

# Parton Model

## 1. What's parton?

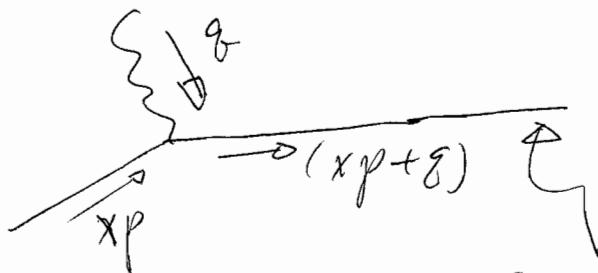
$$\begin{array}{c}
 \text{Diagram: A proton } p \text{ emits a virtual photon } \gamma^* \text{ which decays into an electron } e^- \text{ and a positron } e^+ \\
 = \sum_i \int dx c_i^2 \left[ \text{Diagram: A proton } p \text{ carries fraction } x \text{ of momentum } p \text{ and parton } i. \text{ It emits a virtual photon } \gamma^* \text{ with momentum } q. \text{ The virtual photon decays into an electron } e^- \text{ and a positron } e^+. \text{ The virtual photon has four-momentum } (xp + \varepsilon) \right]
 \end{array}$$

$i = u, d, \bar{u}, \bar{d}, s, \bar{s}, c, \bar{c}, b, \bar{b}, g$

1)

fraction of the proton momentum ( $p$ ) carried by the parton ( $i$ )

Since



and

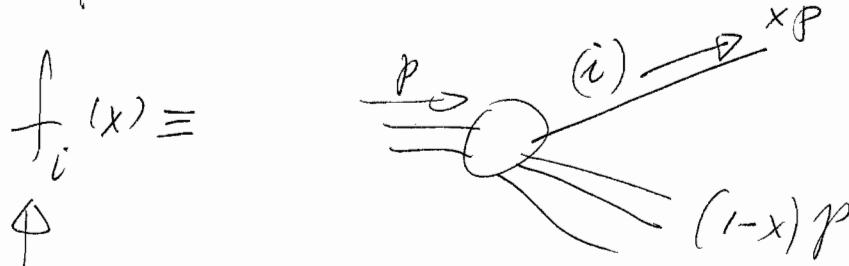
$$(xp + \varepsilon)^2 = 0 \quad (\text{for producing on-shell, massless quark})$$

$$= x^2 p^2 + 2x p \cdot \varepsilon + \varepsilon^2$$

$$\text{For } (\alpha^2 = -\varepsilon^2) \gg (p^2 = M_p^2),$$

$$\text{then } x = \frac{-\varepsilon^2}{2p \cdot \varepsilon} = \frac{\alpha^2}{2p \cdot \varepsilon}$$

## 2) Parton distribution function (PDF)



The probability for the struck parton (i) to carry a fraction  $x$  of the proton momentum.

$$\sum_i \int dx \cdot [x f_i(x)] = 1 \quad \left( \begin{array}{l} \text{Total} \\ \text{Momentum of proton.} \end{array} \right)$$

$\downarrow$

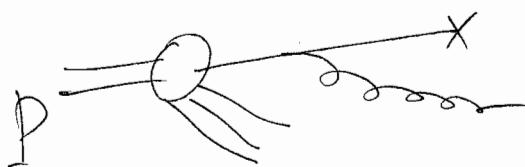
$i = u, \bar{u}, d, \bar{d}, s, \bar{s}, b, \bar{b}, g \quad c, \bar{c}$

3) In parton model,

$$F_2(x) = \sum_i (e_i)^2 x f_i(x) \quad (1969)$$

which is independent of  $Q \Rightarrow$  Bjorken scaling  
 $(x = \frac{t}{Q^2})$

Note: In QCD parton model, the violation of Bjorken scaling is logarithmic, and is a signature of gluon emission.

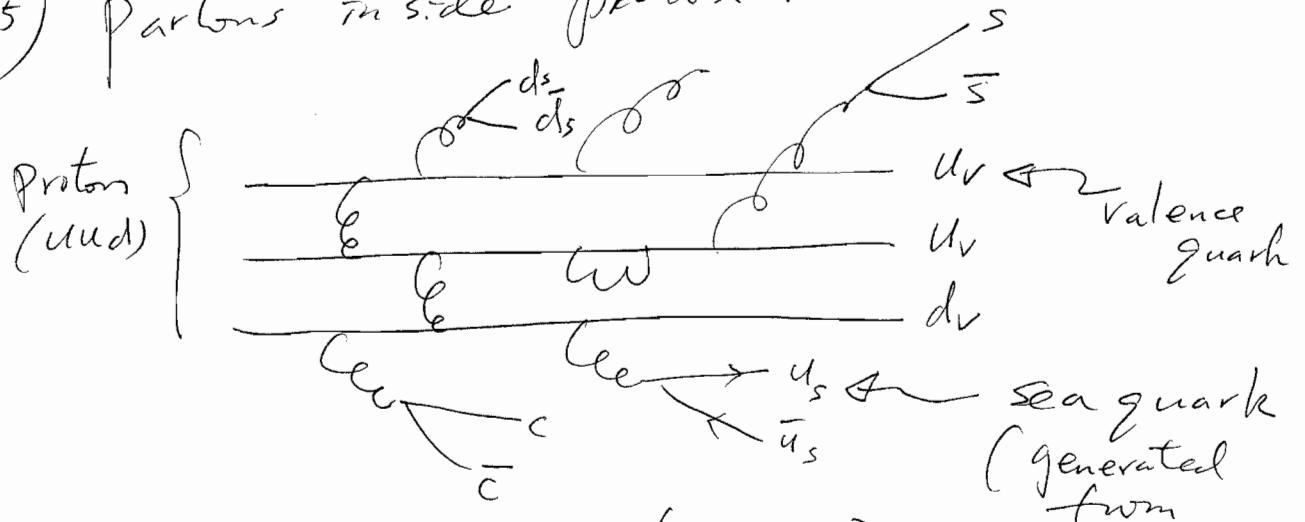


$$4) F_1(x) = \frac{1}{2x} F_2(x) \quad (\text{Callan-Gross relation})$$

proves quark is spin- $\frac{1}{2}$  parton.

[ For a spin-0 parton,  $F_1(x) = 0$ . ]

5) Partons inside proton.



$$u(x) = u_v(x) + u_s(x)$$

$$d(x) = d_v(x) + d_s(x)$$

$$\begin{aligned} & \left( \bar{u}_s = u_s \right) \\ & \left( \bar{d}_s = d_s \right) \end{aligned} \quad \text{gluon splitting}$$

$s, \bar{s}, c, \bar{c}, b, \bar{b}$  are "assumed" to be all coming from gluon splitting (i.e. sea quarks).

$$\int_0^1 dx [u(x) - \bar{u}(x)] = \int_0^1 dx [u_v(x)] = 2$$

$$\int dx [d(x) - \bar{d}(x)] = \int_0^1 dx [d_v(x)] = 1$$

because  $P = (uud)$

6) Where are the gluons?

For  $Q \approx 1.5$  GeV, charm quark is a "heavy" quark.  
Hence, the total momentum of proton gives

$$P = \int_0^1 dx \cdot (xp) [u + \bar{u} + d + \bar{d} + s + \bar{s} + g]$$

$$\Rightarrow \int_0^1 dx \cdot x \cdot (u + \bar{u} + d + \bar{d} + s + \bar{s}) = 1 - \xi_g,$$

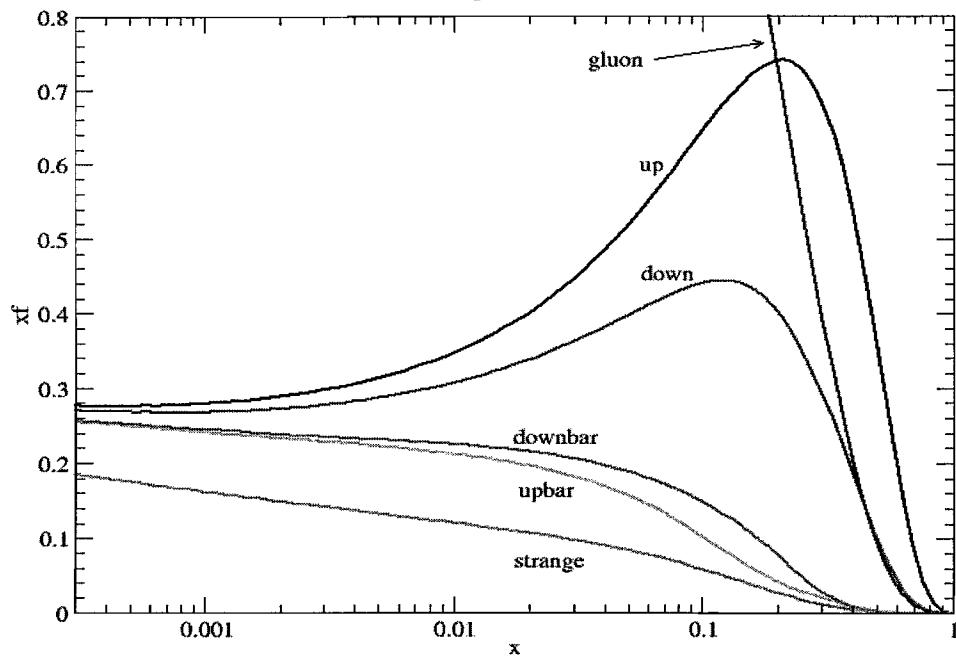
$$\xi_g \equiv \frac{P_g}{P} = \frac{\text{total amount of momentum carried by gluon}}{\text{the momentum of proton}}$$

Experimental data tells us

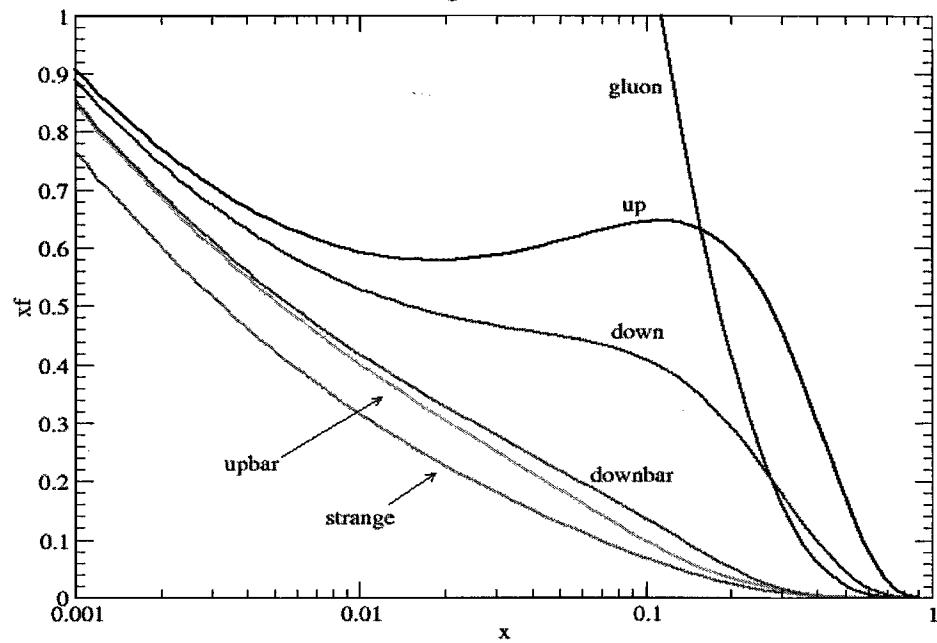
$$\xi_g \approx 0.46$$

Hence, gluon carries about 50% of the proton momentum.

**Parton Distribution Function**  
CTEQ5M1 at 2GeV



**Parton Distribution Function**  
CTEQ5m1 at 100 GeV



# CT10 NNLO PDFs

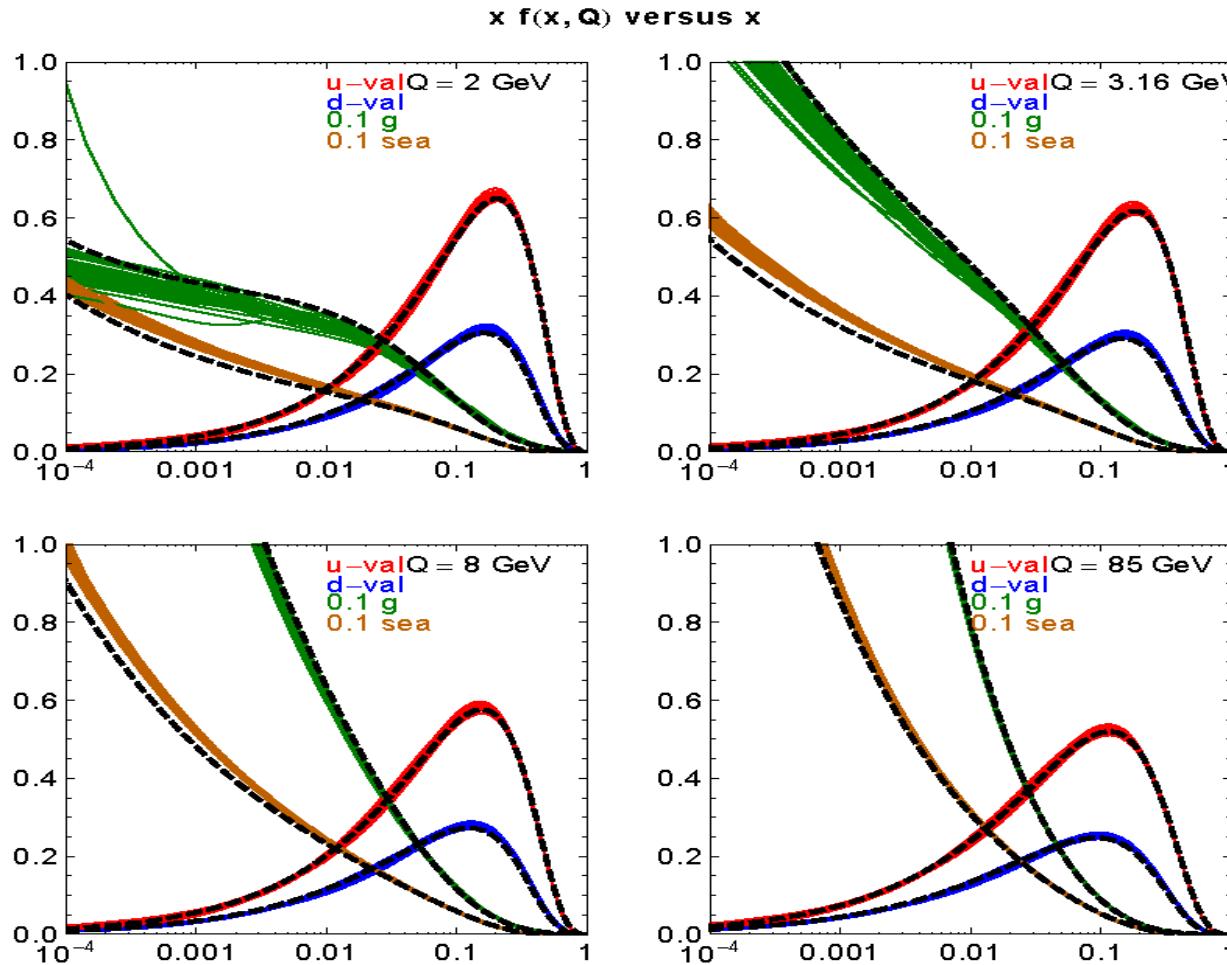


Figure 3: CT10-NNLO parton distribution functions. These figures show the *alternate fits* for the CT10-NNLO analysis. Each graph shows  $x u_{\text{valence}} = x(u - \bar{u})$ ,  $x d_{\text{valence}} = x(d - \bar{d})$ ,  $0.10 x g$  and  $0.10 x \bar{q}\text{sea}$  as functions of  $x$  for a fixed value of  $Q$ . The values of  $Q$  are 2, 3.16, 8, 85 GeV. Sea =  $2(\bar{d} + \bar{u} + \bar{s})$ . The dashed curves are the central NLO fit, CT10.

# Momentum fraction inside proton

