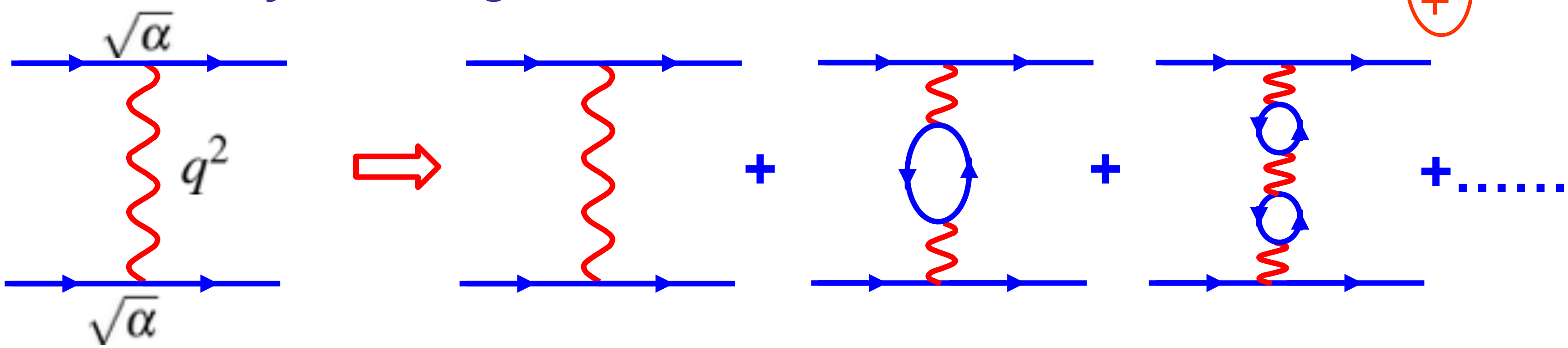
Running Coupling Constants

QED

- "bare" charge of electron screened by virtual e^+e^- pairs
- behaves like a polarizable dielectric



★ In terms of Feynman diagrams:



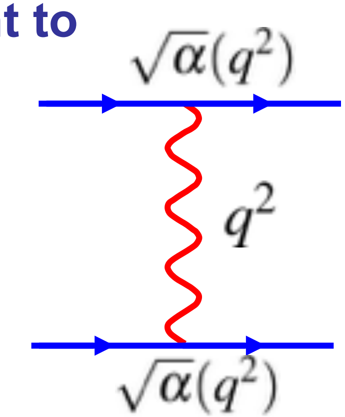
★ Same final state so add matrix element **amplitudes:** $M = M_1 + M_2 + M_3 + \ldots$

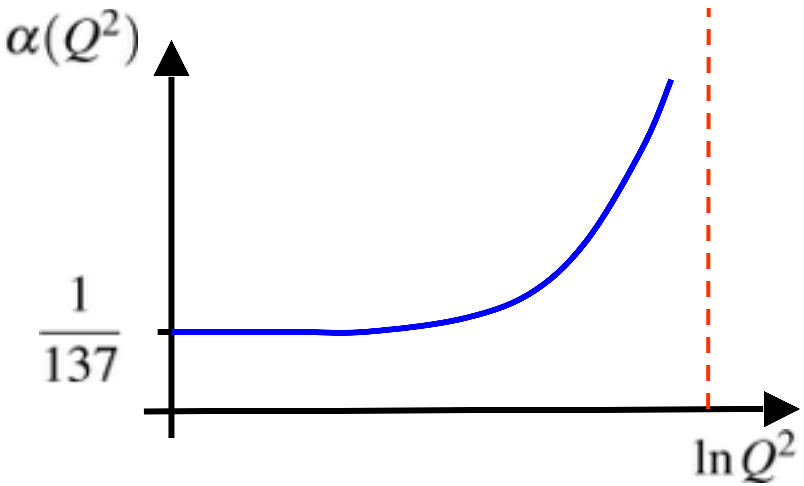
★ Giving an infinite series which can be summed and is equivalent to a single diagram with "running" coupling constant

$$\alpha(Q^2) = \alpha(Q_0^2) \Big/ \left[1 - \frac{\alpha(Q_0^2)}{3\pi} \ln\left(\frac{Q^2}{Q_0^2}\right)\right]$$

$$Q^2 \gg Q_0^2$$

Note sign





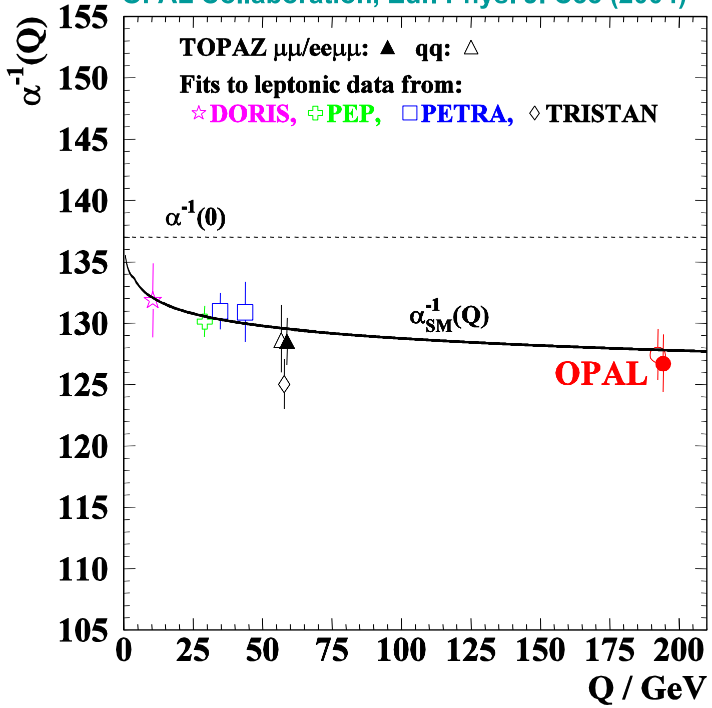
★ Might worry that coupling becomes infinite at

$$\ln\left(\frac{Q^2}{Q_0^2}\right) = \frac{3\pi}{1/137}$$

i.e. at $Q \sim 10^{26}\,\mathrm{GeV}$

- But quantum gravity effects would come in way below this energy and it is highly unlikely that QED "as is" would be valid in this regime

OPAL Collaboration, Eur. Phys. J. C33 (2004)



★ In QED, running coupling increases very slowly

- Atomic physics: $Q^2 \sim 0$

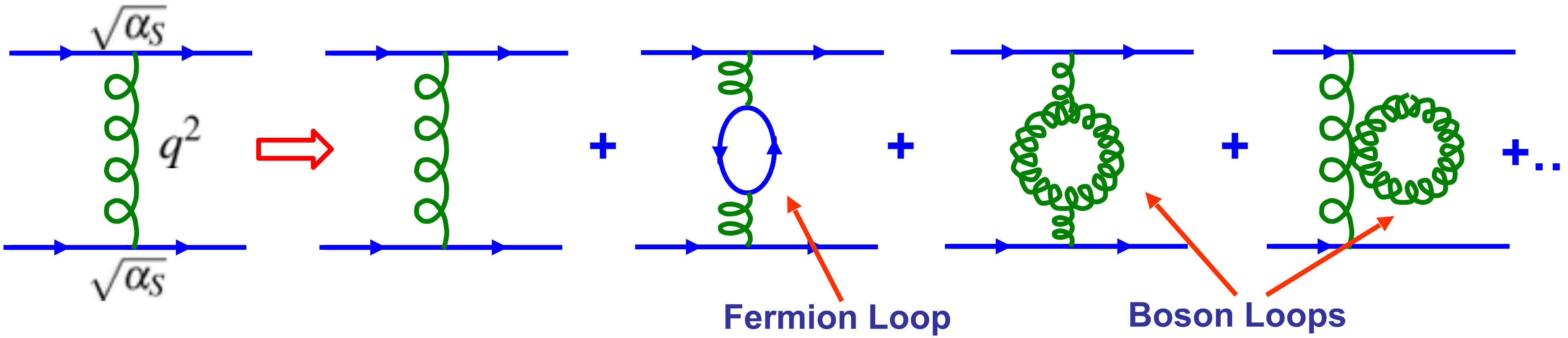
$$1/\alpha = 137.03599976(50)$$

- High energy physics:

$$1/\alpha(193\,\mathrm{GeV}) = 127.4 \pm 2.1$$

Running of α_s

QCD Similar to QED but also have gluon loops



★ Remembering **adding amplitudes**, so can get negative interference and the sum can be smaller than the original diagram alone

★ Bosonic loops "interfere negatively"

$$\alpha_S(Q^2) = \alpha_S(Q_0^2) \Big/ \left[1 + B\alpha_S(Q_0^2)\ln\left(\frac{Q^2}{Q_0^2}\right)\right]$$

with $$B = \frac{11N_c - 2N_f}{12\pi}$$

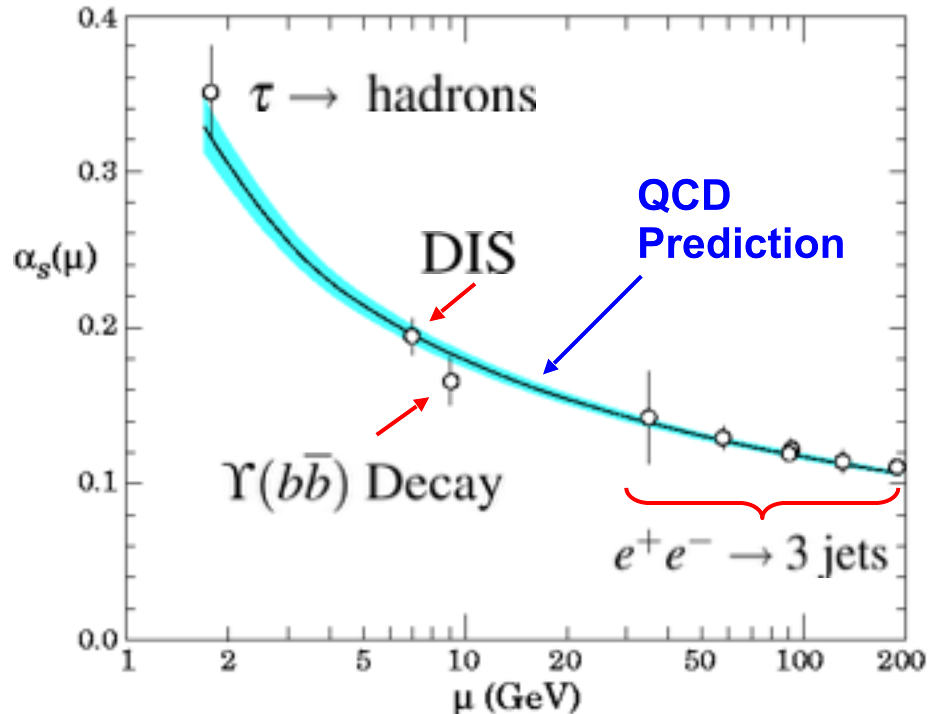
N_c = no. of colours
N_f = no. of quark flavours

$N_c = 3;\ N_f = 6$ ➡ $B > 0$

➡ α_S **decreases with Q^2**

Nobel Prize for Physics, 2004
(Gross, Politzer, Wilczek)

★ Measure α_s in many ways:
- jet rates
- DIS
- tau decays
- bottomonium decays
- +…



★ As predicted by QCD, α_s decreases with Q^2

★ At low Q^2: α_s is large, e.g. at $Q^2 = 1\,\mathrm{GeV}^2$ find $\alpha_s \sim 1$

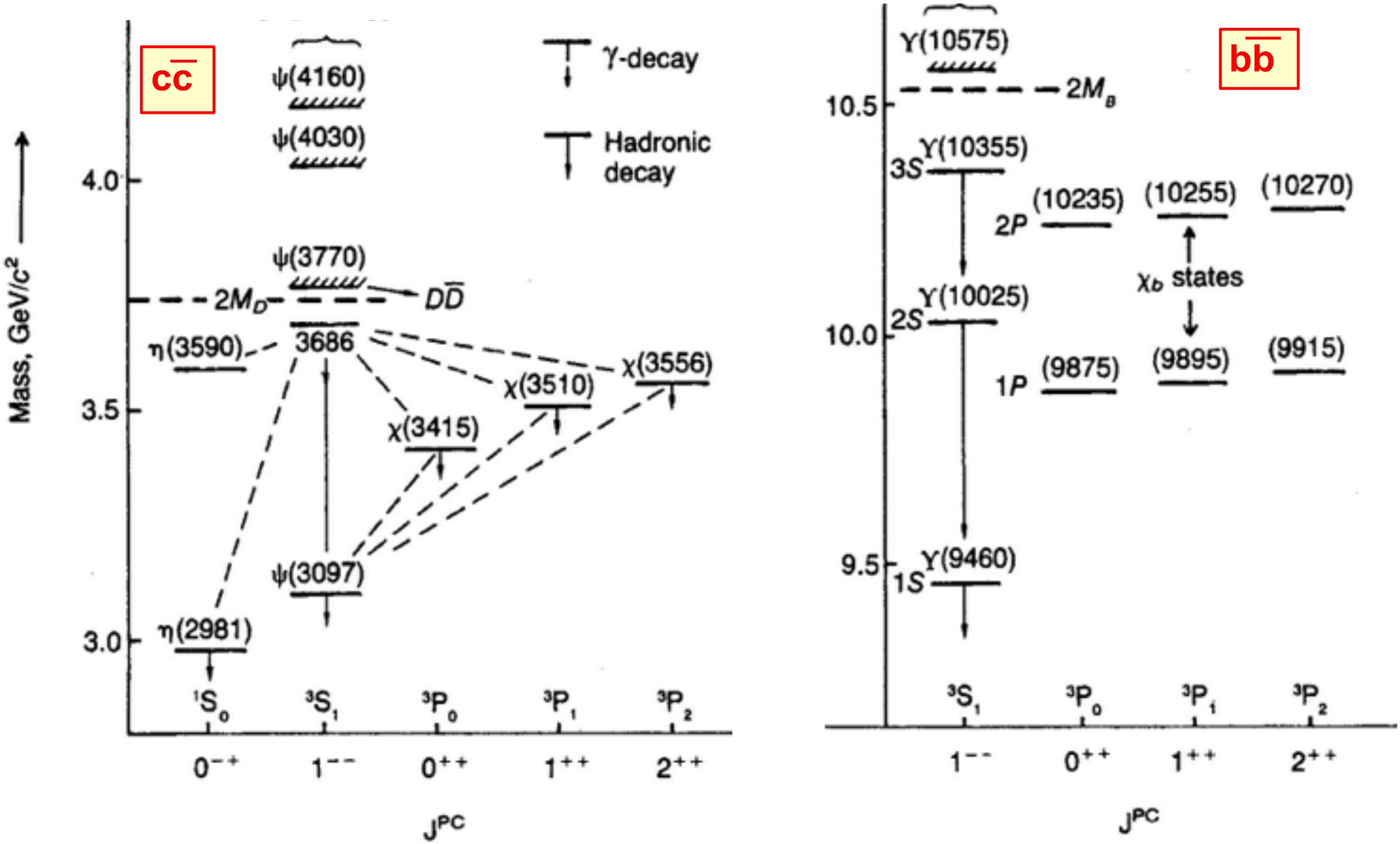
Can't use perturbation theory ! This is the reason why QCD calculations at low energies are so difficult, e.g. properties hadrons, hadronization of quarks to jets,…

★ At high Q^2 : α_s is rather small, e.g. at $Q^2 = M_Z^2$ find $\alpha_s \sim 0.12$

➡ **Asymptotic Freedom**

One can use perturbation theory and this is the reason that in DIS at high Q^2 quarks behave as if they are quasi-free (i.e. only weakly bound within hadrons)

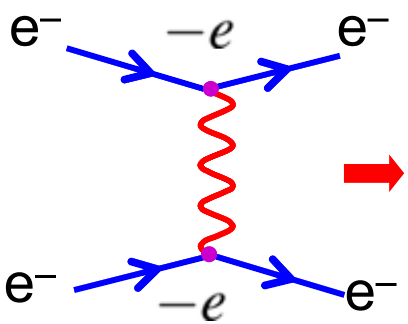
Colour Potentials



Colour Potentials

- Previously argued that gluon self-interactions lead to a $+\lambda r$ long-range potential and that this is likely to explain colour confinement
- Have yet to consider the short range potential – i.e. for quarks in mesons and baryons does QCD lead to an attractive potential?
- Analogy with QED: (NOTE this is very far from a formal proof)

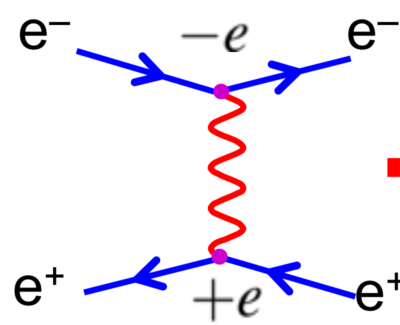
QED



$$V(r) = +\frac{\alpha}{r}$$

Repulsive Potential

Static

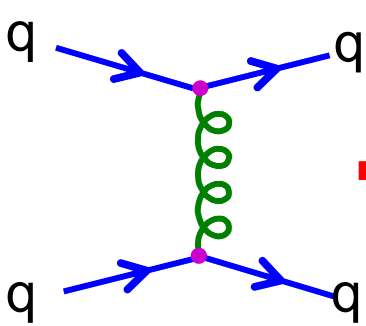


$$V(r) = -\frac{\alpha}{r}$$

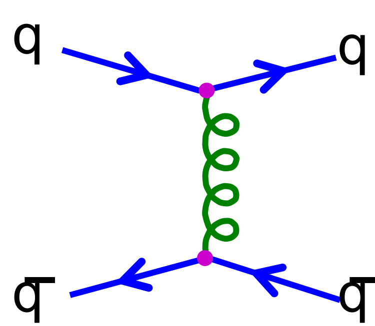
Attractive Potential

QCD

★ by analogy with QED expect potentials of form



$$V(r) = +C\frac{\alpha_S}{r}$$



$$V(r) = -C\frac{\alpha_S}{r}$$

★ Whether it is a attractive or repulsive potential depends on **sign of colour factor**

★ Consider the colour factor for a $q\bar{q}$ system in the colour singlet state:

$$\psi = \frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$$

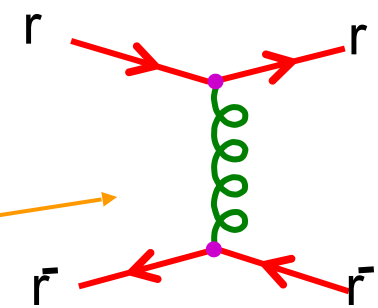
with colour potential $\langle V_{q\bar{q}} \rangle = \langle \psi | V_{\rm QCD} | \psi \rangle$

➡ $$\langle V_{q\bar{q}} \rangle = \frac{1}{3}\left(\langle r\bar{r}|V_{\rm QCD}|r\bar{r}\rangle + \ldots + \langle r\bar{r}|V_{\rm QCD}|b\bar{b}\rangle + \ldots\right)$$

Following the QED analogy:

$$\langle r\bar{r}|V_{\rm QCD}|r\bar{r}\rangle = -C(r\bar{r}\rightarrow r\bar{r})\frac{\alpha_S}{r}$$

which is the term arising from $r\bar{r}\rightarrow r\bar{r}$



Have 3 terms like $r\bar{r}\rightarrow r\bar{r}, b\bar{b}\rightarrow b\bar{b},\ldots$ and 6 like $r\bar{r}\rightarrow g\bar{g}, r\bar{r}\rightarrow b\bar{b},\ldots$

$$\langle V_{q\bar{q}} \rangle = -\frac{1}{3}\frac{\alpha_S}{r}[3\times C(r\bar{r}\rightarrow r\bar{r}) + 6\times C(r\bar{r}\rightarrow g\bar{g})] = -\frac{1}{3}\frac{\alpha_S}{r}\left[3\times\frac{1}{3} + 6\times\frac{1}{2}\right]$$

➡ $$\boxed{\langle V_{q\bar{q}} \rangle = -\frac{4}{3}\frac{\alpha_S}{r}}$$

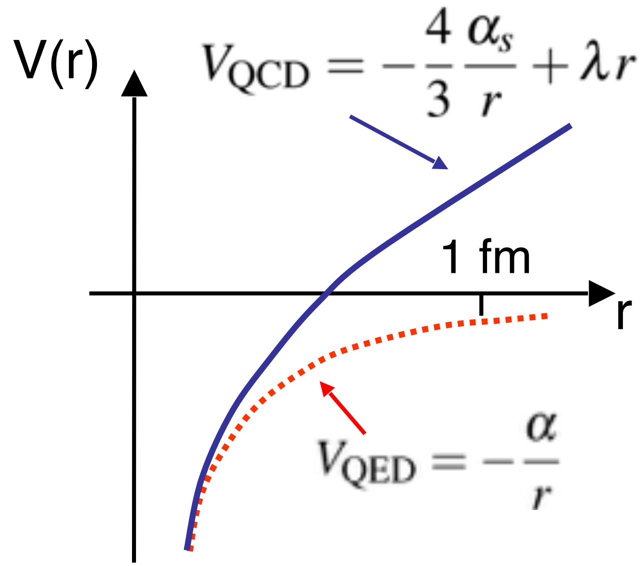
NEGATIVE ➡ ATTRACTIVE

• The same calculation for a $q\bar{q}$ colour octet state, e.g. $r\bar{g}$ gives a positive repulsive potential: $C(r\bar{g}\rightarrow r\bar{g}) = -\frac{1}{6}$

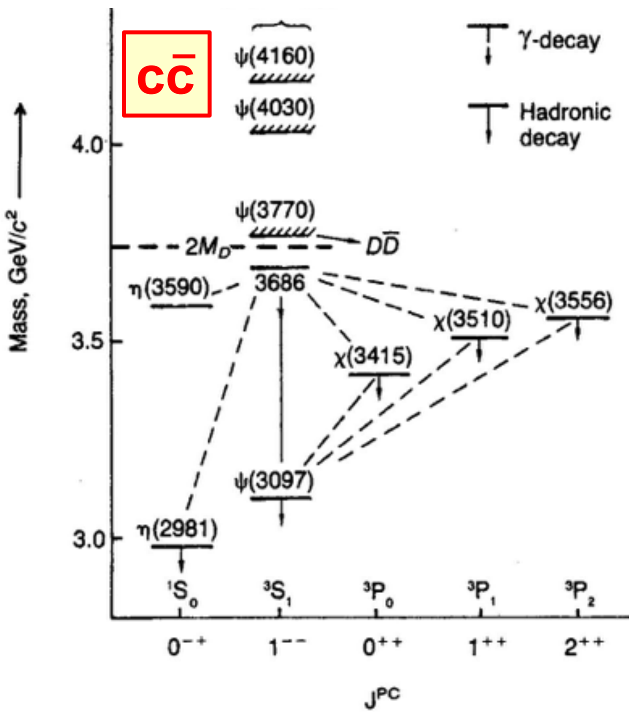
★ Whilst not a formal proof, it is comforting to see that in the colour singlet $q\bar{q}$ state the QCD potential is indeed attractive.

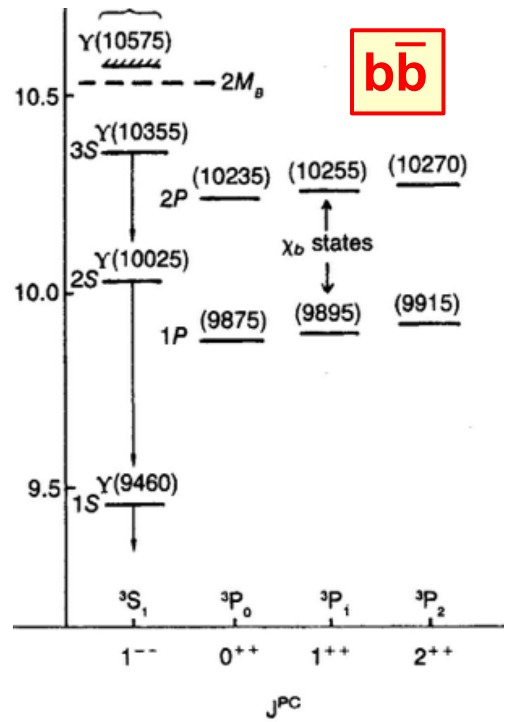
★ **Combining the short-range QCD potential with the linear long-range term discussed previously:**

$$V_{QCD} = -\frac{4}{3}\frac{\alpha_s}{r} + \lambda r$$



★ **This potential is found to give a good description of the observed charmonium (c$\bar{c}$) and bottomonium (b$\bar{b}$) bound states.**





NOTE:

- c, b are heavy quarks
- approx. non-relativistic
- orbit close together
- probe 1/r part of V_{QCD}

Agreement of data with prediction provides strong evidence that V_{QCD} has the Expected form

Summary

★ Superficially QCD very similar to QED

★ But gluon self-interactions are believed to result in colour confinement

★ All hadrons are colour singlets which explains why only observe

Mesons **Baryons**

★ A low energies $\alpha_S \sim 1$

➡ Can't use perturbation theory !

Non-Perturbative regime

★ Coupling constant runs, smaller coupling at higher energy scales

$$\alpha_S(100\,\mathrm{GeV}) \sim 0.1$$

➡ Can use perturbation theory

Asymptotic Freedom

★ Where calculations can be performed, QCD provides a good description of relevant experimental data