Homework #2

1. A free neutron decays via the weak interaction, $n \to pe^-\bar{\nu}$. Please explain why the neutron inside atoms is stable.

2. Consider the scattering process of $A+B \to R \to C+D+\cdots$. The Lorentz invariant form of the cross section is

$$\begin{aligned} &\sigma(A+B \to R \to C+D+\cdots) \\ = &\frac{4\pi s}{k^2}\left[\frac{(2S_R+1)C_R}{(2S_A+1)(2S_B+1)C_A C_B}\right]\frac{\Gamma(R\to AB)\Gamma(R\to C+D+\cdots)}{(s-m_R^2)^2+m_R^2\Gamma_R^2}, \end{aligned}$$

where s denotes the c.m. energy of the system, k labels the wave number of the incoming particle, S_i and C_i is the spin and color of particle i, respectively. m_R (Γ_R) is the mass (width) of the resonance R. $\Gamma(R \to XY)$ represents the partial decay width of the decay channel $R \to XY$. The resonance R contributes maximally to the cross section in the vicinity of $s \approx m_R^2$. Please show the cross section in the limit of $\Gamma_R \ll m_R$. Explain your results.

3. Use CalcHEP to calculate the top-quark decay $t \to be^+\nu_e$ and the μ-lepton decay $\mu^+ \to e^+\bar{\nu}_\mu + \nu_e$. Draw Feynman diagrams of each process. Are they identical if we ignore the color of quarks? Please plot the correlation between m_{be^+} and $m_{e^+\nu_e}$ in the top-quark decay and the correlation between $m_{\bar{\nu}_\mu e^+}$ and $m_{e^+\nu_e}$ in the μ^+-decay. The correlation figures are often called as Dalitz plots. Do you see resonances in both plots? Explain why? Note that: $m_t = 172$ GeV, $m_b = 4.7$ GeV, $m_\mu =$

4. Consider a process in which one proton (p) is at rest and the other collides with it. As a result of collision a particle of rest mass M is produced, in addition to the two protons,
$$p + p \to M + p + p. \qquad (1)$$
   (a) Find the minimum energy the moving proton must have in order to make this reaction possible.
   (b) What could be the corresponding energy if both the protons are moving.

5. The kaon and pion are spineless. Is the decay $K^+ \to \pi^+\gamma$ allowed? Why so? See PDG 2014 for detailed information of both particles, if needed.

6. Consider the decay $\pi^0 \to \gamma\gamma$.
   (a) Show that its is a p-wave decay.
   (b) Show that the decay amplitude can only have the form:
$$F = e^2 F_{\pi^0\gamma\gamma}\vec{k}\cdot(\epsilon^\lambda \times \epsilon^{\lambda'}) \qquad (2)$$
   where $F_{\pi^0\gamma\gamma}$ is a form factor (you can treat it as a real number), $\vec{k}$ the 3-momentum of a photon in the rest frame of π^0, ϵ^λ the polarization vector of

photon where λ denotes the polarization state. Show that in the rest frame of π^0 the decay width

$$\Gamma(\pi^0 \to \gamma\gamma) = 2\pi\alpha^2 F^2_{\pi^0\gamma\gamma} \frac{|\vec{k}|^2}{m_{\pi^0}} = \frac{\pi\alpha^2}{4} F^2_{\pi^0\gamma\gamma} m_{\pi^0}. \qquad (3)$$

Find $F_{\gamma^0\gamma\gamma}$, using the experimental value of τ_{π^0} given in PDG 2014.

7. The figure below shows the Dalitz plot for a process in $p\bar{p}$ annihilation at rest into $\pi^+\pi^-\pi^0$, i.e.

$$p\bar{p} \to \pi^+\pi^-\pi^0$$

from A. Abele, *et al.*, Phys. Lett. B469, 270 (1999). Resonance are apparent. Identify the three resonances as specific hadrons. (Hint: reading the PLB paper might help. Use http://www.gfsoso.com or inspires-hep website to find the journal paper.)

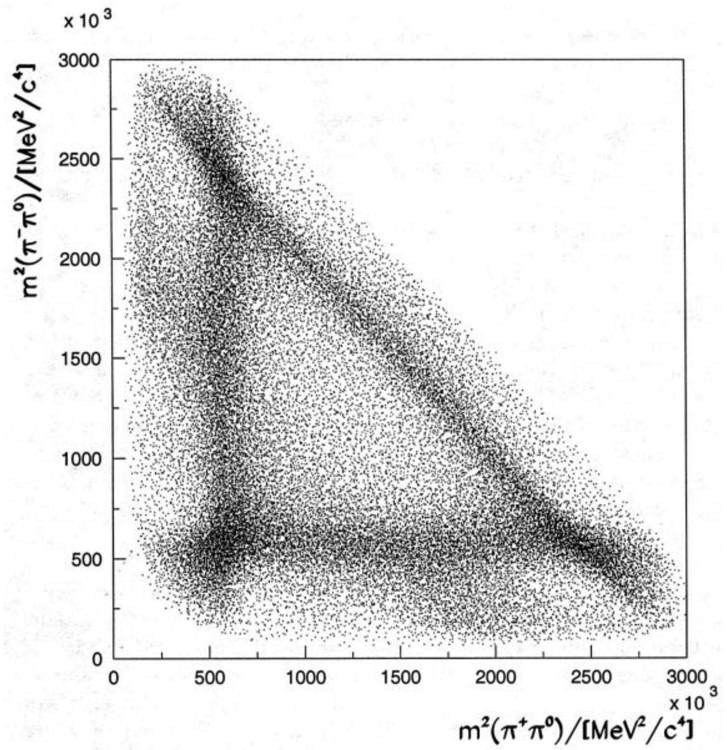

Fig. 1. $\pi^+\pi^-\pi^0$ Dalitz-plot for antiprotons stopping in a liquid H_2 target.

BONUS:

The gluon was discovered in the so-called 'three-jet event' in the TASSO experiment at the Deutsches Elektronen-Synchrotron (DESY) in 1979. The scattering process is

$$e^-(p_1) + e^+(p_2) \to \bar{q}(k_1) + q(k_2) + g(k_3).$$

Define $s = (p_1 + p_2)^2 = q^2$, $q = p_1 + p_2 = k_1 + k_2 + k_3$,

$$x_i = \frac{2k_i \cdot q}{q^2}.$$

Show that the phase space integral is

$$\int \frac{d^3k_1 d^3k_2 d^3k_3}{(2\pi)^9} \frac{1}{2E_1} \frac{1}{2E_2} \frac{1}{2E_3} (2\pi)^4 \delta^4(q - k_1 - k_2 - k_3) = \frac{q^2}{128\pi^3} \int dx_1 \int dx_2 \quad .$$

Textbook: "Particles and Fundamental Interactions: An Introduction to Particle Physics", by Braibant, Giacomeeli and Spurio.