Symmetries and Conservation Laws I
Discreet Symmetries C, P, T

Introduction to Elementary Particle Physics

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Outline

- Symmetries and Conservation Laws
- Parity (P)
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  - Particles and Antiparticles
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Symmetries and Conservation Laws

In Heisenberg representation the time dependence of the operator *Q(t)* is given by:

$$i\hbar \frac{dQ}{dt} = i\hbar \frac{\partial Q}{\partial t} + \left[Q, H\right]$$

An operator with no explicit time dependance is a constant of the motion if it commutes with the hamiltonian operator. In general, conserved quantum numbers are associated to operators commuting with the hamiltonian.

Example: space translations

$$\psi(r+\delta r)=\psi(r)+\delta r\frac{\partial\psi}{\partial r}=\underbrace{\left(1+\delta r\frac{\partial}{\partial r}\right)}\psi$$

For a finite translation:

$$D=\left(1+\frac{ip}{\hbar}\delta r\right)$$

$$D=\lim_{n\to\infty}\left(1+\frac{ip\Delta r}{n\hbar}\right)^n=e^{\frac{ip\Delta r}{\hbar}}$$

P is the generator of the operator D of space translations.
IF *H* is invariant under transations **[D,H] =0** hence:

[p,H] = 0

The following three statements are equivalent:

- Momentum is conserved for an isolated system.
- The hamiltonian is invariant under space translations.
- The momentum operator commutes with the hamiltonian.

Conservation Laws

	Strong	E.M.	Weak
Energy/Momentum	✓	✓	✓
Electric Charge	✓	✓	✓
Baryon Number	✓	✓	✓
Lepton Number	✓	✓	✓
Isospin (I)	✓	✗	✗
Strangeness (S)	✓	✓	✗
Charm (C)	✓	✓	✗
Parity (P)	✓	✓	✗
Charge Conjugation (C)	✓	✓	✗
CP (or T)	✓	✓	✗
CPT	✓	✓	✓

Parity (P)

The operation of spatial inversion of coordinates is produced by the **parity** operator P:

$$P\psi(\vec{r}) = \psi(-\vec{r})$$

Repetition of this operation implies **P^2=1** so that P is a unitary operator.

$$\psi(\vec{r}) \xrightarrow{P} \psi(-\vec{r}) \xrightarrow{P} \psi(\vec{r})$$

Therefore if there are parity eigenvalues they must be: **P = ±1**

Examples:

P = +1 $\quad \psi(x) = \cos x \xrightarrow{P} \cos(-x) = \cos x = \psi(x)$

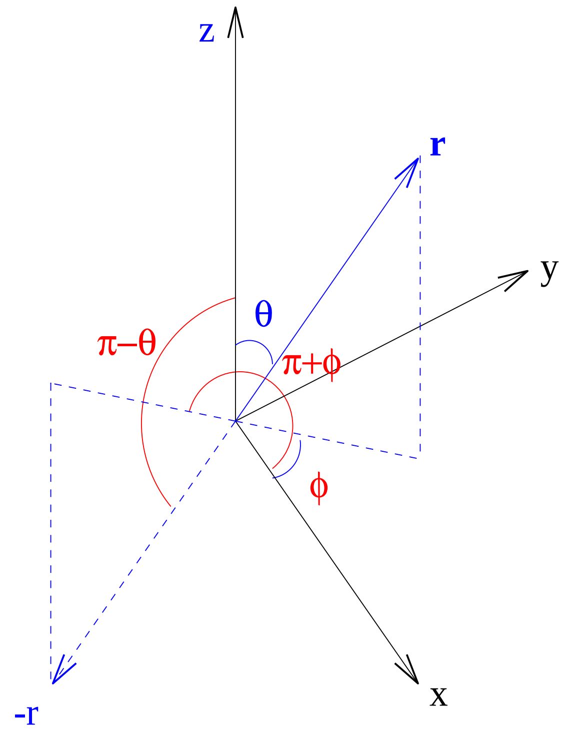
P = -1 $\quad \psi(x) = \sin x \xrightarrow{P} \sin(-x) = -\sin x = -\psi(x)$

$$\psi(x) = \sin x + \cos x \xrightarrow{P} = -\sin x + \cos x \neq \pm\psi(x)$$

Example: <u>**the hydrogen atom**</u>

$$\psi(r,\theta,\varphi) = \chi(r)\sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}}P_m^l(\cos\vartheta)e^{im\varphi}$$

$$\vec{r} \to -\vec{r} \Leftrightarrow \begin{cases} \theta \to \pi - \theta \\ \varphi \to \pi + \varphi \end{cases}$$



$$e^{im\varphi} \to e^{im(\varphi+\pi)} = (-1)^m e^{im\varphi}$$

$$P_l^m(\cos\theta) \to (-1)^{l+m} P_l^m(\cos\theta)$$

$$Y_l^m \to (-1)^l Y_l^m$$

Hence the spherical harmonics have parity ***$P=(-1)^l$.***

For example, in electric dipole transitions, which obey the selection rule $\Delta l = \pm 1$, the atomic parity changes. Therefore the parity of the emitted radiation must be negative, in in order to conserve the total parity of the system atom+photon.

$$P(\gamma) = -1$$

P is a multiplicative quantum number. It is conserved in strong and elettromagnetic interactions, but it is not conserved in weak interactions.

Parity conservation law requires the assignment of an intrinsic parity to each particle.
Protons and neutrons are conventionally assigned positive parity

$$P_p=P_n=+1$$

$\pi^{\pm}$ Parity

The pion (π) is a spin 0 meson. Consider the reaction

$$\pi^- + d \rightarrow n + n$$

(where the deuteron *d* is a *pn* bound state).

In the initial state *l=0*; since $s_\pi=0$, $s_d=1$ the total angular momentum must be J=1 (**J=L+S**). Therefore *also in the final state* we must have J=1. The symmetry of the final state wave function (under interchange of the 2 neutrons) is given by:

$$K = \underbrace{(-1)^{S+1}}_{spin}\underbrace{(-1)^{L}}_{orbitale} = (-1)^{L+S+1}$$

Since we have two identical fermions it must be K=-1, which implies L+S is even.

With the condition J=1 we have the following possibilities:

L=0 S=1 no L=1 S=0 no L=2 S=1 no

L=1 S=1 OK

Therefore the parity of the final state is $P=(-1)^L=-1$. Since the parity of the deuteron is $P_d=+1$ we obtain for the π intrinsic parity $P_\pi = -1$.

The π is therefore a pseudoscalar meson.

Parity of the neutral pion (π^0)

$\pi^0 \to \gamma\,\gamma$ B.R. = (99.798 ± 0.032) %

Let **k** and **-k** be the momentum vectors of the two γ; $\mathbf{e}_1$ and $\mathbf{e}_2$ their polarization vectors. The simplest linear combinations one can form which satisfy requirements of exchange symmetry for identical bosons are:

$$\psi_1(2\gamma) = A(\vec{e}_1 \cdot \vec{e}_2) \propto \cos\phi$$

$$\psi_2(2\gamma) = B(\vec{e}_1 \times \vec{e}_2)\cdot\vec{k} \propto \sin\phi$$

ψ_1 is a scalar and therefore even under space inversion, ψ_2 is a pseudoscalar and therefore it has odd parity.

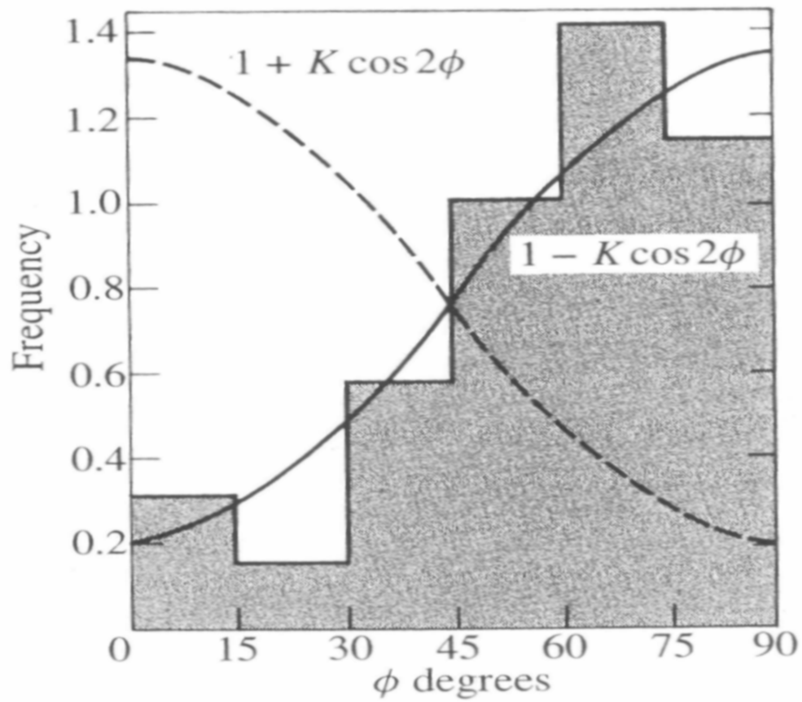
$$P_{\pi^0} = +1 \quad |\psi|^2 \propto \cos^2\phi \qquad\qquad P_{\pi^0} = -1 \quad |\psi|^2 \propto \sin^2\phi$$

where ϕ is the angle between the polarization planes of the two γ. The experiment was done using the decay:

$$\pi^0 \to e^+ + e^- + e^+ + e^-$$

(double Dalitz; B.R. = $(3.14 \pm 0.30)\times 10^{-5}$) in which each Dalitz pair lies predominantly in the polarization plane of the “internally converting” photon. The result is $P_{\pi 0}=-1$.

$$\pi^0 \rightarrow e^+ + e^- + e^+ + e^-$$



The assignment of an intrinsic parity is meaningful when particles interact with one another (as in the case of electric charge).

The nucleon intrinsic parity is a matter of convention.

The relative parity of particle and antiparticle is not a matter of convention.

Fermions and antifermions are created in pairs, for instance:

$$p + p \rightarrow p + p + p + \overline{p}$$

whereas this is not the case for bosons.

<u>Fermions</u>: particle and antiparticle have **opposite parity.**

<u>Bosons</u>: particle and antiparticle have **equal parity.**

$$\left.\begin{aligned} \vec{r} &\rightarrow -\vec{r} \\ \vec{p} &\rightarrow -\vec{p} \end{aligned}\right\} \quad polar \quad vectors$$

$$\vec{\sigma} \rightarrow \vec{\sigma} \qquad axial \quad vector \quad (\vec{r} \times \vec{p})$$

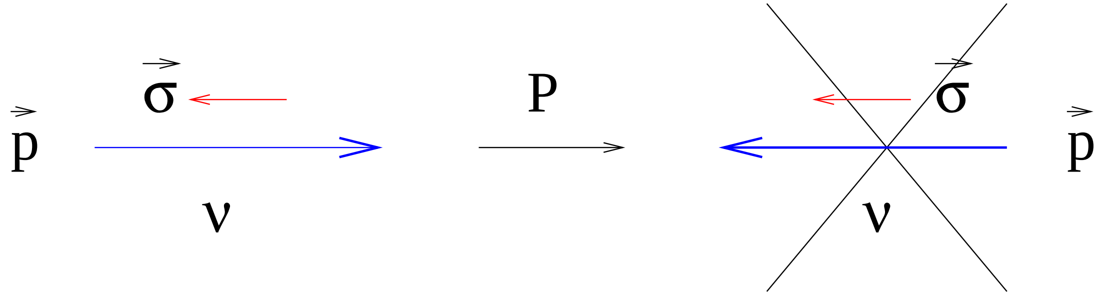
$$\vec{E} \rightarrow -\vec{E}$$

$$\vec{B} \rightarrow \vec{B}$$

Parity Conservation

Parity is conserved in strong and electromagnetic interactions, whereas it is violated in weak interactions. (V-A theory, maximal parity violation)

Example:



In experimental studies of strong and electromagnetic interactions tiny degrees of parity violation are in fact observed, due to contributions from the weak interactions: $H = H_s + H_{em} + H_w$. Atomic transitions:

$$^{16}O^* \longrightarrow {}^{12}C + \alpha$$

$$J^P = 2^- \qquad\qquad J^P = 2^+$$

with total witdth $\Gamma_\alpha = (1.0 \pm 0.3) \times 10^{-10}$ eV, to be compared with $^{16}O^* \rightarrow {}^{16}O + \gamma$ of width 3×10^{-3} eV.

Particles and Antiparticles

The relativistic relation between the total energy *E*, momentum *p* and rest mass *m* of a particle is:

$$E^2 = p^2c^2 + m^2c^4$$

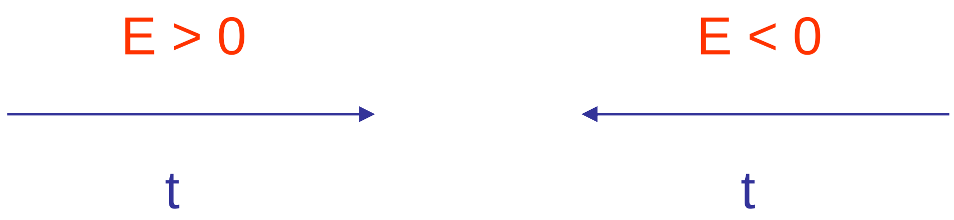
The total energy can assume negative as well as positive values:

$$E = \pm\sqrt{p^2c^2 + m^2c^4}$$

In quantum mechanics we represent the amplitude of an infinite stream of particles, e.g. electrons, travelling along the positive *x*-axis with 3-momentum *p* by the plane wavefunction:

$$\psi = Ae^{-i(Et-px)/\hbar}$$

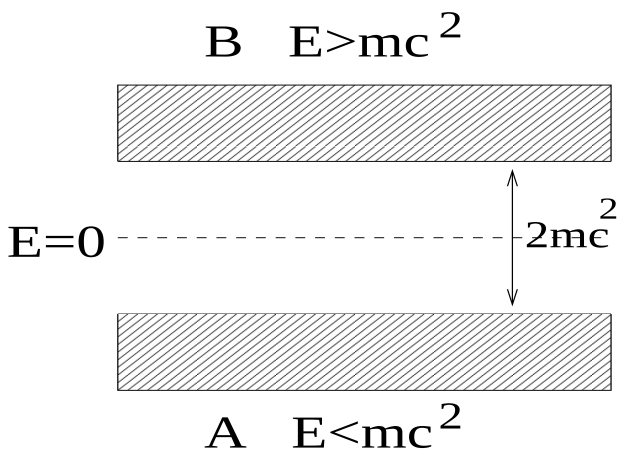
Formally this expression can also represent particles of **energy *-E* and momentum *-p* travelling in the negative *x*-direction and backwards in time**.



Such a stream of negative electrons flowing backwards in time is equivalent to positive charges flowing forward, and thus having E>0.

The negative energy particle states are connected with the existence of positive energy antiparticles of exactly equal but opposite electrical charge and magnetic moment, and otherwise identical.

the positron, the antiparticle of the electron, was discovered experimentally in 1932 in cloud chamber experiments with cosmic rays.



Charge Conjugation (C)

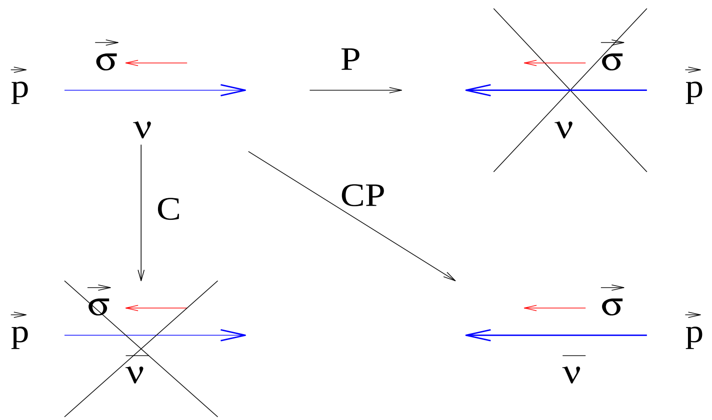
Charge conjugation reverses the charge and magnetic moment of a particle.

In classical physics Maxwell's equations are invariant under:

$$q \rightarrow -q \quad \vec{j} \rightarrow -\vec{j} \quad \vec{E} \rightarrow -\vec{E} \quad \vec{H} \rightarrow -\vec{H}$$

In relativistic quantum mechanics: particle $\leftrightarrow$ antiparticle

	$p \xrightarrow{C} \bar{p}$	
Q	$+e$	$-e$
B	+1	-1
μ	+2.79(e$\hbar$/2mc)	-2.79(e$\hbar$/2mc)
σ	½$\hbar$	½$\hbar$



Eigenstates of the C Operator

Only neutral bosons which are their own antiparticle can be eigenstates of C.

$$C\left|\pi^{+}\right\rangle \rightarrow \left|\pi^{-}\right\rangle \neq \pm\left|\pi^{+}\right\rangle$$

π^+ e π^- are not C eigenstates. For the π°:

$$C\left|\pi^{0}\right\rangle = \eta\left|\pi^{0}\right\rangle$$

$$\eta^{2} = 1 \quad \Rightarrow \quad C\left|\pi^{0}\right\rangle = \pm\left|\pi^{0}\right\rangle$$

$$\pi^{0} \rightarrow \gamma\gamma \quad \Rightarrow \quad C_{\pi^{0}} = +1$$

Electromagnetic interactions conserve C, therefore the decay

$$\pi^{0} \rightarrow 3\gamma$$

should be forbidden. Experimentally we find:

$$\frac{BR(\pi^{0} \rightarrow 3\gamma)}{BR(\pi^{0} \rightarrow 2\gamma)} < 3.1\times10^{-8}$$

Conservation of C

Charge conjugation C is conserved in strong and electromagnetic interactions, but not in weak interaction.

- Spectra of particle and antiparticle, for example:

$$p + \bar{p} \rightarrow \pi^+ + \pi^- + \ldots.$$

$$p + \bar{p} \rightarrow K^+ + K^- + \ldots.$$

- η meson decay ($J^P = 0^-$, M = 550 MeV/c^2)

$\eta \rightarrow \gamma\gamma$	B.R. = (39.21 $\pm$ 0.34) %
$\eta \rightarrow \pi^+\pi^-\pi^0$	B.R. = (23.1 $\pm$ 0.5) %
$\eta \rightarrow \pi^+\pi^-\gamma$	B.R. = (4.77 $\pm$ 0.13) %
$\eta \rightarrow \pi^0 e^+ e^-$	B.R. < 4×10^{-5}

$$B.R. \equiv \frac{\Gamma_{ch}}{\Gamma_{tot}}$$

Since $\eta \rightarrow \gamma\gamma$ we must have $C_\eta = +1$. Hence the decay $\eta \rightarrow \pi^0 e^+ e^-$ is forbidden by C conservation.

Positronium Decay

Positronium is an e^+e^- bound state which possesses energy levels similar to the hydrogen atom (with about half the spacing). Wave function:

$\psi(e^+e^-) = \phi(\text{space}) \times \alpha(\text{spin}) \times \chi(\text{charge})$

ϕ(space) Particle interchange is equivalent to space inversion introducing a factor $(-1)^L$ where L is orbital angular momentum

α(spin)

$$\begin{cases} \alpha(1,1) = \psi_1(\tfrac{1}{2},\tfrac{1}{2})\psi_2(\tfrac{1}{2},\tfrac{1}{2}) \\ \alpha(1,0) = \tfrac{1}{\sqrt{2}}\left[\psi_1(\tfrac{1}{2},\tfrac{1}{2})\psi_2(\tfrac{1}{2},-\tfrac{1}{2}) + \psi_1(\tfrac{1}{2},-\tfrac{1}{2})\psi_2(\tfrac{1}{2},\tfrac{1}{2})\right] \\ \alpha(1,-1) = \psi_1(\tfrac{1}{2},-\tfrac{1}{2})\psi_2(\tfrac{1}{2},-\tfrac{1}{2}) \end{cases}$$

Triplet S=1
Symmetric

$$\alpha(0,0) = \tfrac{1}{\sqrt{2}}\left[\psi_1(\tfrac{1}{2},\tfrac{1}{2})\psi_2(\tfrac{1}{2},-\tfrac{1}{2}) - \psi_1(\tfrac{1}{2},-\tfrac{1}{2})\psi_2(\tfrac{1}{2},\tfrac{1}{2})\right]$$

Singlet S=0
Antisymmetric

The symmetry of α is therefore $(-1)^{S+1}$

Let the charge wave function acquire a factor C.

The total symmetry of the wave function for the interchange of e^+ and e^- is

$$K = (-1)^L(-1)^{S+1}C$$

Two decays are observed for positronium annihilation from L=0:

$(e^+e^-) \rightarrow 2\gamma$ $(e^+e^-) \rightarrow 3\gamma$

The two-photon decay must have J=0, so the three-photon decays has to be assigned J=1.

	S=J	L	C	K
2γ	0	0	+1	-1
3γ	1	0	-1	-1

(C = $(-1)^n$ for a system consisting of n photons).

In QED the widths of these states can be calculated very accurately:

	Γ	τ (theory)	τ (experiment)
2γ	$\frac{1}{2}mc^2\alpha^5$	$1.252\times10^{-10}s$	$(1.252\pm0.017)\times10^{-10}s$
3γ	$\frac{2}{9\pi}(\pi^2-9)\alpha^6mc^2$	$1.374\times10^{-7}s$	$(1.377\pm0.004)\times10^{-7}s$

Charge Conservation and Gauge Invariance

Electric charge is known to be very accurately conserved in all processes.

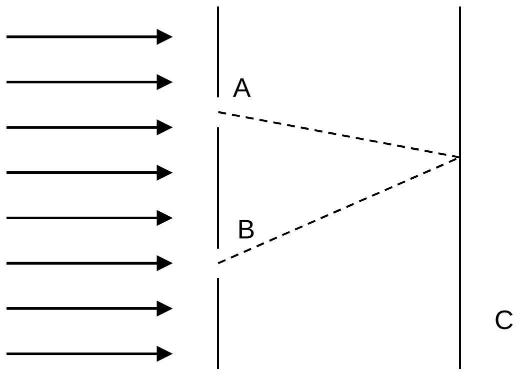
$$\frac{n \to p\nu_e\overline{\nu}_e}{n \to pe^-\overline{\nu}_e} < 9 \times 10^{-24}$$

The conservation of electric charge is related to the gauge invariance of the electromagnetic interaction.

Wigner(1949): Suppose we create a charge *Q* at a point where the potential is ϕ. Let us now move the charge to a point where the potential is ϕ'. $\Delta W = Q(\phi-\phi')$. Suppose we destroy the charge in this point. If W was the work done to create the charge, this work will be recovered when the charge is destroyed. Therefore we gain a net energy $W-W+\phi-\phi'$ because W does not depend on ϕ.

The conservation of energy implies that we cannot create or destroy charge if the scale of electrostatic potential is arbitrary.

$$\psi = e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$



Suppose we send a beam of electrons on a screen in which there are two slits *A* and *B* and that we observe the interference on a second screen *C* located at a distance d from the first.

$$\psi = e^{i(\vec{p}\cdot\vec{x}-Et)} = e^{ipx} \qquad p \equiv (E,\vec{p}) \quad x \equiv (t,\vec{x}) \quad \hbar = c = 1$$

Let us redefine ψ by adding a phase -$e\alpha$.

$$\psi = e^{(ipx - e\alpha)}$$

The interference pattern on *C* depends only on phase differences and it is independent on the global phase ***e***α. If however $e\alpha = e\alpha(x)$:

$$\frac{\partial}{\partial x} i(\ px - e\alpha\ ) = i(\ p - e\frac{\partial\alpha}{\partial x}\ )$$

And the result *would seem to depend on the local phase transformation*.

Electrons however are charged and they interact via an electromagnetic potential, which we write as a 4-vector *A*:

$$A \equiv (\phi, \vec{A})$$

The effect of the potential is to change the phase of an electron:

$$p \rightarrow p + eA$$

So the derivative now becomes:

$$\frac{\partial}{\partial x} i\big(px + eAx - e\alpha(x)\big) = i\left(p + eA - e\frac{\partial \alpha}{\partial x}\right)$$

The potential scale is also arbitrary and we can change it by adding to *A* the gradient of any scalar function (Gauge transformation):

$$A \rightarrow A + \frac{\partial \alpha}{\partial x}$$

With this transformation the derivative becomes *ip,* independent of $\alpha(x)$.

The effect of the original local phase transformation is cancelled exactly by the gauge transformation.

$$A \rightarrow A + \frac{\partial \alpha(x)}{\partial x}$$

Time Reversal (T)

$$p + {}^{27}Al \rightleftarrows \alpha + {}^{24}Mg$$

$$t \rightarrow -t$$

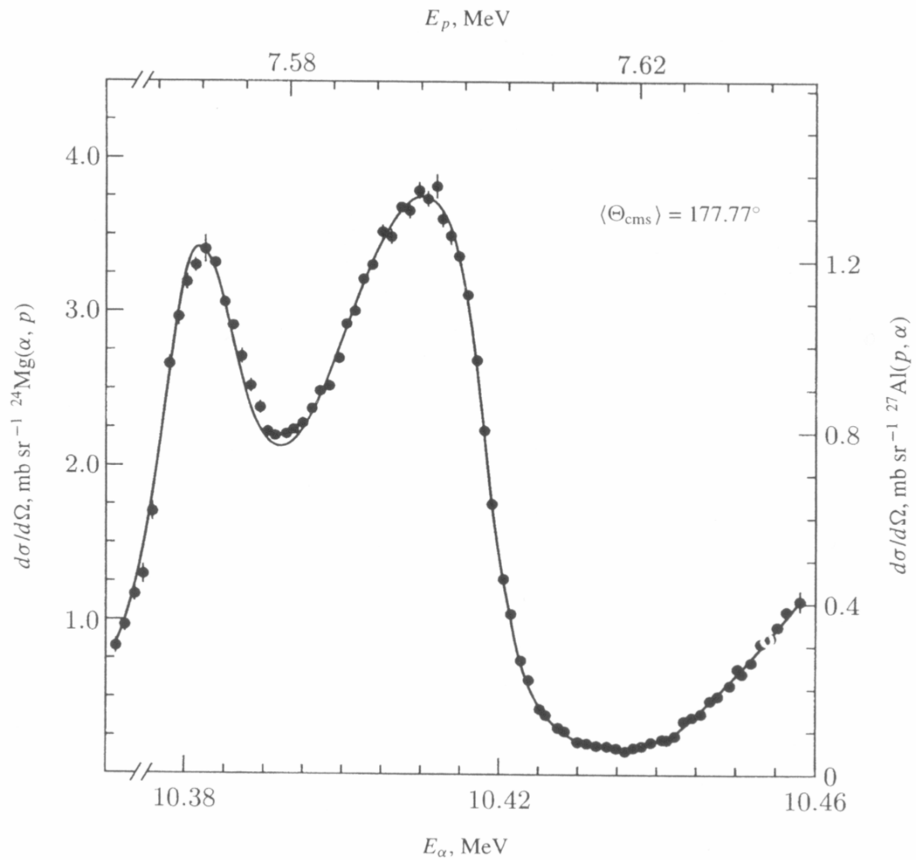
$$\vec{r} \longrightarrow +\vec{r}$$

$$\vec{p} \longrightarrow -\vec{p}$$

$$\vec{\sigma} \longrightarrow -\vec{\sigma}$$

$$\vec{E} \longrightarrow +\vec{E}$$

$$\vec{B} \longrightarrow -\vec{B}$$



T violation < 5×10^{-4}

CPT

CPT theorem:

All interactions are invariant under the succession of the three operation C, P and T taken in any order.

m(particle) = *m*(antiparticle)

$$\frac{m_{K^0} - m_{\overline{K^0}}}{m_{K^0} + m_{\overline{K^0}}} < 10^{-19}$$

mass

τ (particle) = τ (antiparticle)

$$\frac{\tau_{\mu^+} - \tau_{\mu^-}}{\tau_{\mu^+} + \tau_{\mu^-}} < 10^{-4}$$

lifetime

μ (particle)=-μ (antiparticle)

$$\frac{\mu_{e^+} - \mu_{e^-}}{\mu_{e^+} + \mu_{e^-}} < 10^{-12}$$

magnetic moment

CP

In 1964 it was discovered that the long lived K^0_L, which normally decays into three pions (CP = -1), could occasionally decay into two pions (CP=+1). This result represents the discovery of CP violation.

CP violation is at the origin of the asymmetry between matter and antimatter in our universe.

CP violation is equivalent to T violation (via the CPT theorem).

The following observables are sensitive to T violation:

- Transverse polarization $\boldsymbol{\sigma}\cdot(\mathbf{p_1}\times\mathbf{p_2})$ in weak decays such as $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$. Upper limits from these studies $< 10^{-3}$.
- Electric dipole moment $\boldsymbol{\sigma}\cdot\mathbf{E}$. Upper limit for the neutron:

$$\mathrm{EDM}(n) < 1.0 \times 10^{-25}\ e\cdot\mathrm{cm}$$

Fermi's Golden Rule

$$\frac{d^2N}{dAdt} = \sigma \cdot n_b \cdot n_a v_i \qquad W = \sigma \cdot n_a \cdot v_i$$

W = number of interactions per unit time per target particle

The cross section contains information on the interacting particles and on the interaction dynamics. If we write the hamiltonian as:

$$H = H_0 + H'$$

it can be shown that in first order perturbation theory:

$$W = \frac{2\pi}{\hbar} \left| M_{if} \right|^2 \frac{dn}{dE_0}$$

Fermi's Golden Rule

$$M_{if} = \int \psi_f^* H' \psi_i d\tau$$

it contains the interaction dynamics

$$\frac{dn}{dE_0}$$

final state density (phase space factor)

$$\sigma(a + b \rightarrow c + d) = \frac{1}{\pi\hbar^4} \left| M_{if} \right|^2 \frac{(2s_c + 1)(2s_d + 1)}{v_i v_f} p_f^2$$

Principle of Detailed Balance

$$a + b \rightleftarrows c + d$$

From T and P invariance we obtain:

$$\left|M_{if}\right|^2 = \left|M_{fi}\right|^2$$

$$\left\langle f(\vec{p}_c, \vec{p}_d, s_c, s_d)\middle|H'\middle|i(\vec{p}_a, \vec{p}_b, s_a, s_b)\right\rangle$$

$$\Big\downarrow T$$

$$\left\langle i(-\vec{p}_a, -\vec{p}_b, -s_a, -s_b)\middle|H'\middle| f(-\vec{p}_c, -\vec{p}_d, -s_c, -s_d)\right\rangle$$

$$\Big\downarrow P$$

$$\left\langle i(\vec{p}_a, \vec{p}_b, -s_a, -s_b)\middle|H'\middle| f(\vec{p}_c, \vec{p}_d, -s_c, -s_d)\right\rangle$$

Summing over all *(2s+1)* spin projections yields $|M_{if}|^2=|M_{fi}|^2$.

If T and P invariance does not hold (e.g. weak interactions) the same result can be obtained using first order perturbation theory and the hermiticity of H′.

Spin of the Charged Pion ($\pi^{\pm}$)

The spin of the charged pion was determined by applying detailed balance to the reversible reaction:

$$p + p \rightleftarrows \pi^+ + d$$

$$\sigma_{pp\to\pi^+d} = \left|M_{if}\right|^2 \frac{(2s_\pi+1)(2s_d+1)}{v_i v_f} p_\pi^2$$

$$\sigma_{\pi^+d\to pp} = \frac{1}{2}\left|M_{fi}\right|^2 \frac{(2s_p+1)^2}{v_f v_i} p_p^2$$

(the factor ½ comes from the integration over half solid angle, due to the fact that there are two identical bosons in the final state).

$$\frac{\sigma_{pp\to\pi^+d}}{\sigma_{\pi^+d\to pp}} = 2\frac{(2s_\pi+1)(2s_d+1)}{(2s_p+1)^2}\frac{p_\pi^2}{p_p^2}$$

Measuring the cross sections for the direct and reverse reactions one obtains:

$$\boxed{s_\pi = 0}$$

Spin of the Neutral Pion (π^0)

For the neutral pion the decay $\pi^0 \rightarrow \gamma\gamma$ proves that the spin must be integral and that it cannot be one.

For a photon (zero mass, spin 1) $s_z = \pm 1$. Taking the common line of flight of the photons in the π^0 rest frame as the quantization axis, if S is the total spin of the two photons we can have: $S_z = 0$ oppure $S_z = 2$.

If the π^0 spin is 1, then $S_z = 0$. In this case the two-photon amplitude must behave under spatial rotations like the polynomial $P^0{}_1(\cos\theta)$, which is odd under the interchange of the two photons.

But the wave function must be symmetric under the interchange of the two identical bosons, hence the π^0 spin cannot be 1.

In conclusion $s_\pi = 0$ or $s_\pi \geq 2$.