

粒子物理

25. 中微子质量 (群论应用示例)

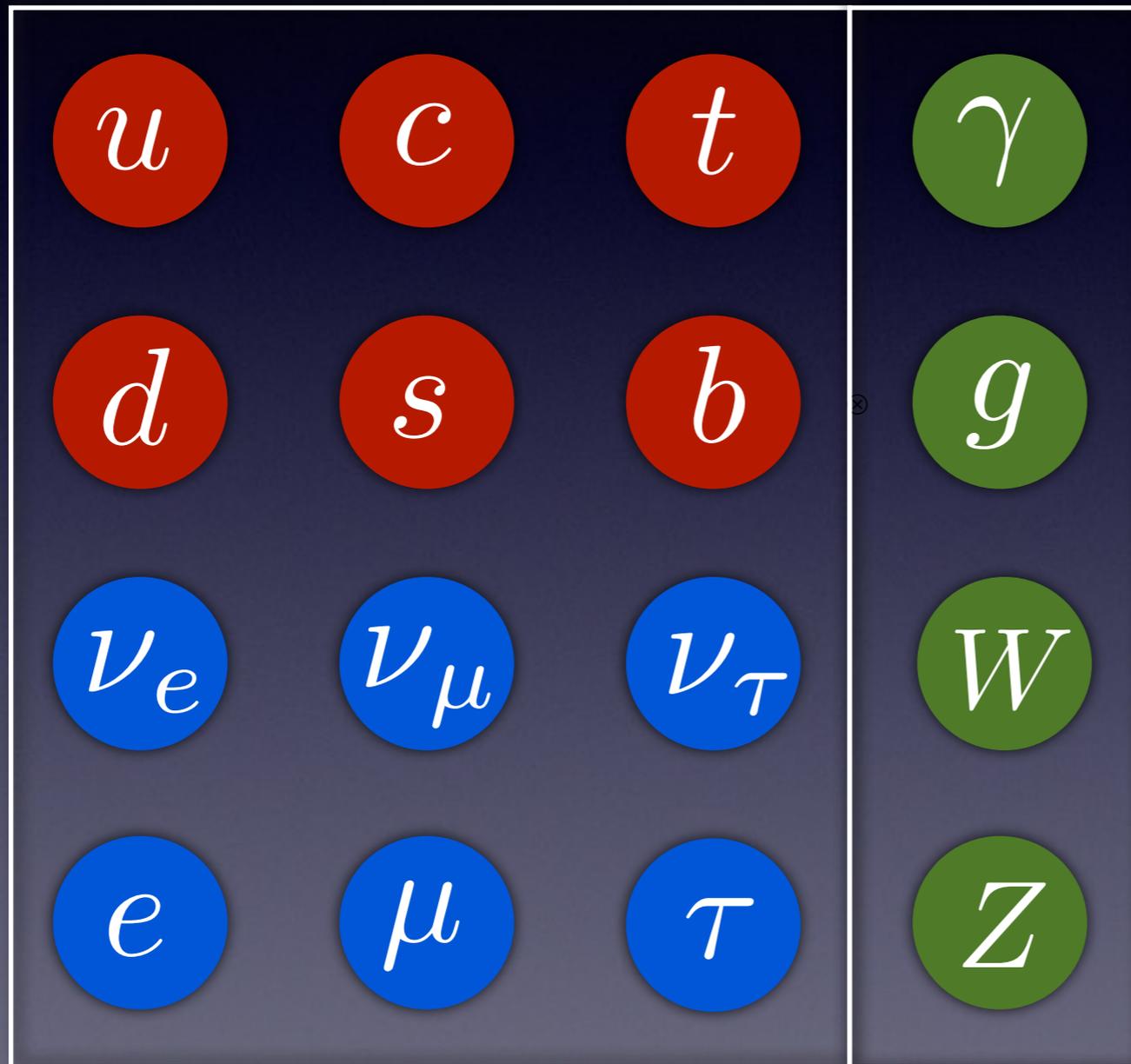
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粒子物理的标准模型

已知的基本粒子谱

夸克



自旋1/2

自旋1

电磁

强

弱

希格斯粒子

h 自旋0

$SU(3) \times SU(2) \times U(1)$
规范对称性

$$\begin{aligned}
\mathcal{L}_{\text{SM}} = & -\frac{1}{2}\partial_\nu g_\mu^a \partial_\nu g_\mu^a - g_s f^{abc} \partial_\mu g_\nu^a g_\mu^b g_\nu^c - \frac{1}{4}g_s^2 f^{abc} f^{ade} g_\mu^b g_\nu^c g_\mu^d g_\nu^e + \frac{1}{2}ig_s^2 (\bar{q}_i^\sigma \gamma^\mu q_j^\sigma) g_\mu^a + \bar{G}^a \partial^2 G^a + g_s f^{abc} \partial_\mu \bar{G}^a G^b g_\mu^c \\
& -\partial_\nu W_\mu^+ \partial_\nu W_\mu^- - M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\nu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2c_w^2} M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H - \frac{1}{2}m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- \\
& -M^2 \phi^+ \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \frac{1}{2c_w^2} M \phi^0 \phi^0 - \beta_h \left[\frac{2M^2}{g^2} + \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-) \right] + \frac{2M^4}{g^2} \alpha_h \\
& -igc_w \left[\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - Z_\nu^0 (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + Z_\mu^0 (W_\nu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\nu W_\mu^+) \right] \\
& -igs_w \left[\partial_\nu A_\mu (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - A_\nu (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + A_\mu (W_\nu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\nu W_\mu^+) \right] \\
& -\frac{1}{2}g^2 W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- + \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\mu^+ W_\nu^- + g^2 c_w^2 (Z_\mu^0 W_\mu^+ Z_\nu^0 W_\nu^- - Z_\mu^0 Z_\nu^0 W_\mu^+ W_\nu^-) + g^2 s_w^2 (A_\mu W_\mu^+ A_\nu W_\nu^- - A_\mu A_\nu W_\mu^+ W_\nu^-) \\
& +g^2 s_w c_w \left[A_\mu Z_\nu^0 (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - 2A_\mu Z_\mu^0 W_\nu^+ W_\nu^- \right] - g\alpha \left[H^3 + H\phi^0 \phi^0 + 2H\phi^+ \phi^- \right] \\
& -\frac{1}{8}g^2 \alpha_h \left[H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2 \right] - gM W_\mu^+ W_\mu^- H - \frac{1}{2}g \frac{M}{c_w^2} Z_\mu^0 Z_\mu^0 H \\
& -\frac{1}{2}ig \left[W_\mu^+ (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W_\mu^- (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0) \right] + \frac{1}{2}g \left[W_\mu^+ (H \partial_\mu \phi^- - \phi^- \partial_\mu H) - W_\mu^- (H \partial_\mu \phi^+ - \phi^+ \partial_\mu H) \right] \\
& +\frac{1}{2}g \frac{1}{c_w} Z_\mu^0 (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) - ig \frac{s_w^2}{c_w} M Z_\mu^0 (W_\mu^+ \phi^- - W_\mu^- \phi^+) + igs_w M A_\mu (W_\mu^+ \phi^- - W_\mu^- \phi^+) - ig \frac{1-2c_w^2}{2c_w} Z_\mu^0 (\phi^+ \partial_\mu \phi^- \\
& -\phi^- \partial_\mu \phi^+) + igs_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \frac{1}{4}g^2 W_\mu^+ W_\mu^- \left[H^2 + (\phi^0)^2 + 2\phi^+ \phi^- \right] - \frac{1}{4}g^2 \frac{1}{c_w^2} Z_\mu^0 Z_\mu^0 [H^2 + (\phi^0)^2 \\
& -2(2s_w^2 - 1)^2 \phi^+ \phi^-] - \frac{1}{2}g^2 \frac{s_w^2}{c_w} Z_\mu^0 \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+) - \frac{1}{2}ig^2 \frac{s_w^2}{c_w} Z_\mu^0 H (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+) \\
& +\frac{1}{2}ig^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+) - g^2 \frac{s_w}{c_w} (2c_w^2 - 1) Z_\mu^0 A_\mu \phi^+ \phi^- - g^1 s_w^2 A_\mu A_\mu \phi^+ \phi^- - e^\lambda (\gamma \partial + m_e^\lambda) e^\lambda - \nu^\lambda \gamma \partial \nu^\lambda \\
& -\bar{u}_j^\lambda (\gamma \partial + m_u^\lambda) u_j^\lambda - \bar{d}_j^\lambda (\gamma \partial + m_d^\lambda) d_j^\lambda + igs_w A_\mu [-(\bar{e}^\lambda \gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}_j^\lambda \gamma^\mu u_j^\lambda) - \frac{1}{3}(\bar{d}_j^\lambda \gamma^\mu d_j^\lambda)] \\
& +\frac{ig}{4c_w} Z_\mu^0 \left[(\nu^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (e^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{u}_j^\lambda \gamma^\mu (\frac{4}{3}s_w^2 - 1 - \gamma^5) u_j^\lambda) + (\bar{d}_j^\lambda \gamma^\mu (1 - \frac{8}{3}s_w^2 - \gamma^5) d_j^\lambda) \right] \\
& +\frac{ig}{2\sqrt{2}} W_\mu^+ \left[(\nu^\lambda \gamma^\mu (1 + \gamma^5) e^\lambda) + (\bar{u}_j^\lambda \gamma^\mu (1 + \gamma^5) C_{\lambda\kappa} d_j^\kappa) \right] + \frac{ig}{2\sqrt{2}} W_\mu^- \left[(e^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{d}_j^\kappa C_{\lambda\kappa}^\dagger \gamma^\mu (1 + \gamma^5) u_j^\lambda) \right] \\
& +\frac{ig}{2\sqrt{2}} \frac{m_e^\lambda}{M} \left[-\phi^+ (\nu^\lambda (1 - \gamma^5) e^\lambda) + \phi^- (e^\lambda (1 + \gamma^5) \nu^\lambda) \right] - \frac{g}{2} \frac{m_e^\lambda}{M} \left[H(e^\lambda e^\lambda) + i\phi^0 (e^\lambda \gamma^5 e^\lambda) \right] \\
& +\frac{ig}{2M\sqrt{2}} \phi^+ \left[-m_d^\kappa (\bar{u}_j^\lambda C_{\lambda\kappa} (1 - \gamma^5) d_j^\kappa) + m_u^\lambda (\bar{u}_j^\lambda C_{\lambda\kappa} (1 + \gamma^5) d_j^\kappa) \right] + \frac{ig}{2M\sqrt{2}} \phi^- \left[m_d^\lambda (\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger (1 + \gamma^5) u_j^\kappa) - m_u^\kappa (\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger (1 - \gamma^5) u_j^\kappa) \right] \\
& -\frac{g}{2} \frac{m_u^\lambda}{M} H(\bar{u}_j^\lambda u_j^\lambda) - \frac{g}{2} \frac{m_d^\lambda}{M} H(\bar{d}_j^\lambda d_j^\lambda) + \frac{ig}{2} \frac{m_u^\lambda}{M} \phi^0 (\bar{u}_j^\lambda \gamma^5 u_j^\lambda) - \frac{ig}{2} \frac{m_d^\lambda}{M} \phi^0 (\bar{d}_j^\lambda \gamma^5 d_j^\lambda) + \bar{X}^+ (\partial^2 - M^2) X^+ + \bar{X}^- (\partial^2 - M^2) X^- \\
& +\bar{X}^0 \left(\partial^2 - \frac{M^2}{c_w^2} \right) X^0 + \bar{Y} \partial^2 Y + igc_w W_\mu^+ (\partial_\mu \bar{X}^0 X^- - \partial_\mu \bar{X}^+ X^0) + igs_w W_\mu^+ (\partial_\mu \bar{Y} X^- - \partial_\mu \bar{X}^+ Y) + igc_w W_\mu^- (\partial_\mu \bar{X}^- X^0 - \partial_\mu \bar{X}^0 X^+) \\
& +igs_w W_\mu^- (\partial_\mu \bar{X}^- Y - \partial_\mu \bar{Y} X^+) + igc_w Z_\mu^0 (\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) + igs_w A_\mu (\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) - \frac{1}{2}gM[\bar{X}^+ X^+ H + \bar{X}^- X^- H \\
& +\frac{1}{c_w^2} \bar{X}^0 X^0 H] + \frac{1-2c_w^2}{2c_w} igM[\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-] + \frac{1}{2c_w} igM[\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + igMs_w[\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] \\
& +\frac{1}{2}igM[\bar{X}^+ X^+ \phi^+ - \bar{X}^- X^- \phi^-] + \frac{1}{2}igM[\bar{X}^0 X^0 \phi^0] + \frac{1}{2}igM[\bar{X}^+ X^+ \phi^0] + \frac{1}{2}igM[\bar{X}^- X^- \phi^0] + \frac{1}{2}igM[\bar{X}^0 X^0 \phi^0]
\end{aligned}$$

麦克斯韦方程组

$$\oiint \mathbf{E} \cdot d\mathbf{S} = 4\pi Q$$

$$\oiint \mathbf{B} \cdot d\mathbf{S} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{r} = -\frac{1}{c} \frac{d}{dt} \iint \mathbf{B} \cdot d\mathbf{S}$$

$$\oint \mathbf{B} \cdot d\mathbf{r} = \frac{4\pi}{c} I$$

$$-\frac{1}{c} \frac{d}{dt} \iint \mathbf{E} \cdot d\mathbf{S}$$

Faraday

$$\nabla \cdot \mathbf{E} = 4\pi\rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial}{\partial t} \mathbf{B}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{\partial}{\partial t} \mathbf{E}$$

Maxwell

$$\partial_\mu F^{\mu\nu} = -\frac{4\pi}{c} j^\nu$$

$$\partial_\mu \tilde{F}^{\mu\nu} = 0$$

Einstein

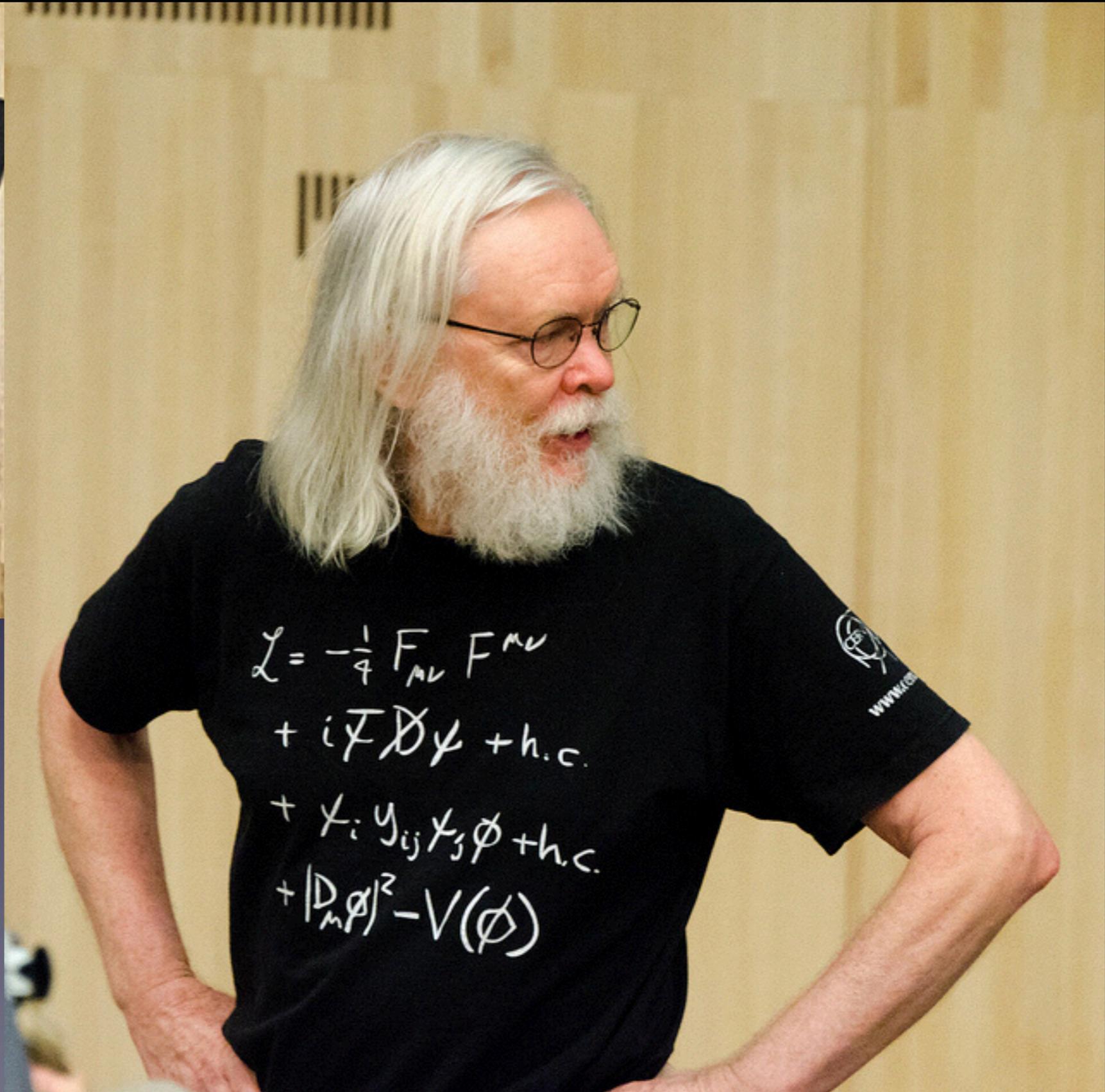
1864年10月27日，麦克斯韦写下方程组：
283种符号，20个变量，20个方程

粒子物理的标准模型



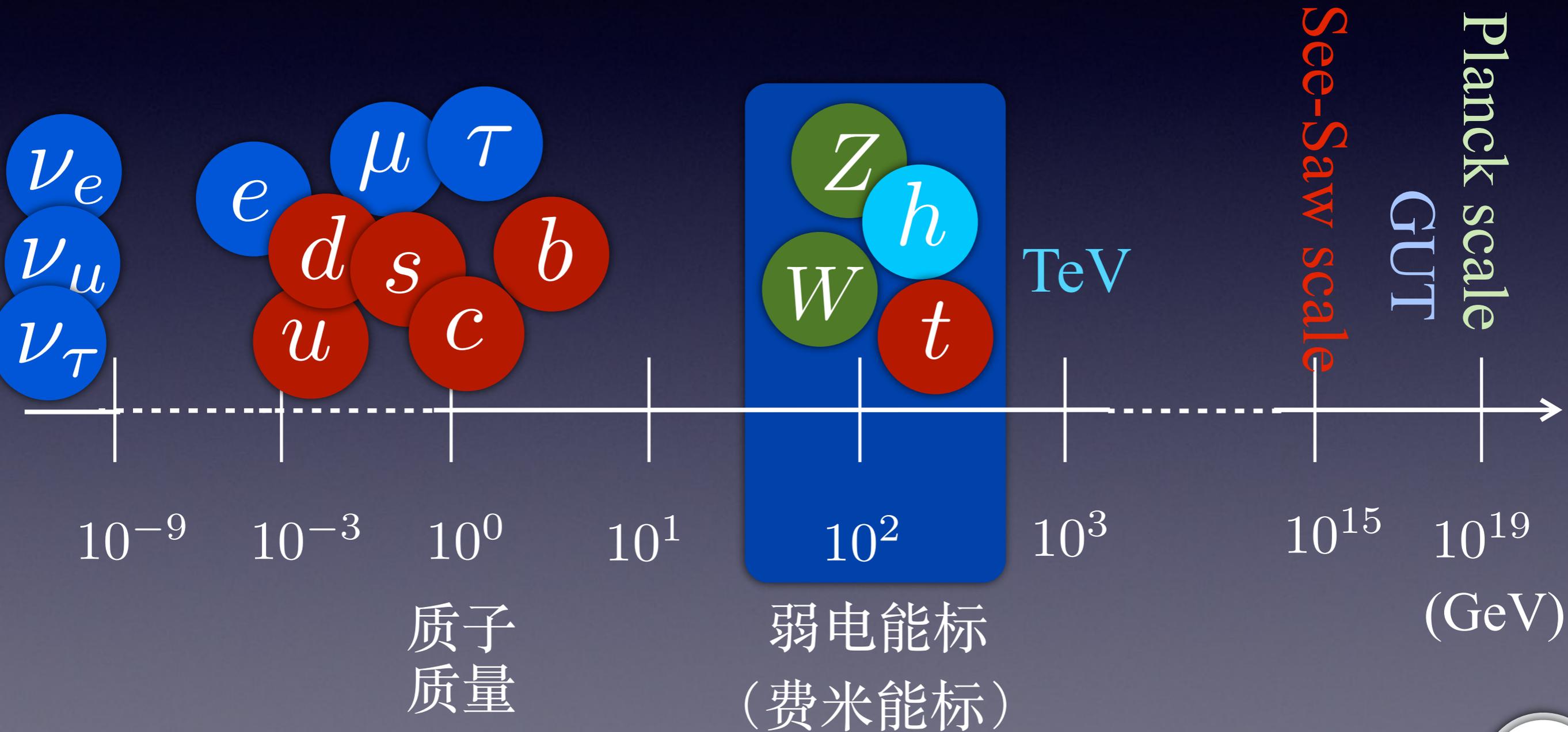
$$\bar{Q}_L \Phi u_R$$

$$2 \times \bar{2} = 1 \oplus 3$$



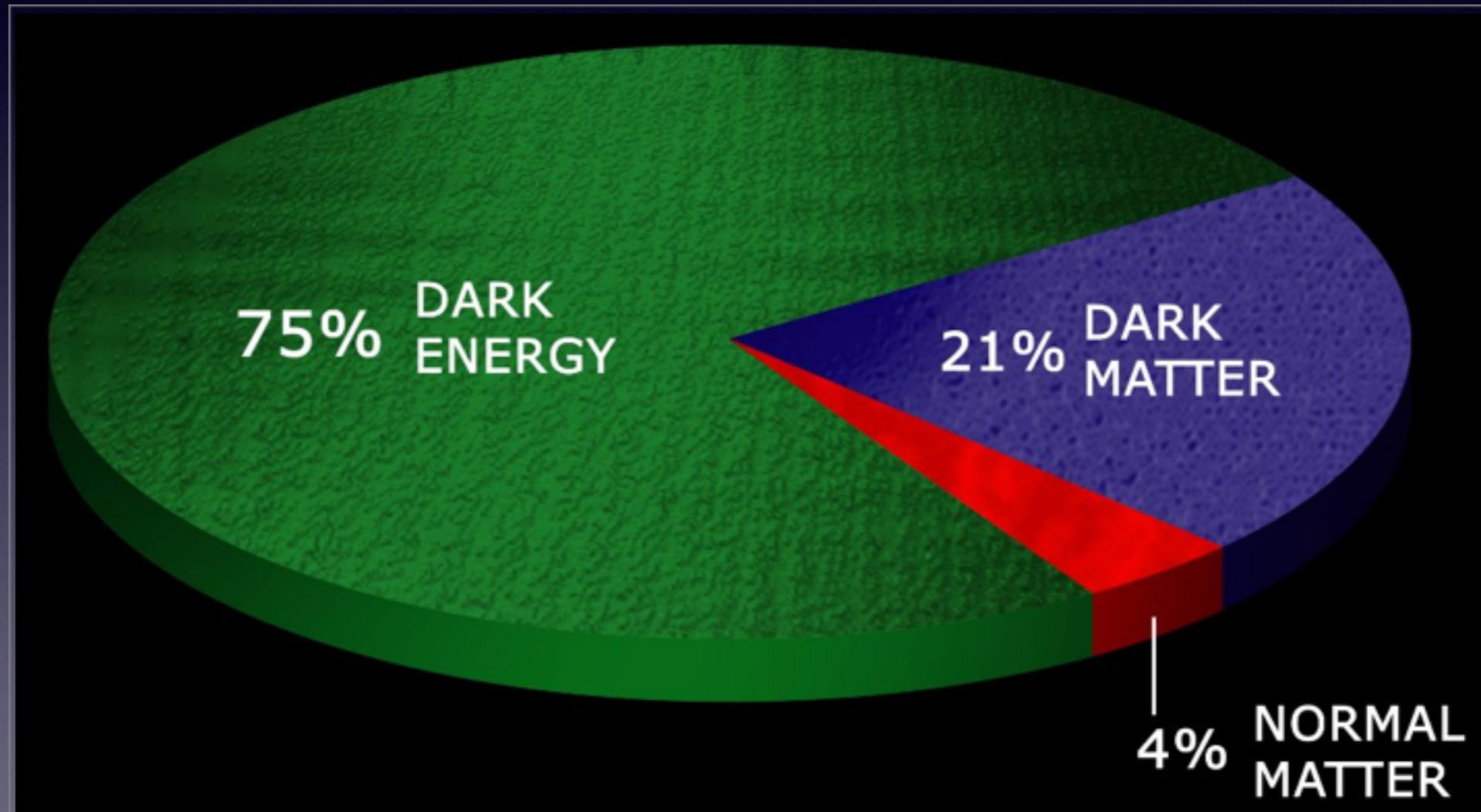
标准模型的不足之处

- 1) 标准模型的粒子谱具有非常大的质量差异和中微子质量起源



标准模型的不足之处

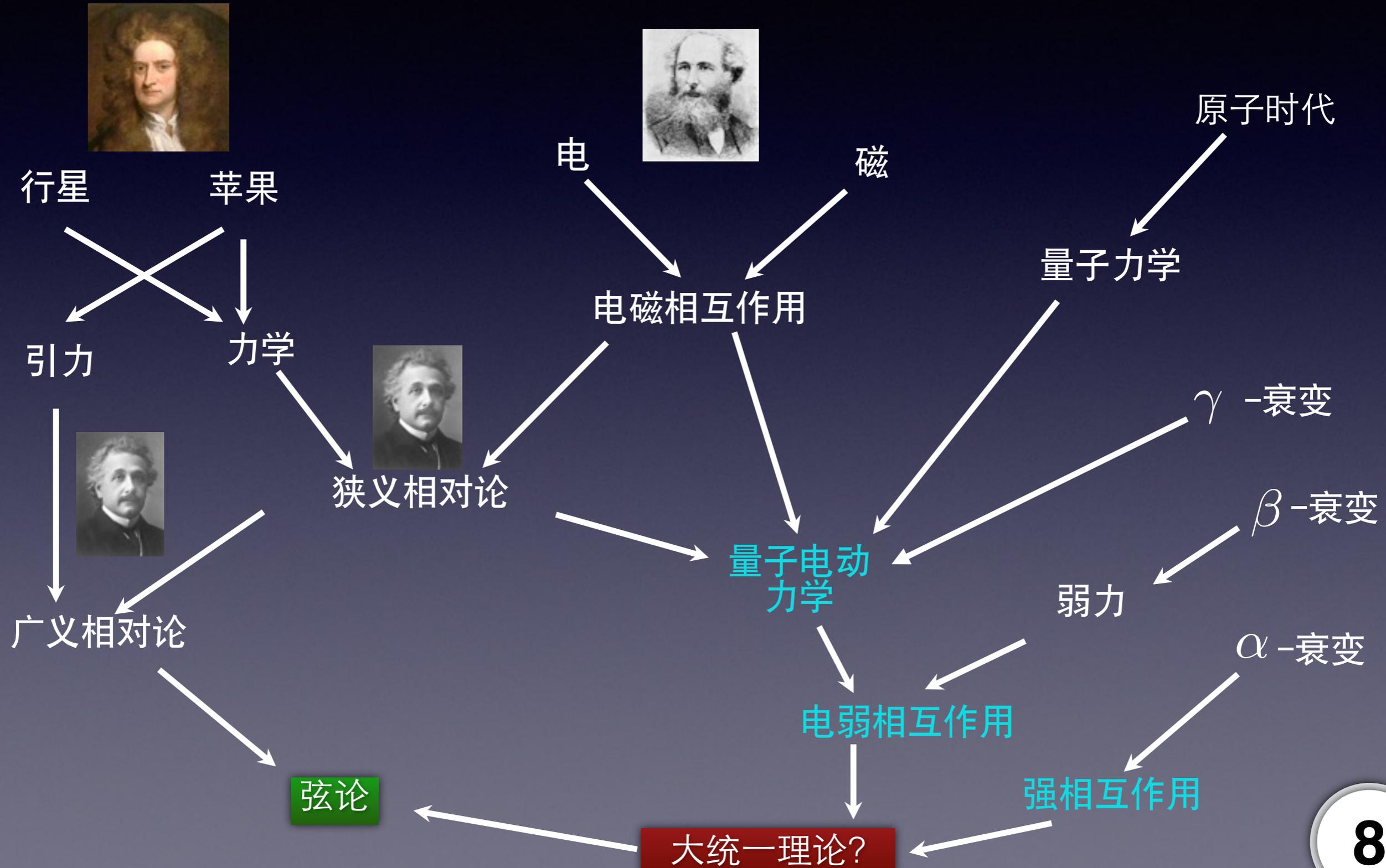
2) 暗物质的粒子物理起源



标准模型中没有暗物质候选者

标准模型的不足之处

3) 大统一理论

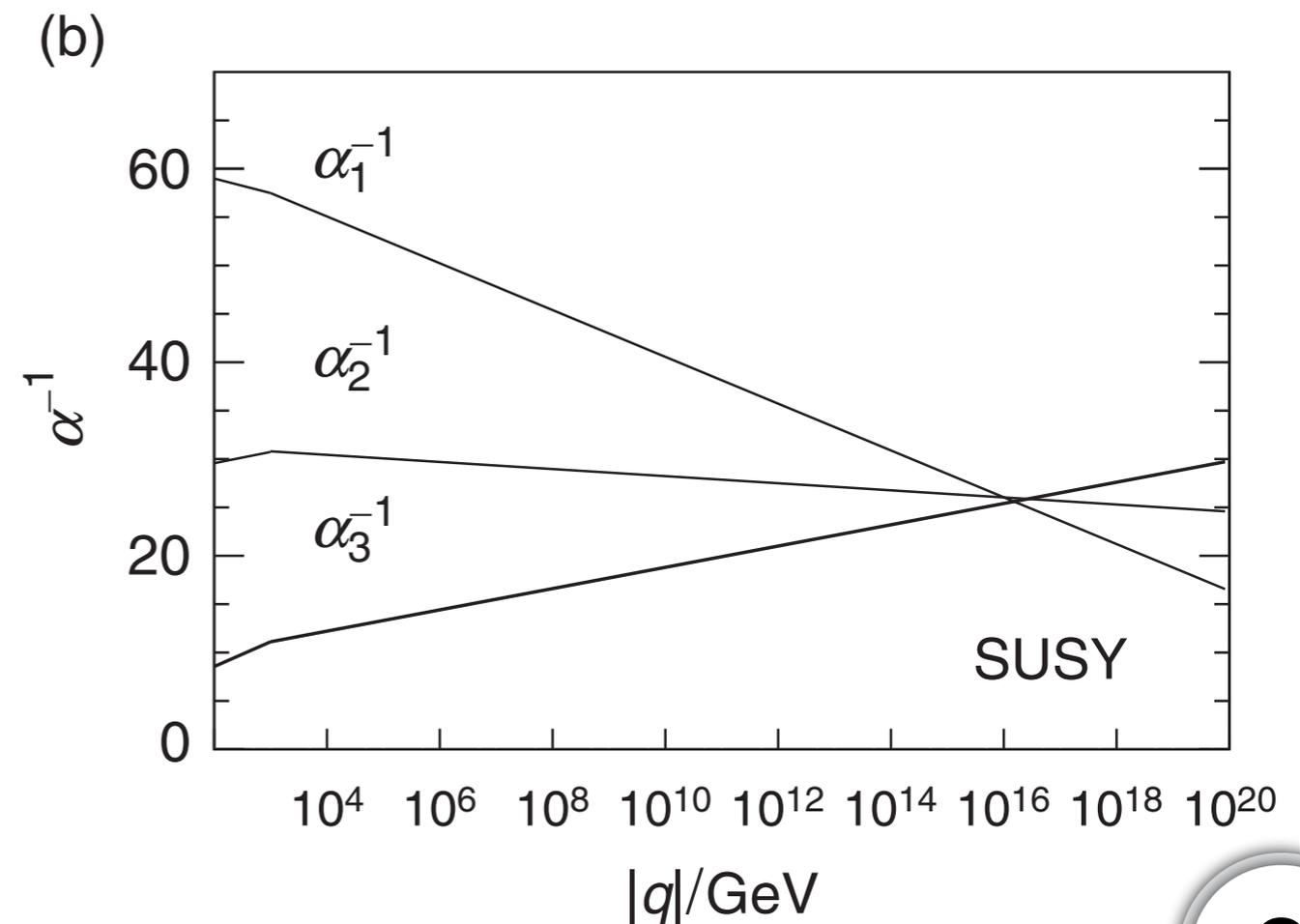
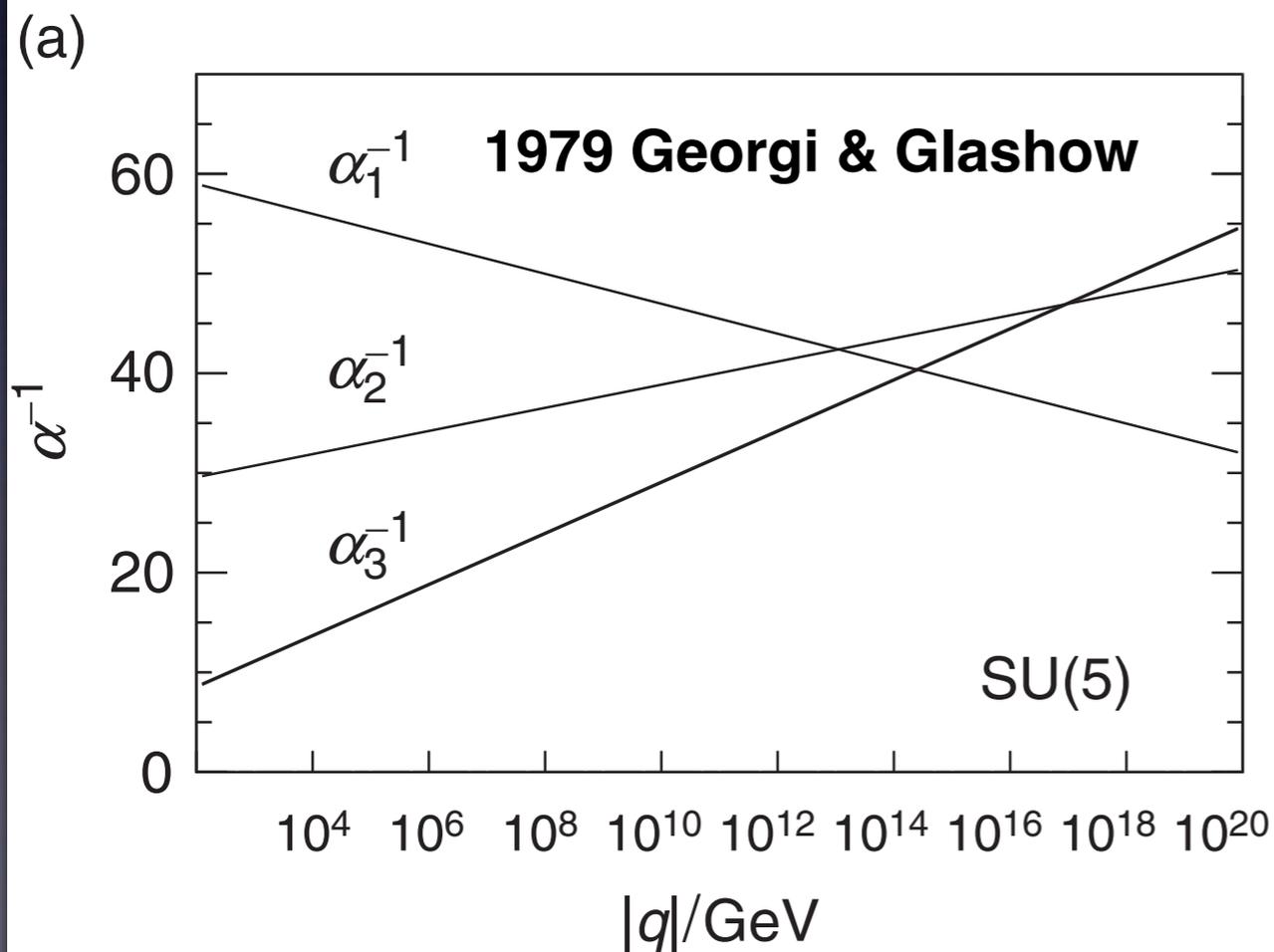


标准模型的不足之处

3) 大统一理论：标准模型三种作用力无法统一

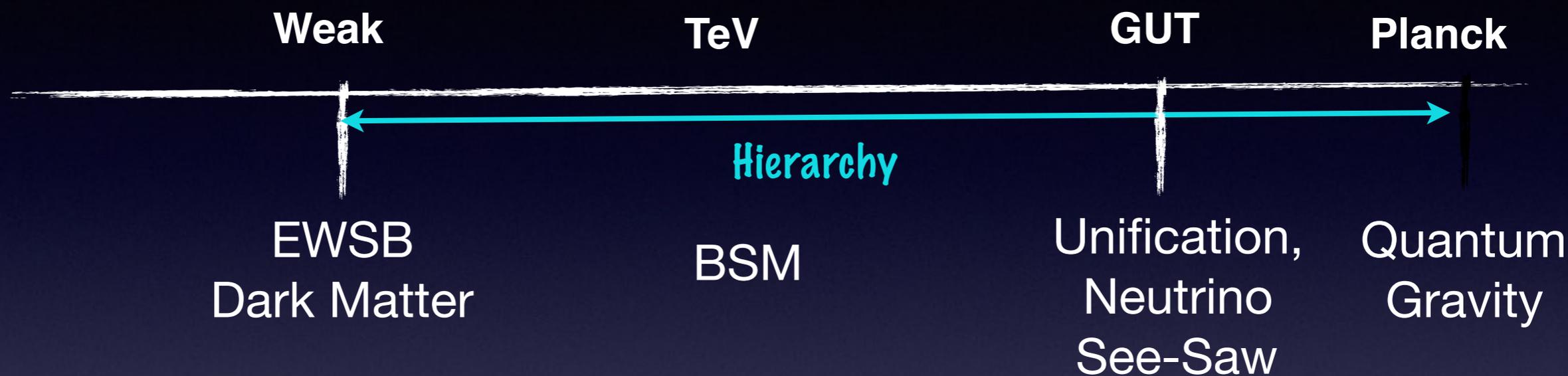
$$\alpha^{-1} : \alpha_W^{-1} : \alpha_S^{-1} \approx 128 : 30 : 9$$

$$[\alpha_i(q^2)]^{-1} = [\alpha_i(\mu^2)]^{-1} + \beta \ln\left(\frac{q^2}{\mu^2}\right)$$



标准模型的不足之处

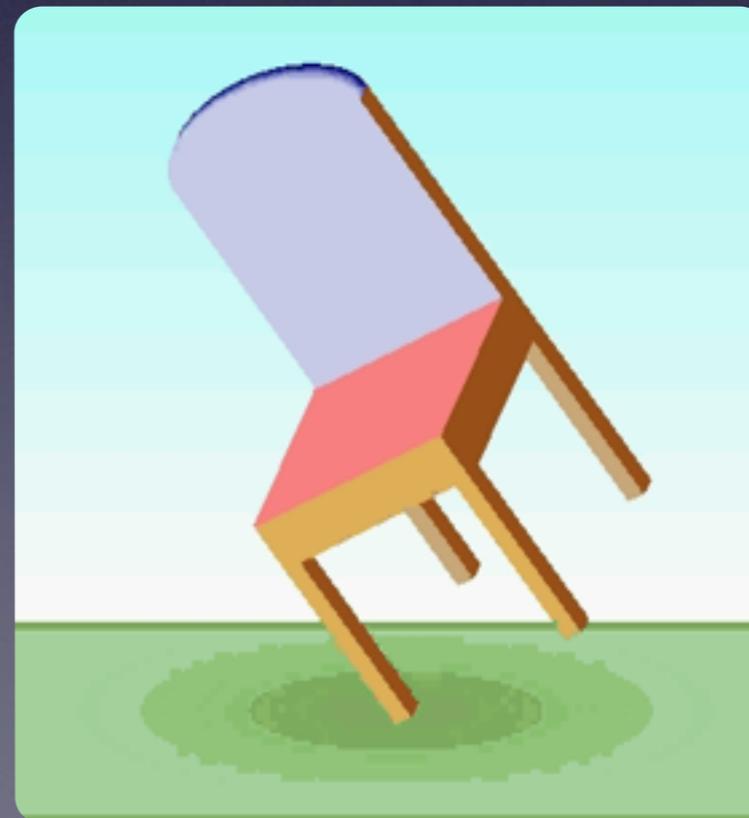
4) 等级性问题 (精细调节问题)



If SM valid up to GUT scale,
the theory has
extreme fine-tuning !

$$m_h^2 = m_0^2 - \delta m_h^2$$

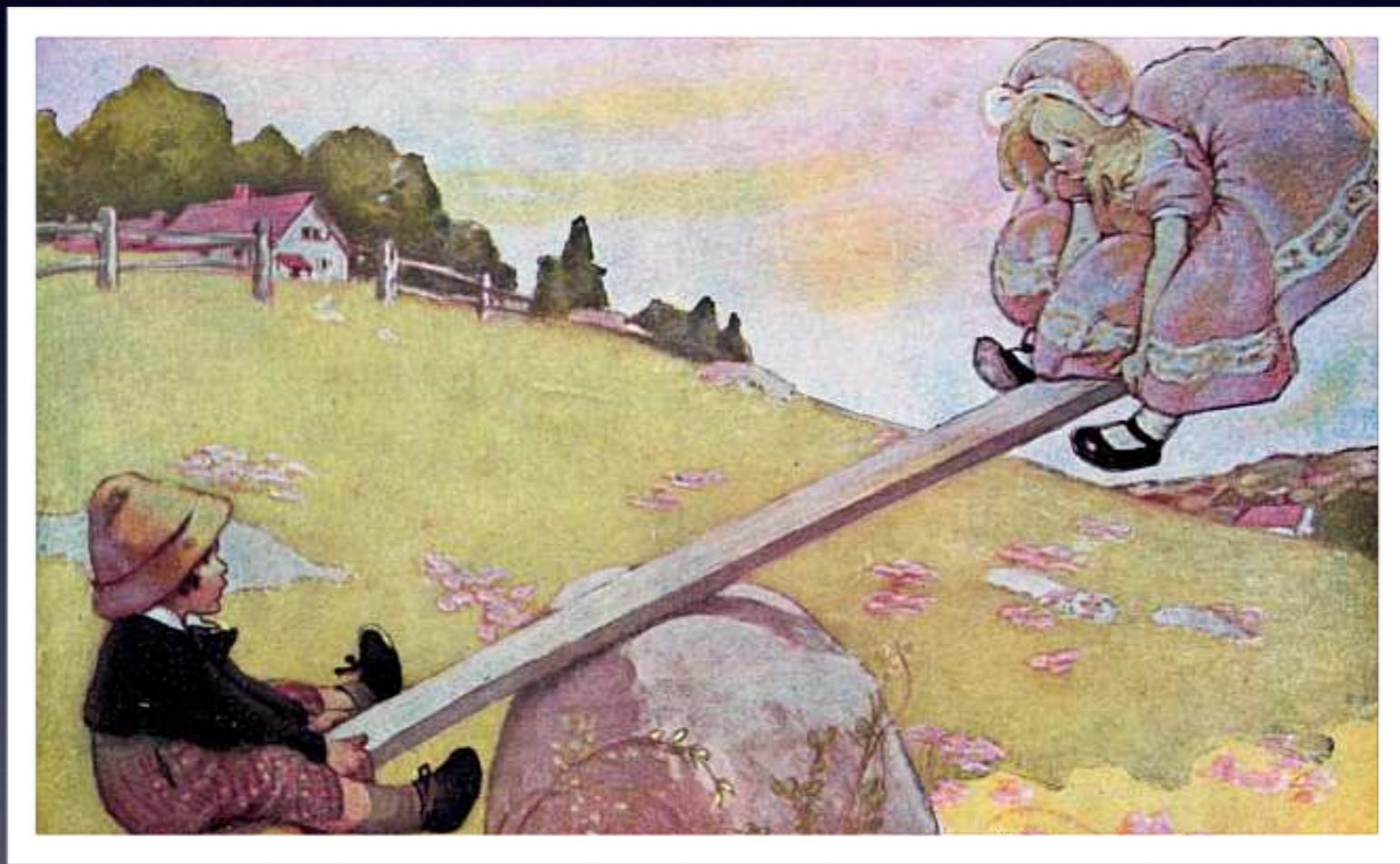
125 **bare** $\Lambda^2 / 52$



今天任务：中微子质量起源

—— 跷跷板机制 See-Saw Mechanism

简单
群论
应用



2



2

=

3



1

为何中微子质量如此之小?

跷跷板机制 —— 简单优雅解决方案

1) 加入一个新的 ν_R , (SM + ν_R)

SM neutral not gauged under SU(2)xU(1)

2) 对角化中微子质量矩阵

$$\begin{pmatrix} \nu_L & \nu_R \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$$

➔ $m_\nu = \frac{m_D^2}{M} \sim \frac{\text{TeV}^2}{M} \sim \text{eV}$

➔ $M \sim \frac{\text{TeV}^2}{\text{eV}} = \frac{(10^3 \text{ GeV})^2}{10^{-9} \text{ GeV}} = 10^{15} \text{ GeV}$

跷跷板机制的种类

探测新物理的有力工具 — 有效场论

BSM (Λ)

SM (m_W)



$$\mathcal{L} = \mathcal{L}_{\text{SM}}^{(4)} + \frac{O^{(5)}}{\Lambda} + \frac{O^{(6)}}{\Lambda^2} + \dots$$

High dimensional operator

$$O^{(5,6,\dots)}$$

are made of SM fields
with respect to the SM symmetry
 $SU(2) \times U(1)_Y$

温伯格中微子质量算符

1979年温伯格指出标准模型中存在唯一一个量纲为5的算符可以给中微子质量

$$\frac{(L\Phi)^2}{\Lambda} \quad L = \begin{pmatrix} \nu \\ e \end{pmatrix} \quad [L] = \frac{3}{2}$$
$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad [\Phi] = 1$$

$$\mathcal{L}_{m_\nu} = -\frac{f_{ij}}{2\Lambda} (\nu_i \phi^0 - \ell_i \phi^+) (\nu_j \phi^0 - \ell_j \phi^+) + h.c.$$

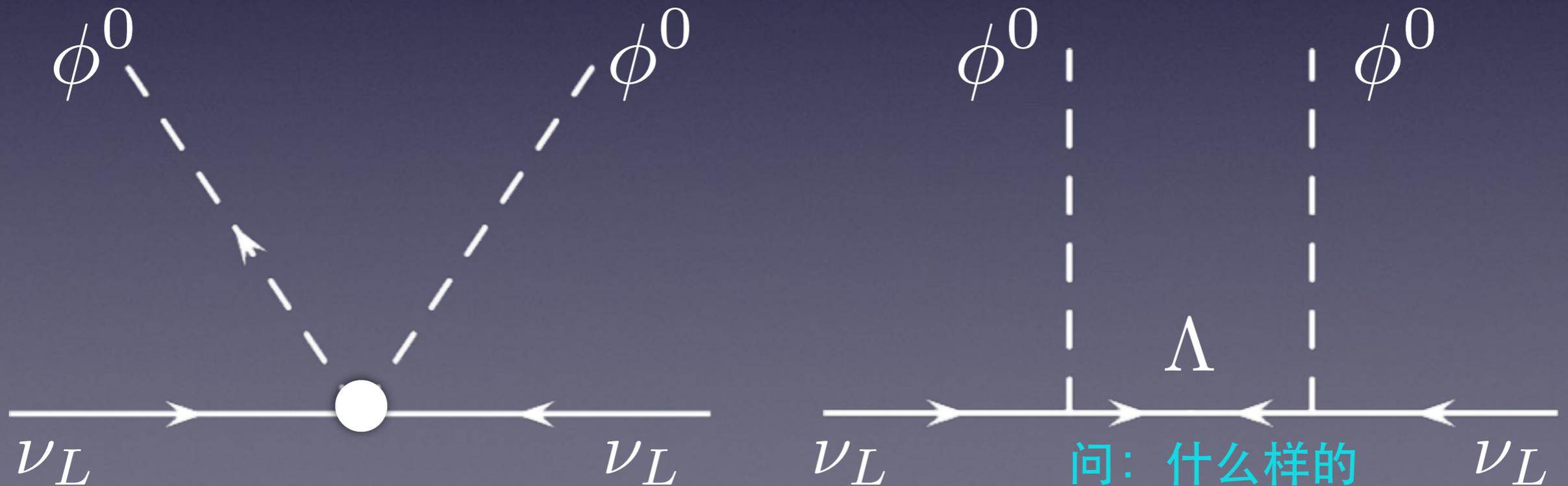
$$\Rightarrow (m_\nu)_{ij} = \frac{f_{ij} \langle \phi^0 \rangle^2}{\Lambda} = \frac{f_{ij} v^2}{\Lambda}$$

温伯格中微子质量算符

1979年温伯格指出标准模型中存在唯一的量纲为5的算符可以给中微子质量

$$\mathcal{L}_{m_\nu} = -\frac{f_{ij}}{2\Lambda} (\nu_i \phi^0 - \ell_i \phi^+) (\nu_j \phi^0 - \ell_j \phi^+) + h.c.$$

$$\Rightarrow (m_\nu)_{ij} = \frac{f_{ij} \langle \phi^0 \rangle^2}{\Lambda} = \frac{f_{ij} v^2}{\Lambda} \quad \begin{array}{l} [\phi] = 1 \\ [\nu] = 3/2 \end{array}$$



问：什么样的新物理？

温伯格算符

$$\frac{(L\Phi)^2}{\Lambda}$$

角动量耦合

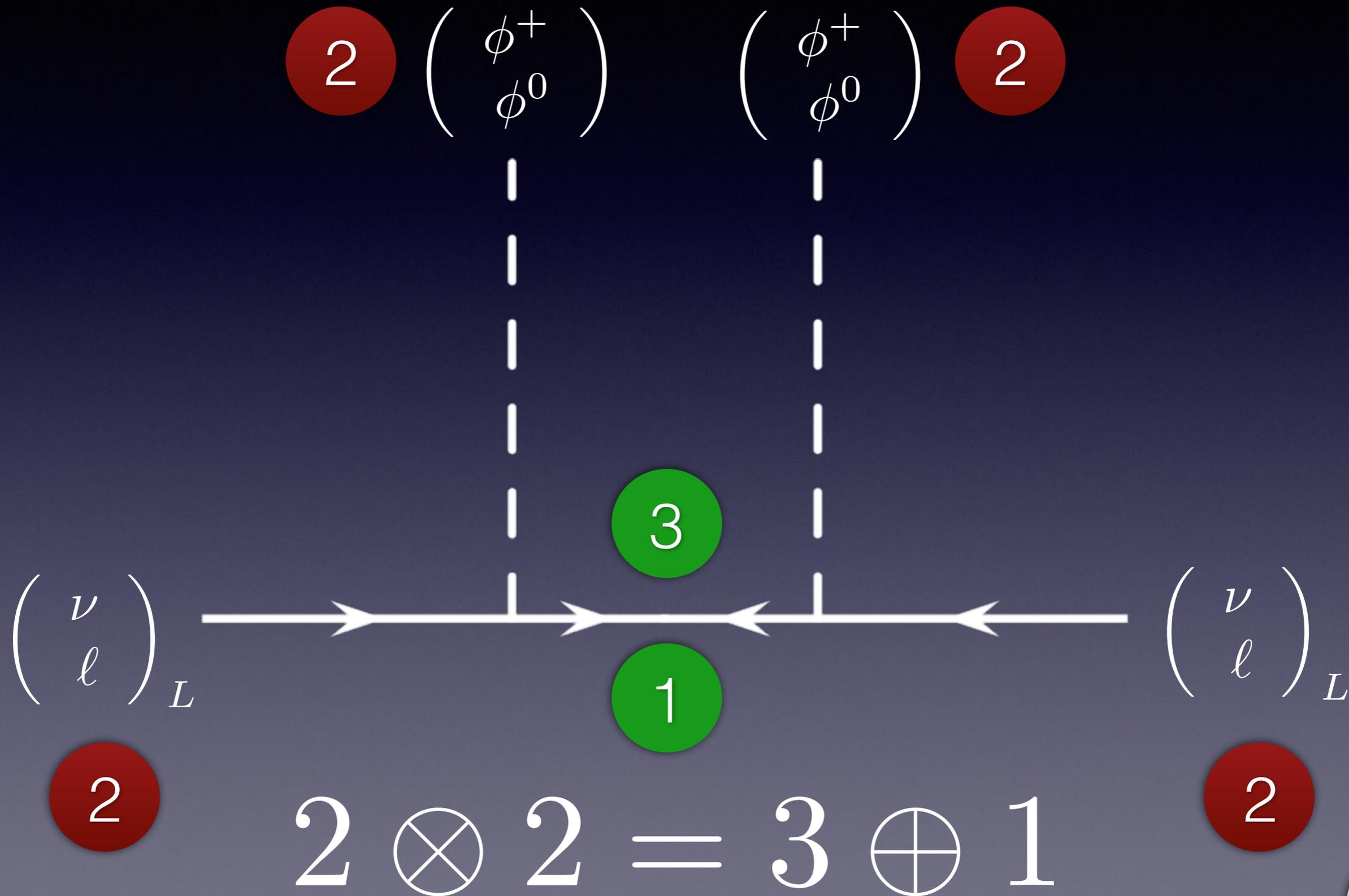
$$\begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix}_1 \otimes \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix}_2 = \begin{pmatrix} \uparrow_1 \uparrow_2 \\ \frac{1}{\sqrt{2}} (\uparrow_1 \downarrow_2 + \downarrow_1 \uparrow_2) \\ \downarrow_1 \downarrow_2 \end{pmatrix} \oplus \frac{1}{\sqrt{2}} (\uparrow_1 \downarrow_2 - \downarrow_1 \uparrow_2)$$

标准模型 SU(2)xU(1)

$$(L\Phi) = \begin{pmatrix} \nu \\ \ell \end{pmatrix} \otimes \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \sqrt{2}\nu\phi^+ \\ \underline{\nu\phi^0 + \ell\phi^+} \\ \sqrt{2}\ell\phi^0 \end{pmatrix} \oplus \begin{pmatrix} \underline{\nu\phi^0} - \ell\phi^+ \end{pmatrix}$$

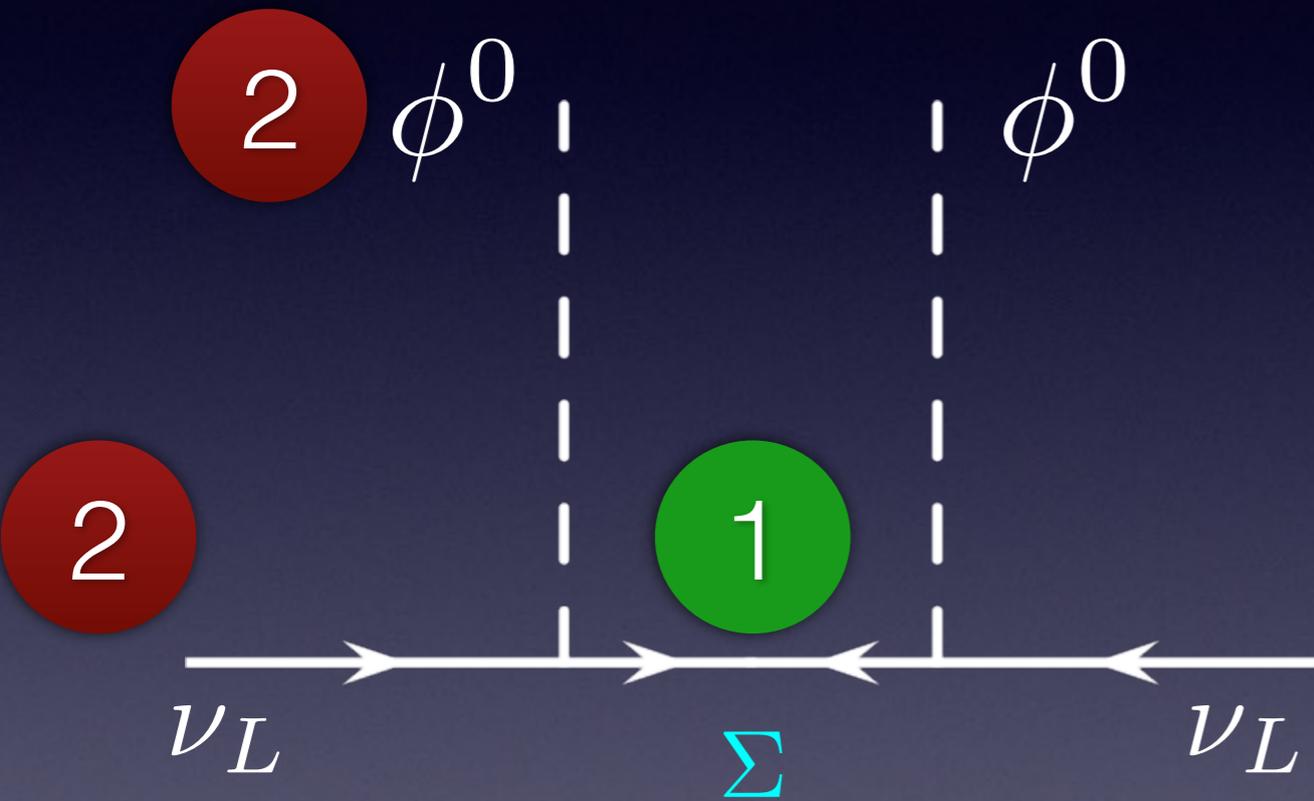
$$2 \otimes 2 = 3 \oplus 1$$

树图跷跷板机制

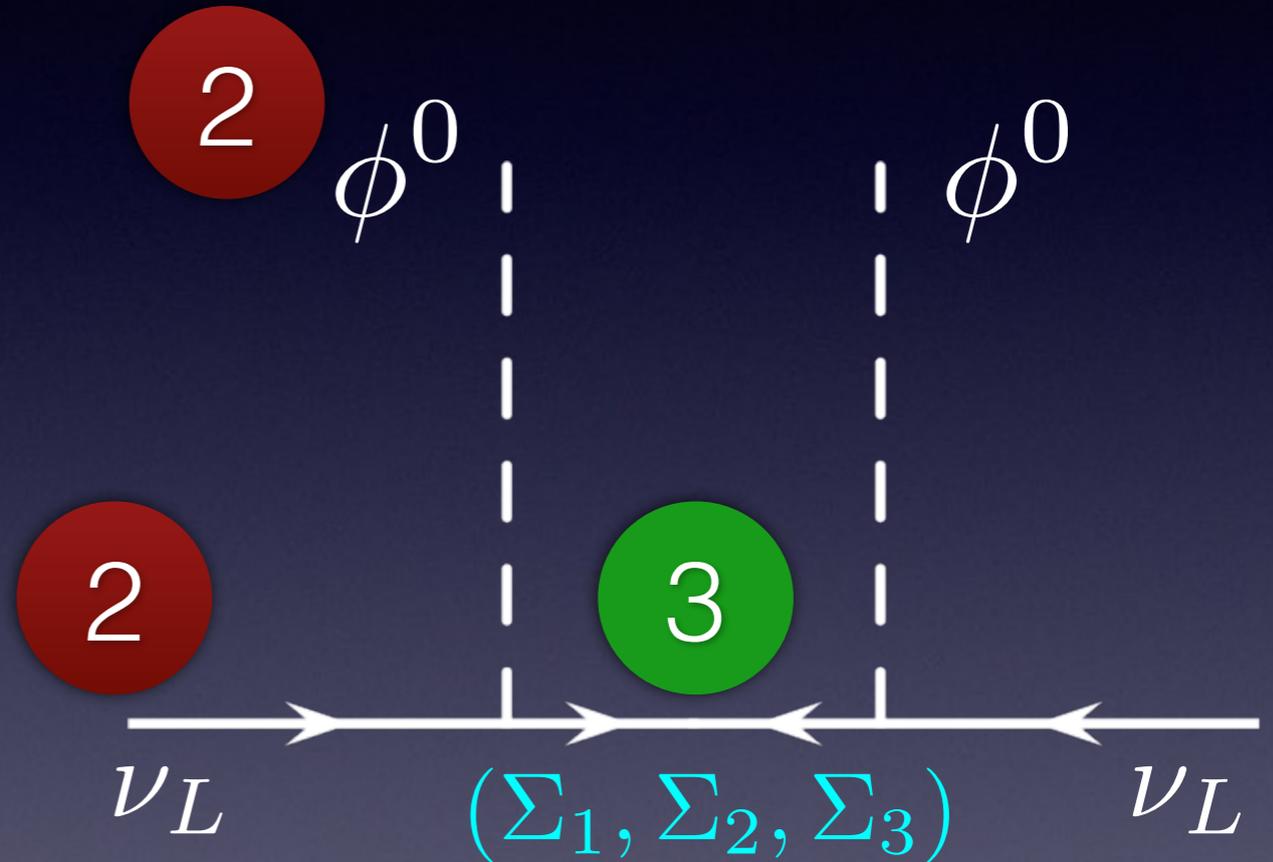


树图跷跷板机制：第1和第3类

第1类



第3类



角动量耦合

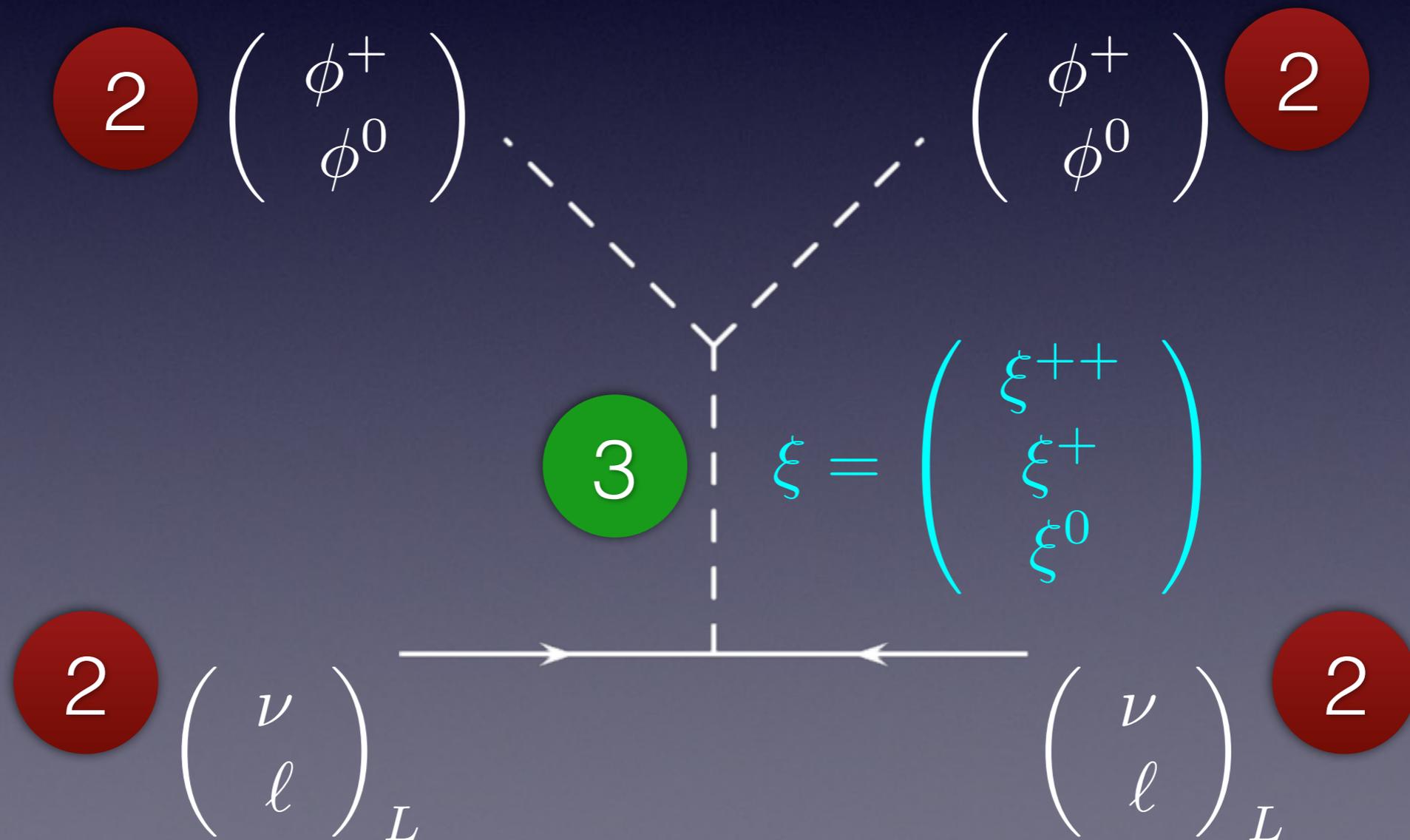
$$\vec{J} = \vec{L}_1 + \vec{L}_2$$

$$|J| = |L_1 - L_2|, \dots, L_1 + L_2$$

树图跷跷板机制：第2类

$$(LL) = \begin{pmatrix} \nu \\ \ell \end{pmatrix}_i \otimes \begin{pmatrix} \nu \\ \ell \end{pmatrix}_j = \begin{pmatrix} \overline{\nu_i \nu_j} \\ \nu_i \ell_j + \ell_j \nu_i \\ \ell_i \ell_j \end{pmatrix} \oplus (\nu_i \ell^j - \ell_i \nu_j)$$

相互作用
非质量项



粒子物理

26. 么正性和电弱理论起源

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作为一门实验科学,在物理学发展史上,理论和实验处于共生状态。通常人们会遇到现有理论无法解释的实验现象,从而使得人们构造新的物理规律。但如果设有实验数据支持的情况下,人们也可以从理论角度来研究当前物理规律的不足之处。

⇒ 公正性 可以帮助我们判断理论的高能行为

↳ 高能安全性 (High Energy Safety)

所谓公正性,仅仅是几率相加等于1的时髦说法。

基本原则是: What comes out should not be more than what goes in

* 基本粒子在高能区的行为

a) 有质量规范玻色子 —— 3种极化

$$\left. \begin{aligned} \epsilon_L^M &= (0, 1, -i, 0) \\ \epsilon_R^M &= (0, 1, +i, 0) \end{aligned} \right] [\epsilon_{L,R}^M] = 0$$

$$\epsilon_0^M \sim \frac{k^M}{M_V} \quad \longrightarrow \quad [\epsilon_0^M] = E$$

• Goldstone 等价性定理

高能极限下 ($E \gg M$), W/Z 规范玻色子的纵向极化分量
的行为等价于其“吃掉”的 Goldstone 粒子

$$\epsilon_L^M(k) = \left(\frac{|\vec{k}|}{m}, 0, 0, \frac{E}{m} \right), \quad \epsilon_L \cdot k = 0$$

b) 法则

自旋	λ/\hbar	差	端尖
0	1	$\frac{1}{E^2}$	$\frac{1}{E^2}$
$\frac{1}{2}$	\sqrt{E}	$\frac{1}{E}$	$\frac{1}{E}$
1	E	1	$\frac{1}{E^2}$

端尖: 该粒子两端都连接在质壳上的粒子

规范理论：各种图的坏高能行为彼此相消

单个图都有坏的行为，但自然界安排如此巧妙，
最后不同图之间的坏行为相互抵消

→ 有时人们需要引入新的粒子，而这一旦被证实
就引起整个领域的巨大变革

* 费米理论

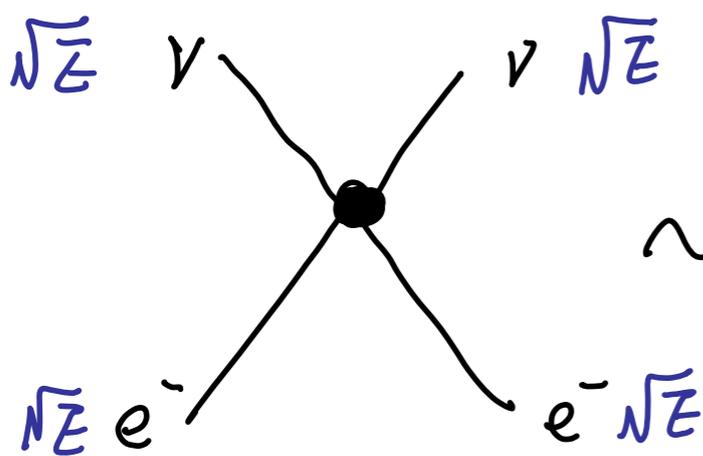
$$n \rightarrow p e^- \bar{\nu} : G_F \bar{\Psi}_n \gamma^\mu \Psi_p \bar{\Psi}_{e^-} \gamma^\mu \Psi_\nu$$

$$[G_F] = \text{GeV}^{-2}$$

非重整理论
(不完整)

- Feynman - Gell-mann V-A 理论

$$G_F \bar{\Psi}_n \gamma^\mu (1 - \gamma_5) \Psi_p \bar{\Psi}_{e^-} \gamma^\mu (1 - \gamma_5) \Psi_\nu$$

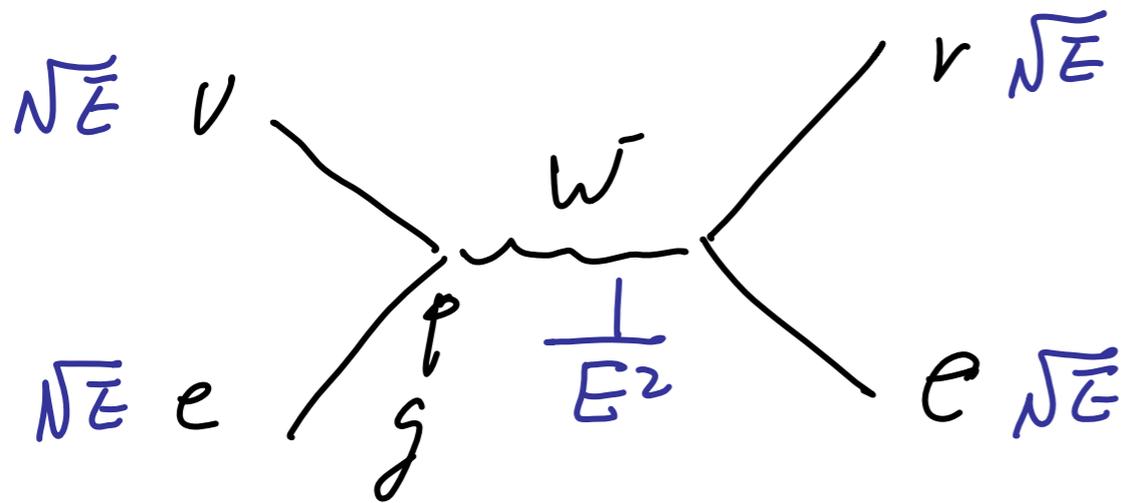


$$\sim G_F (\sqrt{E})^4 \sim G_F S$$

$$S = (2E)^2$$

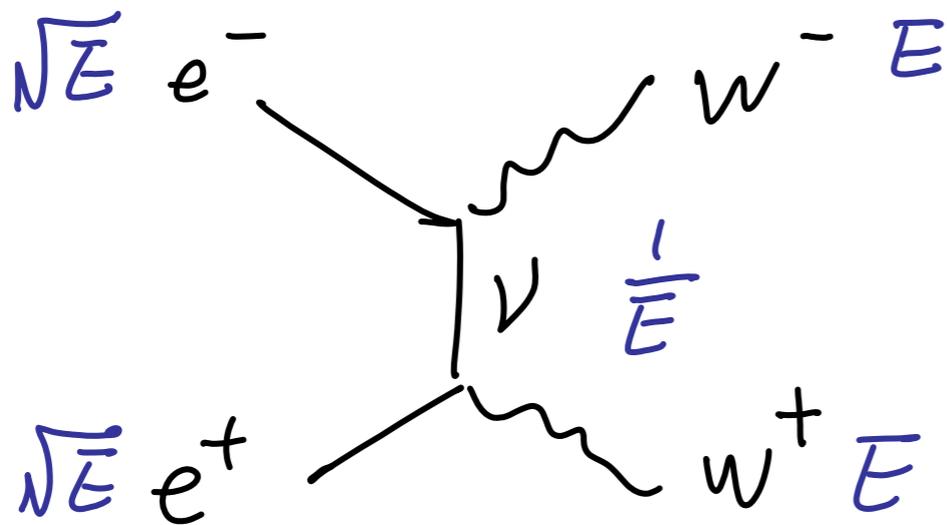
$$G_F = \text{常数} \Rightarrow \sigma(e^- \nu \rightarrow e^- \nu) \xrightarrow{S \rightarrow \infty} \text{发散}$$

• 中间玻色子模型 (intermediate gauge boson model)



$$M \sim g^2 \frac{(\sqrt{E})^4}{E^2} \sim (E)^0 \quad \text{ok}$$

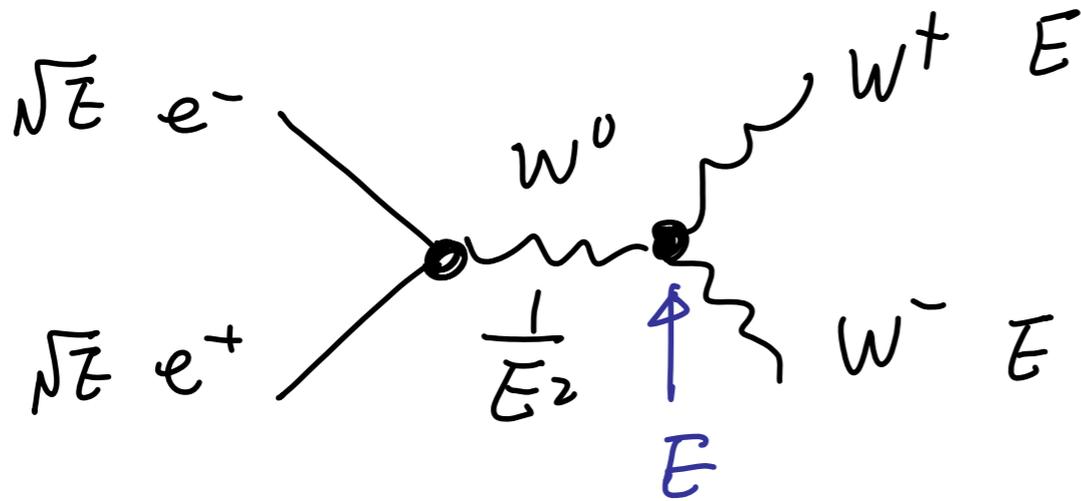
但是



$$M \sim g^2 \frac{E^2 \cdot (\sqrt{E})^2}{E} \sim E^2 \quad \text{bad}$$

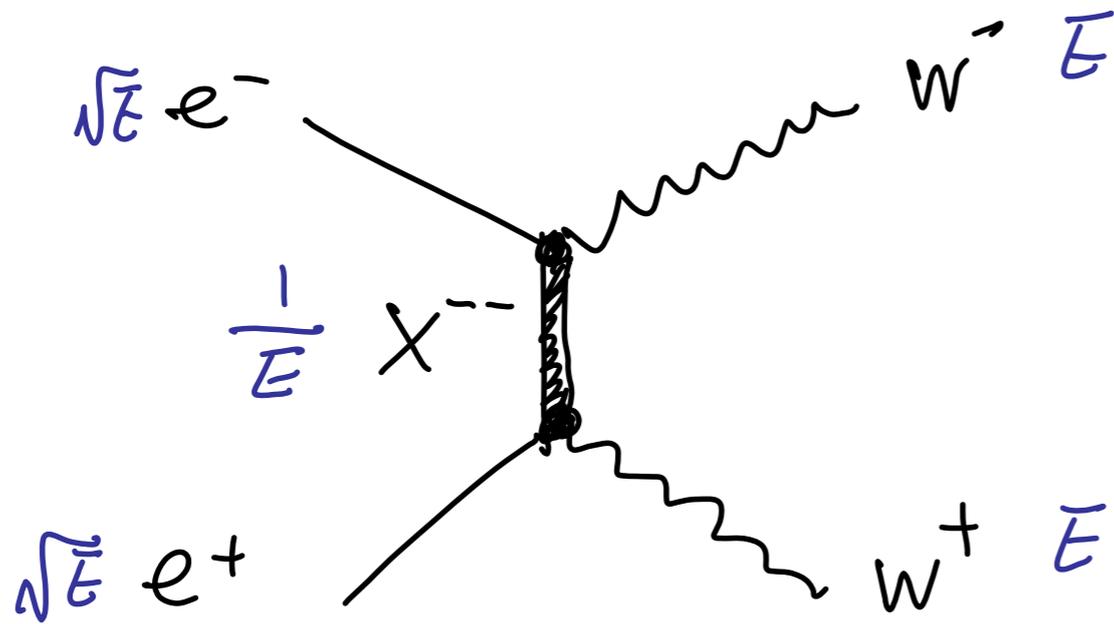
解决方案

(1) 中性流



$$m \sim g^2 \frac{(\sqrt{E})^2 E^{-2} \times E}{E^2} \sim E^2$$

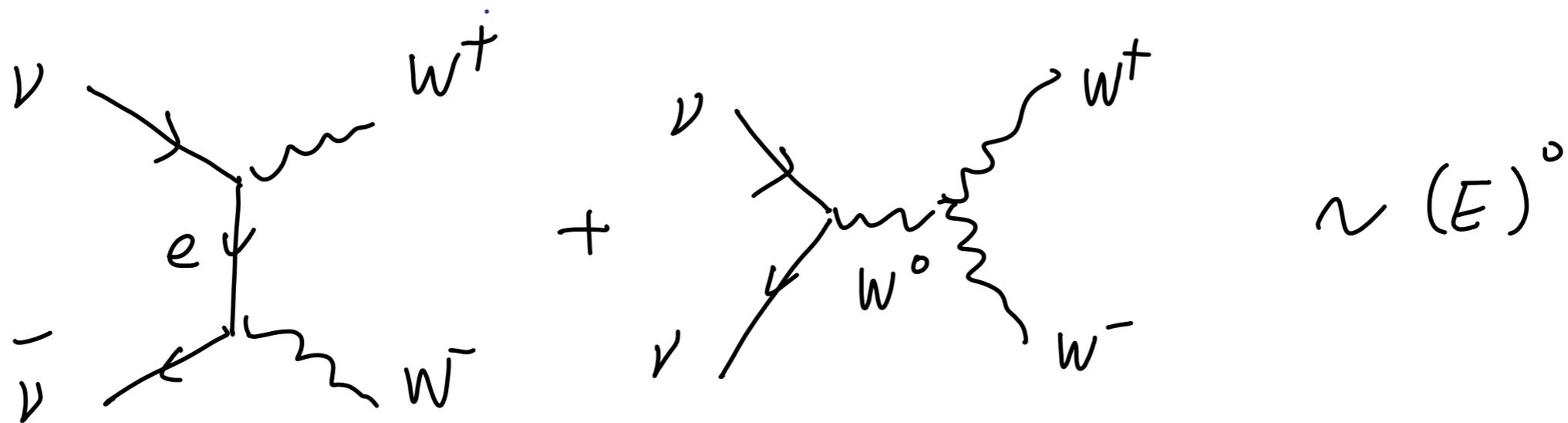
(2)



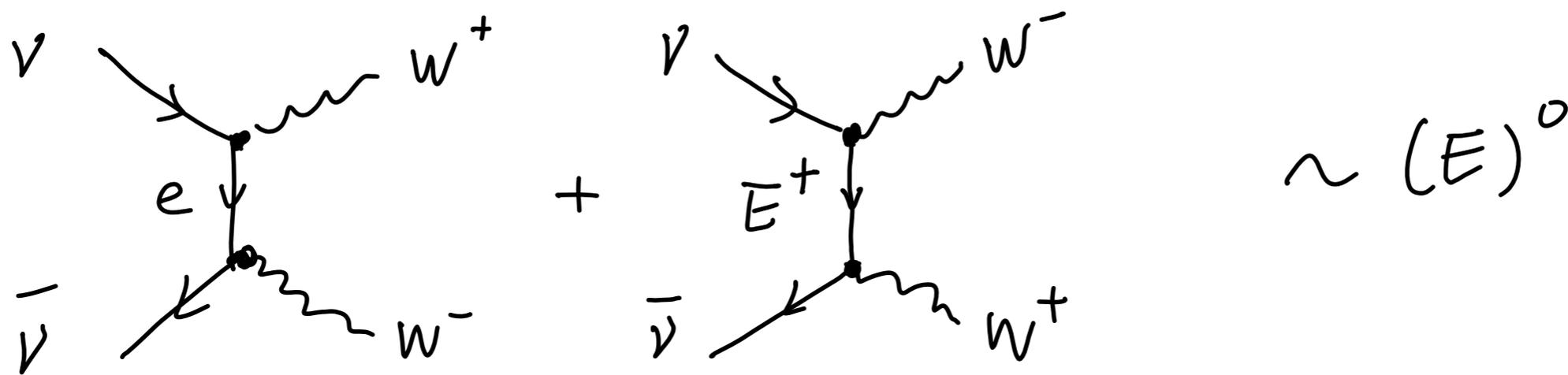
$$m \sim \frac{g^2 (\sqrt{E})^2 E^2}{E} \sim E^2$$

Exp says NO!

*)



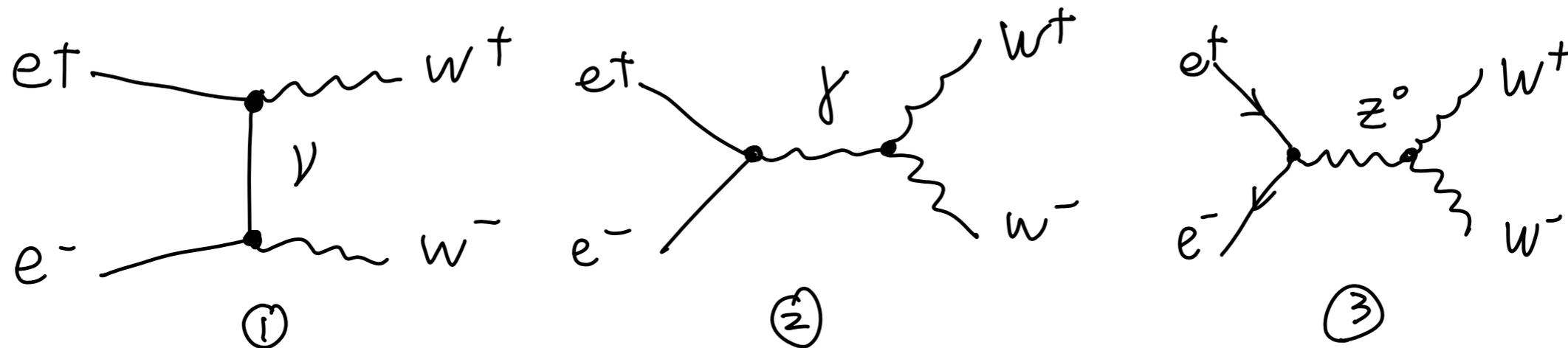
⇒ 中性流要和中微子相互作用



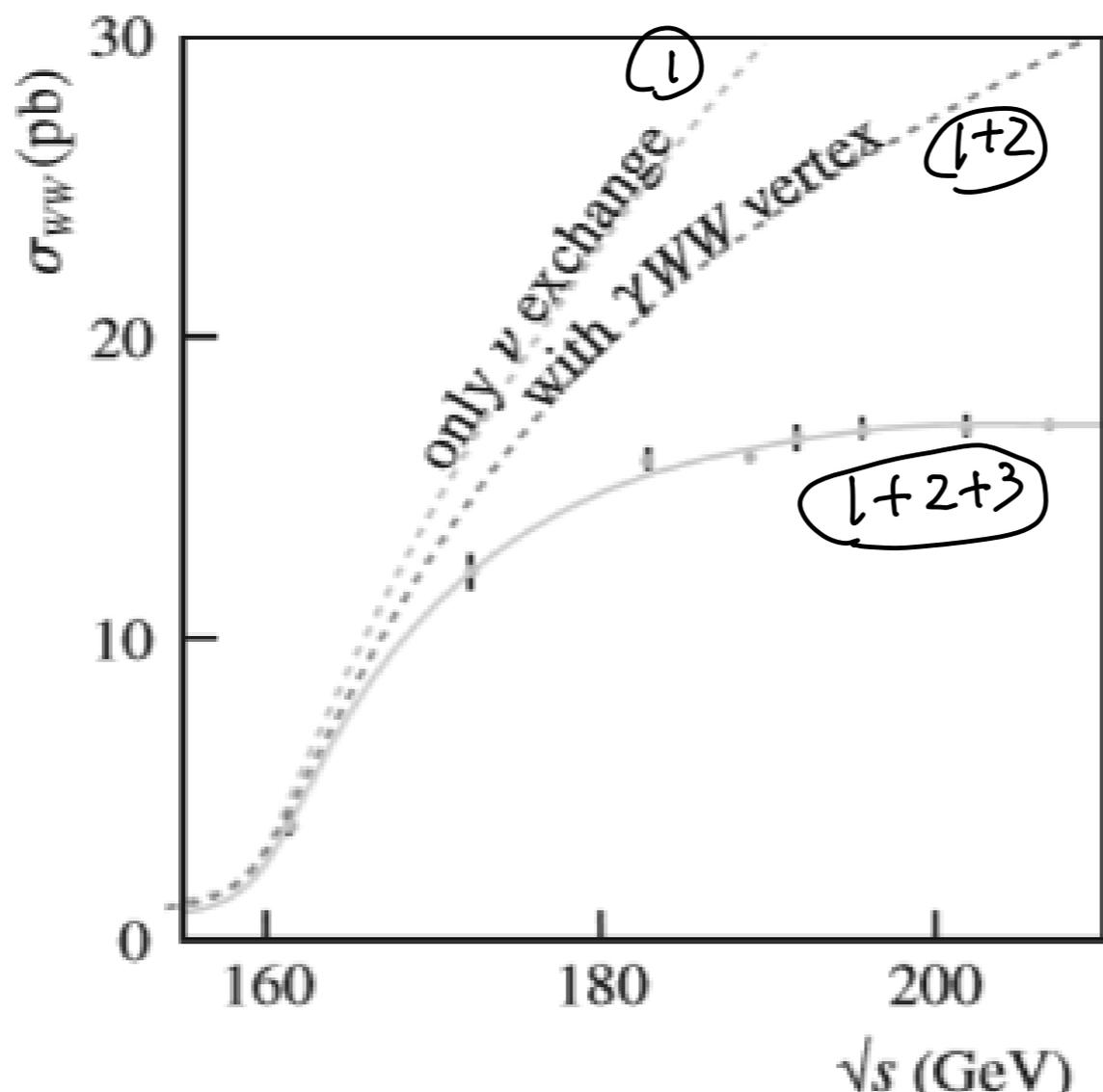
⇒ Paralepton E^+

H. Georgi, Glashow
 Phys. Rev. Lett. 28, 1494 (1972)
 Unified Weak and EM Interactions
 without Neutral currents

*) $e^+e^- \rightarrow w^+w^-$

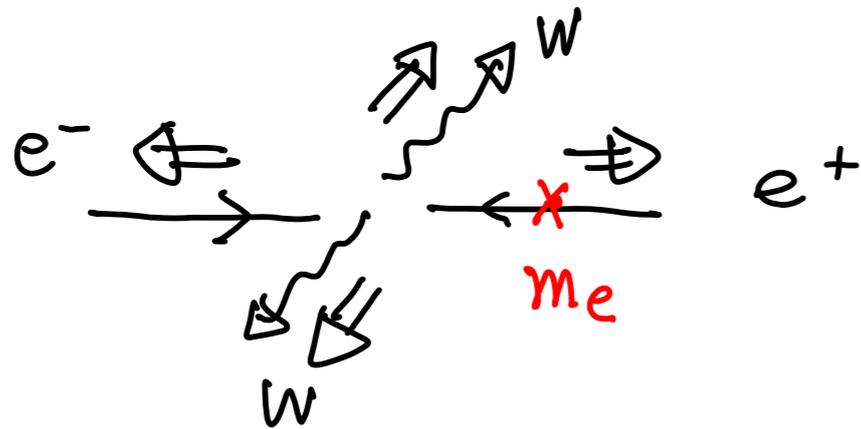


主要散射通过 p-波来进行 (s道的传播子是光子或 Z 玻色子 $J=1$)



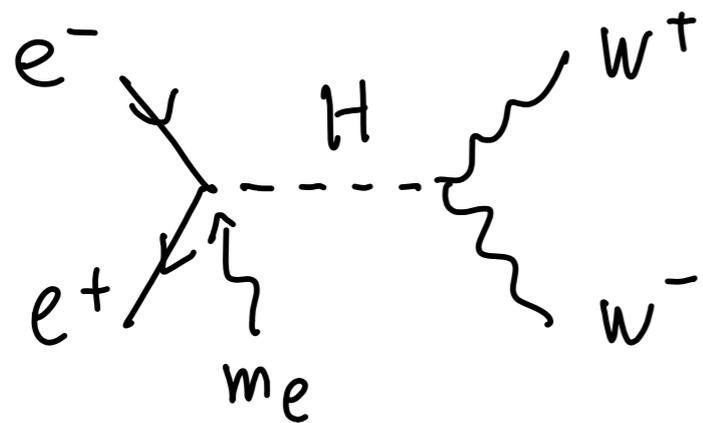
作业: 使用 CALCHEP
重复此条曲线

但除主导的P-波散射之外, $e^+e^- \rightarrow W^+W^-$ 还有S波贡献



虽然 S-波散射相比于P-波散射有 $(\frac{m_e}{m_W})^2$ 压低, 但 $E_{cm} \rightarrow \infty$ 时此分波振幅发散!

如何得到女子的高能行为?



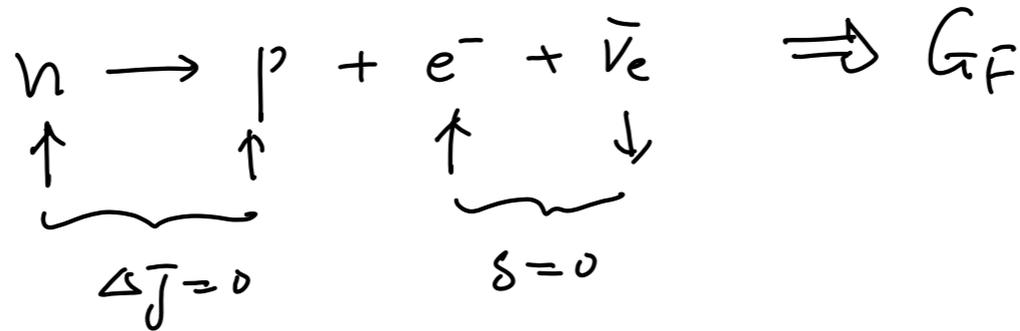
给出相反的高能区行为

* 中子衰变和 μ^- 衰变 (Cabibbo 混合)

β 衰变: $n \rightarrow p + e^- + \bar{\nu}_e$

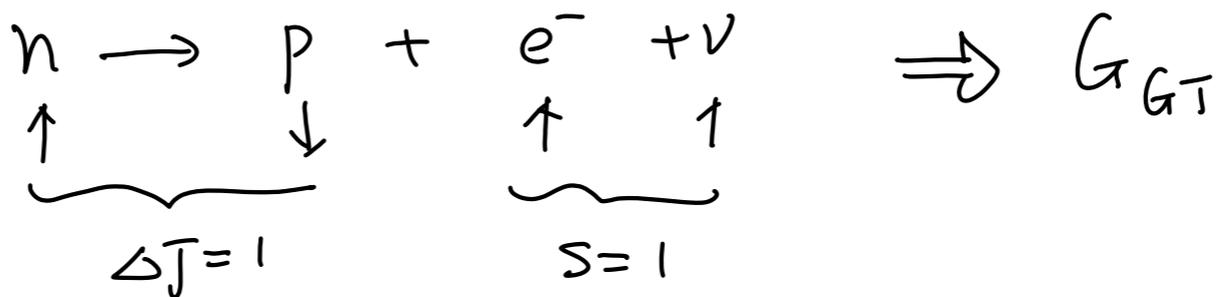
允许两种衰变:

1) 费米跃迁: 核自旋不变



$$T = \frac{2\pi}{\hbar} G_F^2 \left| \int (\psi_p^* \psi_n) (\psi_e^* \psi_{\bar{\nu}}) d\tau \right|^2 f(E)$$

2) GT 跃迁: 核自旋改变



$$T = \frac{2\pi}{\hbar} G_{GT}^2 \left| \int (\psi_p^* \sigma \psi_n) (\psi_e^* \sigma \psi_{\nu}) d\tau \right|^2 f(E)$$

实验表明

$$G_{GT} = 1.18 G_F$$

$$T = \frac{2\pi}{h} G_F^2 \left| \int (\psi_p^* \psi_n) (\psi_e^* \psi_\nu) d\tau \right|^2 \rho(E)$$

$$T = \frac{2\pi}{h} G_{GT}^2 \left| \int (\psi_p^* \sigma \psi_n) (\psi_e^* \sigma \psi_\nu) d\tau \right|^2 \rho(E)$$

实验表明 $G_{GT} = 1.18 G_F$

注意: $(\psi^* \psi)$ 和 $(\psi^* \sigma \psi)$ 并非相对论性协变 $\frac{\partial}{\partial x}$

矢量流 $V_\mu = \bar{\psi} \gamma_\mu \psi = (\bar{\psi} \gamma_0 \psi, \bar{\psi} \vec{\gamma} \psi) \xrightarrow[\text{极限}]{NR} (\psi^* \psi, \vec{0})$

轴矢量流 $A_\mu = \bar{\psi} \gamma_\mu \gamma_5 \psi = (\bar{\psi} \gamma_0 \gamma_5 \psi, \bar{\psi} \vec{\gamma} \gamma_5 \psi) \xrightarrow{NR} (0, \psi^* \vec{\sigma} \psi)$

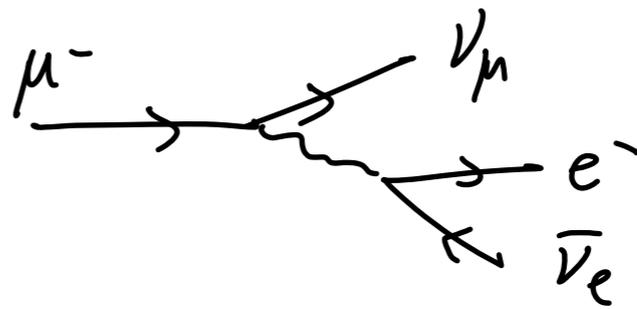
\Rightarrow V-A理论 V_μ 和 A_μ 同时出现且占相同比重 ($G_F = G_{GT}$)

\Rightarrow Exp data 表明 (V-1.18A) 理论

$$H = G_F (\bar{\psi}_p \gamma_\mu (1 - 1.18 \gamma_5) \psi_n) (\bar{\psi}_e \gamma^\mu (1 - \gamma_5) \psi_\nu)$$

μ -衰变 (V-A 理论)

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$



$$H_\mu = G_\mu (\bar{\Psi}_\nu \gamma_\rho (1 - \gamma_5) \Psi_\mu) (\bar{\Psi}_e \gamma^\rho (1 - \gamma_5) \Psi_\nu)$$

Exp data: $G_\mu \approx G_F$

实验数据.

$$\Delta S = 0$$

$$G_F: \begin{aligned} n &\rightarrow p + e^- + \bar{\nu}_e \\ \pi^- &\rightarrow \pi^0 + e^- + \bar{\nu}_e \\ \Sigma^+ &\rightarrow \Sigma^0 + e^+ + \nu_e \end{aligned}$$

$$|\Delta S| = 1$$

$$G'_F: \begin{aligned} \Lambda &\rightarrow p + e^- + \bar{\nu}_e \\ \Sigma^- &\rightarrow n + \mu^- + \bar{\nu}_\mu \\ K^+ &\rightarrow \pi^0 + e^+ + \nu_e \\ \Xi^- &\rightarrow \Lambda + e^- + \bar{\nu}_e \end{aligned}$$

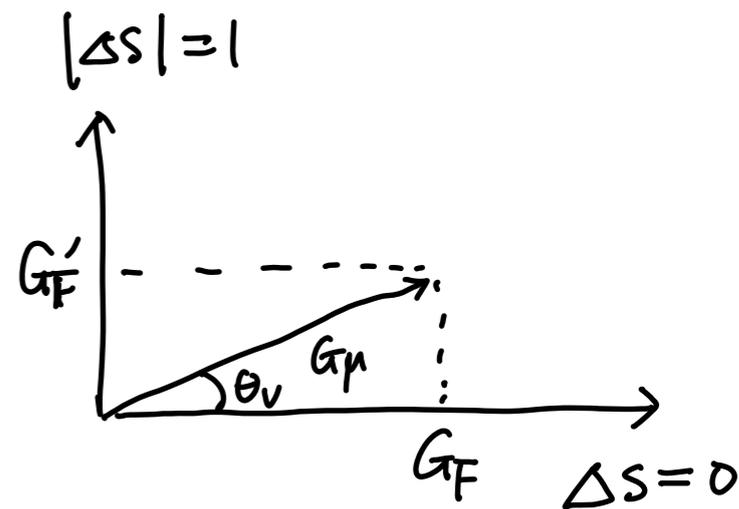
$$G'_F \approx \frac{1}{10} G_F$$

普适性要求 $G'_F = G_F = G_\mu$, 但与数据矛盾

Cabibbo 建议如下普适性:

$$\sqrt{G_F^2 + (G'_F)^2} = G_\mu \Rightarrow \begin{cases} G_F = G_\mu \cos \theta_V \\ G'_F = G_\mu \sin \theta_V \\ \frac{G'_F}{G_F} = \tan \theta_V \end{cases}$$

建立了轻子和强子之间的联系



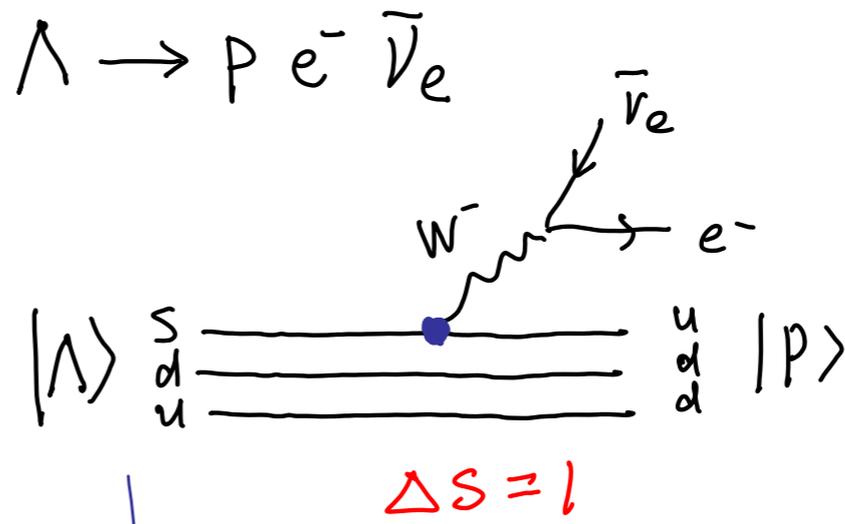
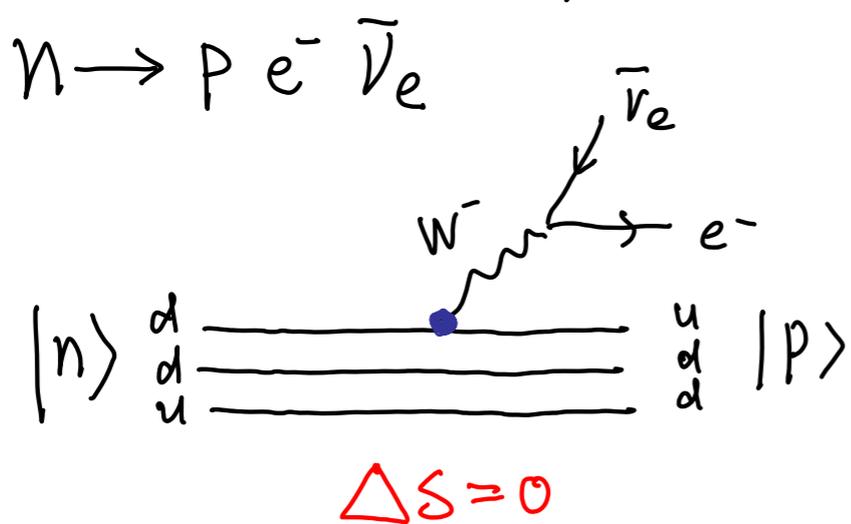
费米跃迁 $\frac{k^+ \rightarrow \pi^0 e^+ \nu_e}{\pi^+ \rightarrow \pi^0 e^+ \nu_e} \Rightarrow \theta_V = 0.247 \pm 0.008$

GT 跃迁 $\frac{k^+ \rightarrow \mu^+ \nu_\mu}{\pi^+ \rightarrow \mu^+ \nu_\mu} \Rightarrow \theta_A = \arctan \left(\frac{G'_{GT}}{G_{GT}} \right) = 0.226 \pm 0.005$

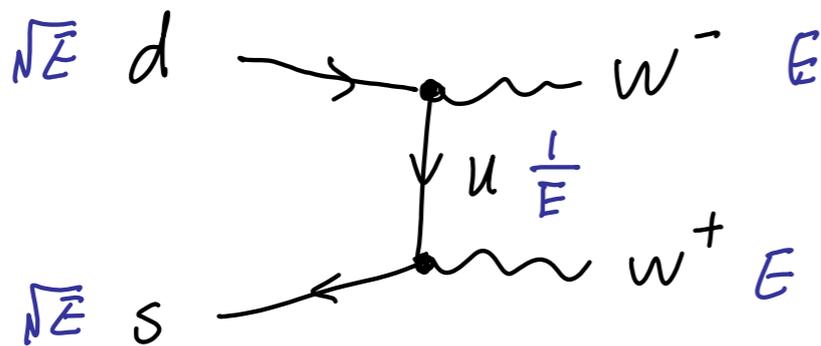
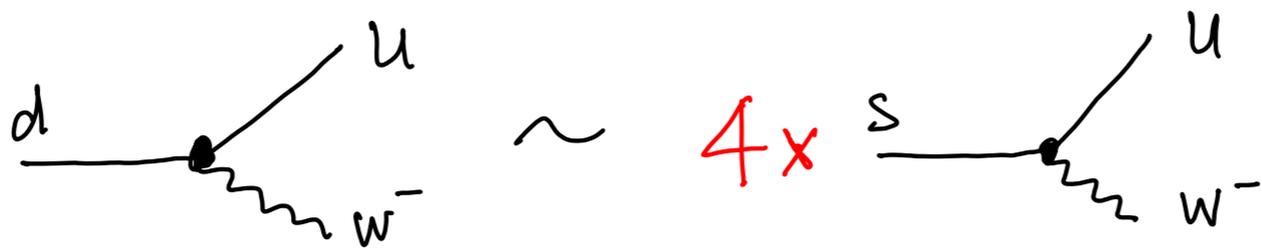
所以, $\frac{G'_{GT}}{G_{GT}} \approx \frac{G'_F}{G_F}; \theta_A \approx \theta_V \sim 13^\circ$

*) Charm 夸克

实验上发现：荷电流在轻子部分是普适的，
但在夸克部分却不是



observed in beta decay of strange hadron



$$m \sim \frac{E^2 \cdot (\sqrt{E})^2}{E} \sim E^2$$

如何解释 $G_{udW} \neq G_{usW}$ 以及 $ds \rightarrow W^+W^-$ 的么正破坏？

• Cabibbo 混合理论 (1963)

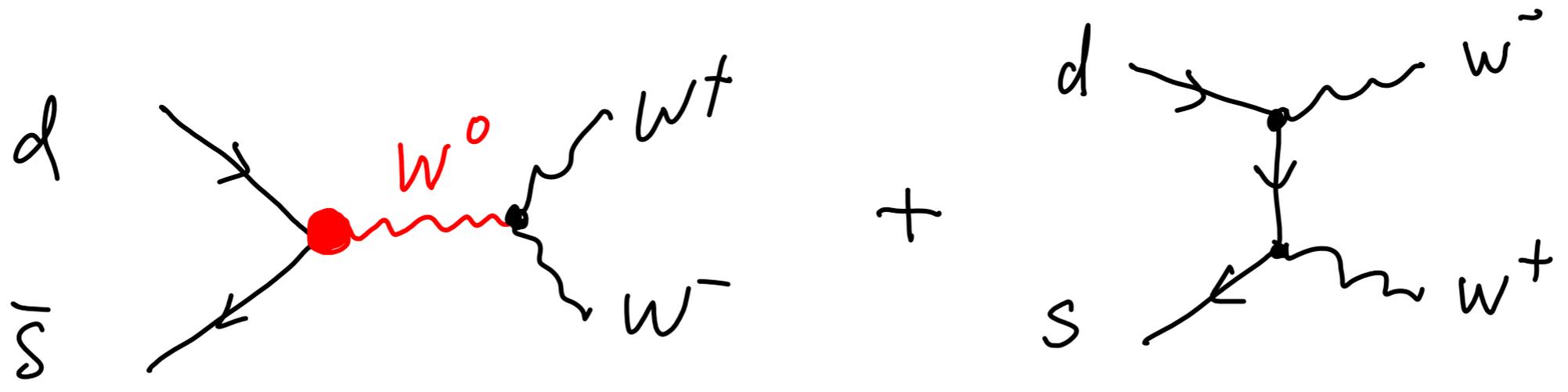
$$d'_W = \cos\theta_c d_L + \sin\theta_c s_L$$

\longleftarrow Cabibbo angle

$$\begin{aligned}
 M &\propto G_F \bar{e}_L \gamma_\alpha \nu_{eL} \bar{d}'_L \gamma^\alpha u_L \\
 &= G_F \bar{e}_L \gamma_\alpha \nu_{eL} \bar{d}_L \gamma^\alpha u_L \cos\theta_c \\
 &\quad + G_F \bar{e}_L \gamma_\alpha \nu_{eL} \bar{s}_L \gamma^\alpha u_L \sin\theta_c
 \end{aligned}$$

$$\left. \begin{aligned}
 \sin\theta_c &= 0.221 \\
 \cos\theta_c &= 0.974
 \end{aligned} \right\} \text{可解释实验结果}$$

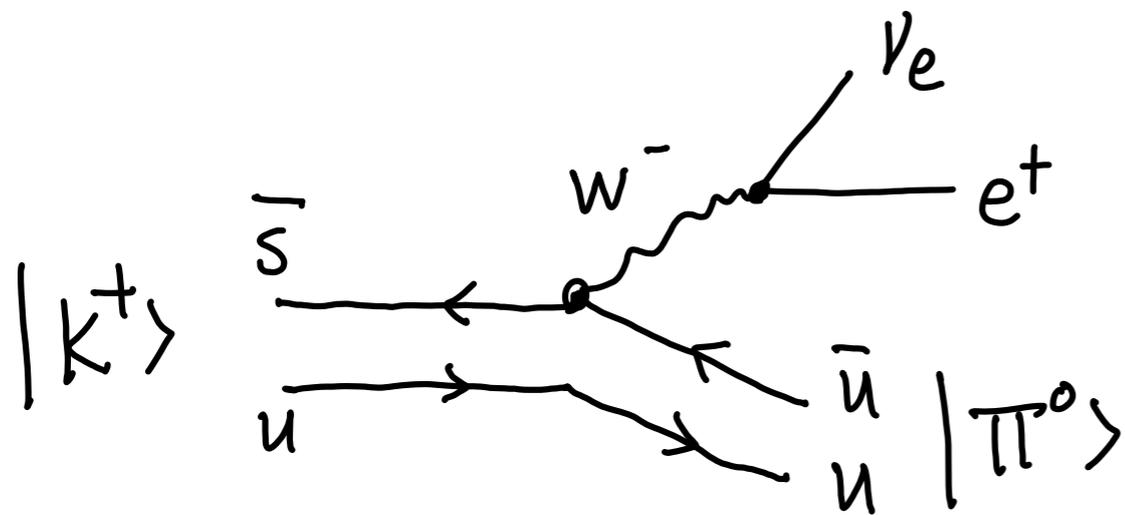
方案1: 高能安全的理论



这种味改变中性流 (Flavor-changing Neutral Current, FCNC)

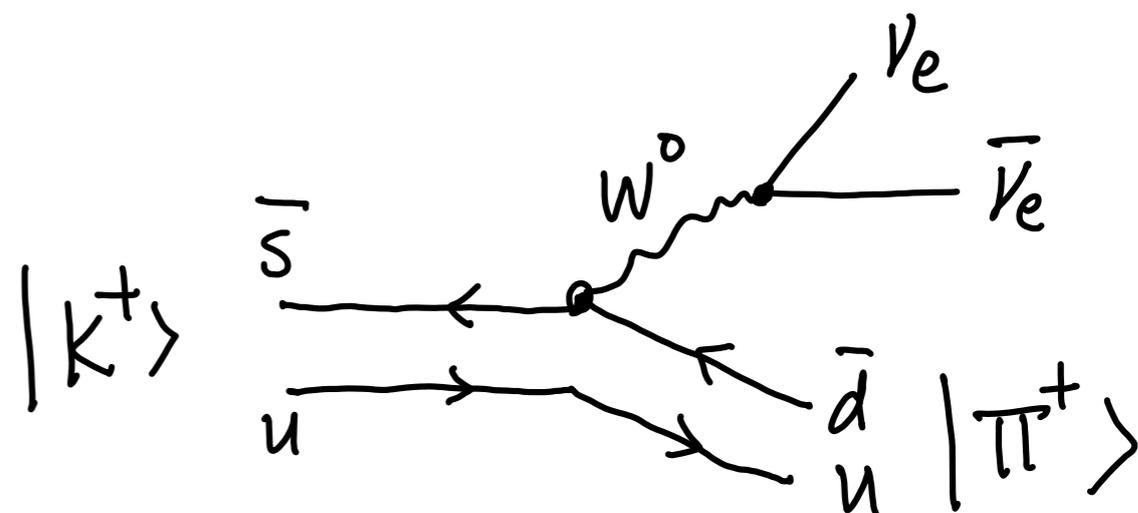
正是 Cabibbo 理论的不足之处

$$\bar{d}'_L \gamma^\alpha d'_L = C_\theta^2 \bar{d}_L \gamma^\alpha d_L + S_\theta^2 \bar{s}_L \gamma^\alpha s_L + C_\theta S_\theta [\bar{d}_L \gamma^\alpha s_L + \bar{s}_L \gamma^\alpha d_L]$$

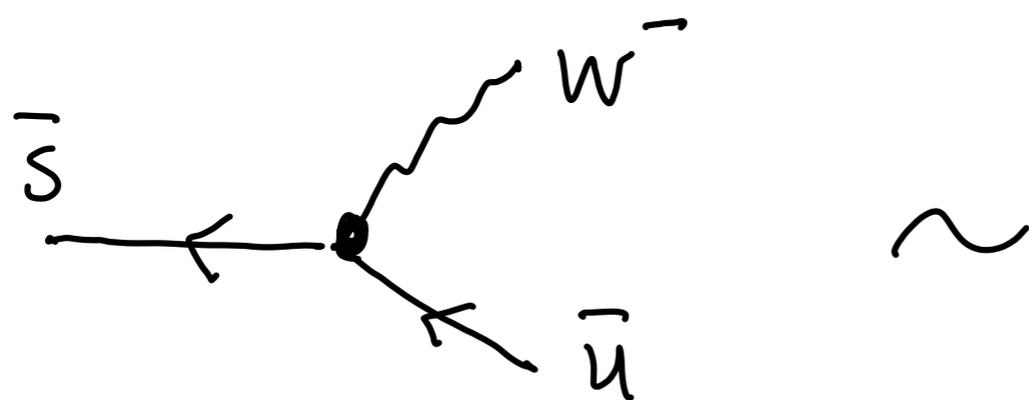


$$Br \sim 10^{-2}$$

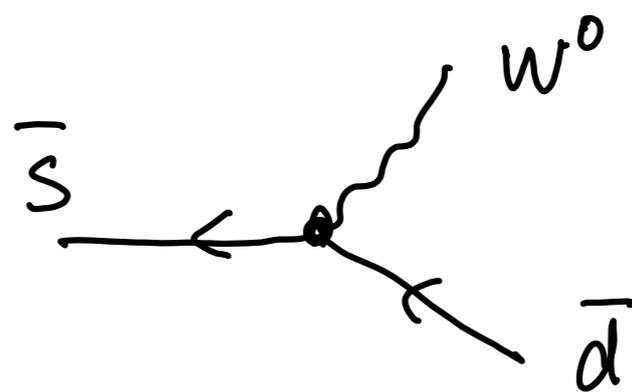
\gg



$$Br \sim 10^{-10}$$



$$G_F \bar{s}_L \gamma^\alpha u S_\theta$$



$$G_F C_\theta S_\theta \bar{s}_L \gamma^\alpha d_L + h.c.$$

Cabibbo混合角 $S_\theta \simeq 0.22$, $C_\theta \simeq 0.974$

\Rightarrow 上面两个顶点不应该有这么大的差异

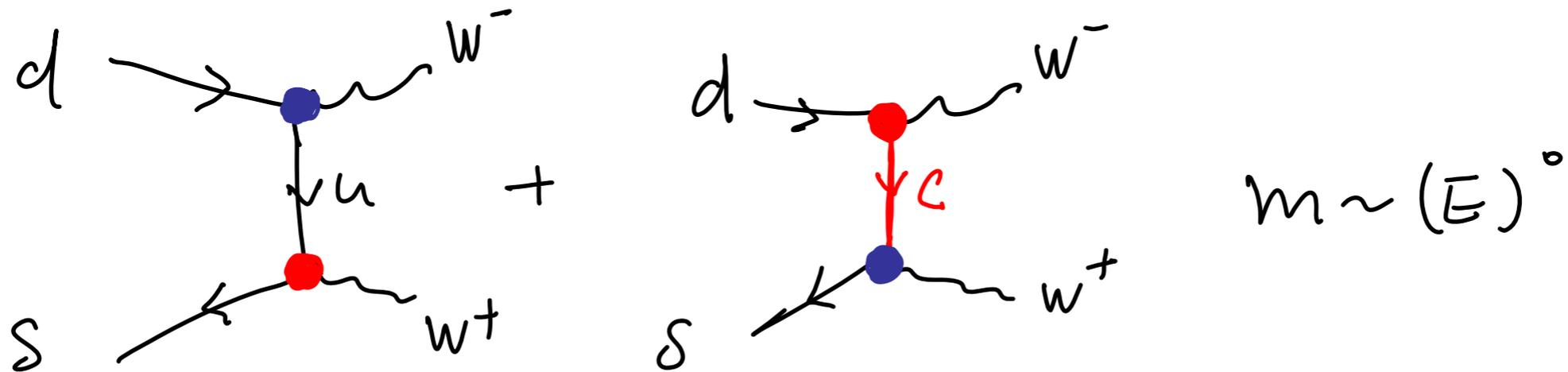
如何解释“不存在 $\Delta S=1$ 的中性流”以及调节



的高能行为?

Big Puzzle !!!

方案2: 高能安全理论



1970年 GIM 机制 —— 引入尚未发现的第4种夸克 - charm

$$\begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix}$$

$$\bar{S}_\alpha \gamma_\alpha S_L = S_\theta^2 \bar{d}_L \gamma_\alpha d_L + C_\theta^2 \bar{S}_L \gamma_\alpha S_L - S_\theta C_\theta [\bar{d}_L \gamma_\alpha S_L + \bar{S}_L \gamma_\alpha d_L]$$

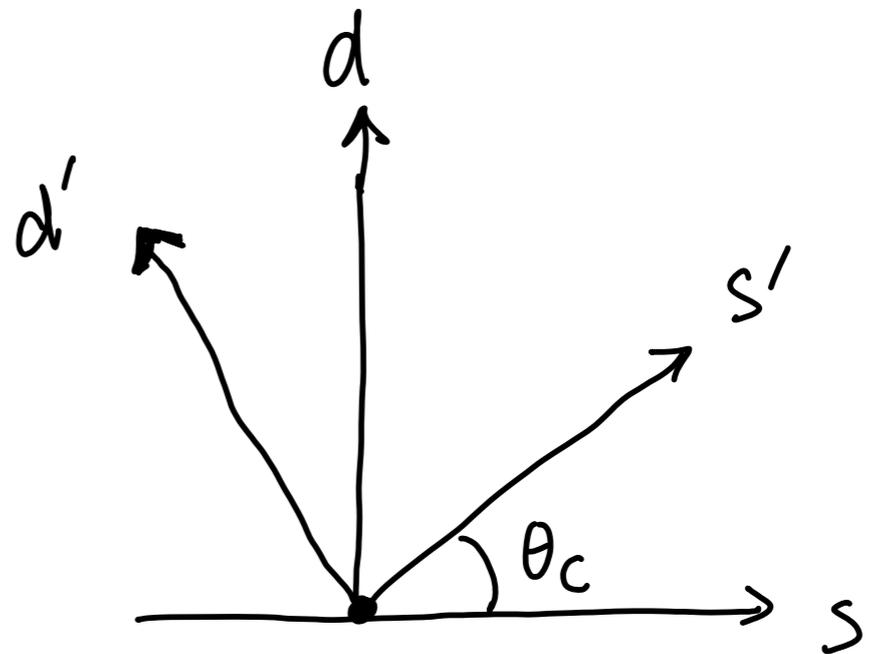
1970年 GIM 机制 — 引入尚未发现的第4种夸克 — charm

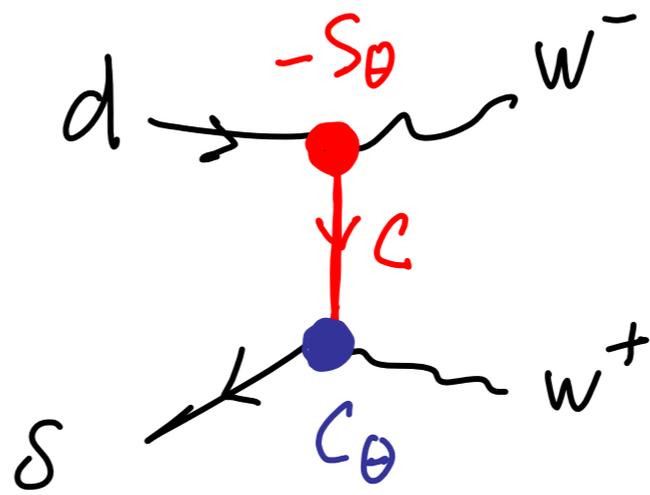
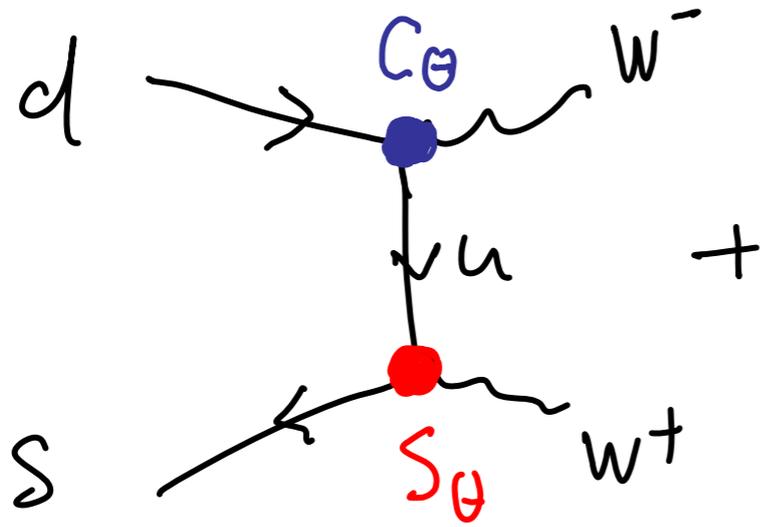
$$\begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix}$$

$$\begin{aligned} \bar{S}_\alpha \gamma_\alpha S_L &= S_\theta^2 \bar{d}_L \gamma_\alpha d_L + C_\theta^2 \bar{S}_L \gamma_\alpha S_L \\ &\quad - S_\theta C_\theta [\bar{d}_L \gamma_\alpha S_L + \bar{S}_L \gamma_\alpha d_L] \end{aligned}$$

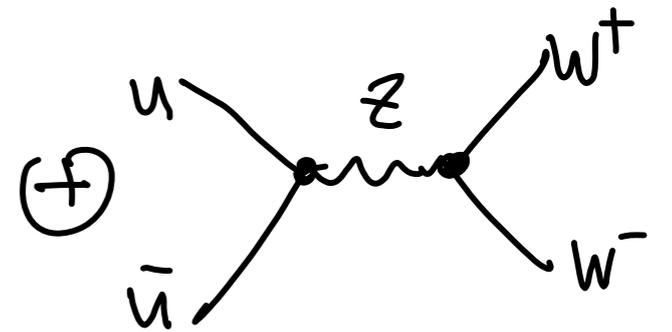
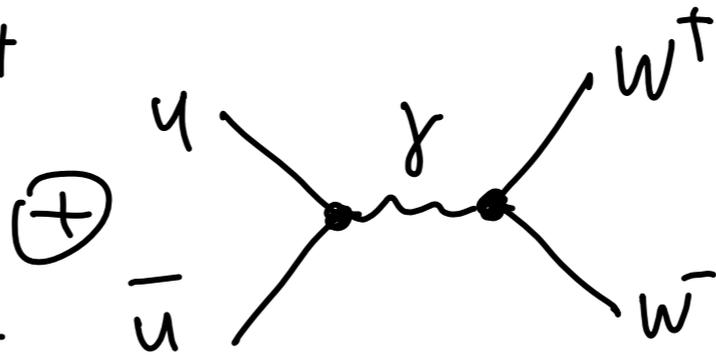
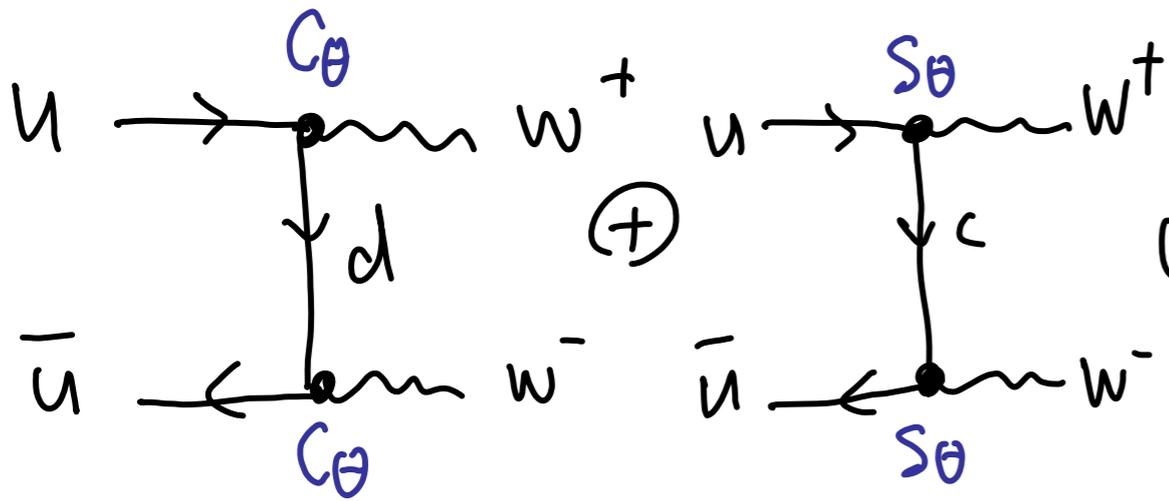
$$\begin{aligned} \Rightarrow \quad &\bar{S}'_\alpha \gamma_\alpha S'_L + \bar{d}'_L \gamma_\alpha d'_L \\ &= \bar{d}_L \gamma_\alpha d_L + \bar{S}_L \gamma_\alpha S_L \end{aligned}$$

neutral current
conserves flavor!

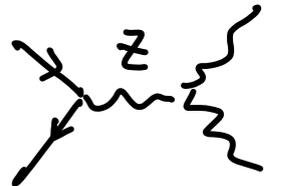
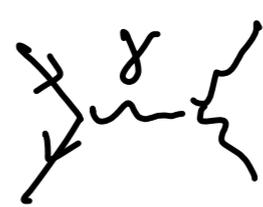
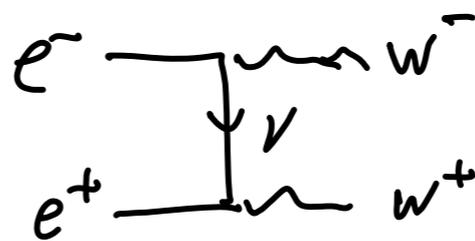




\Rightarrow 高能安全



$C_{\theta}^2 + S_{\theta}^2 = 1$



*) Higgs boson



$$m \sim E^4$$

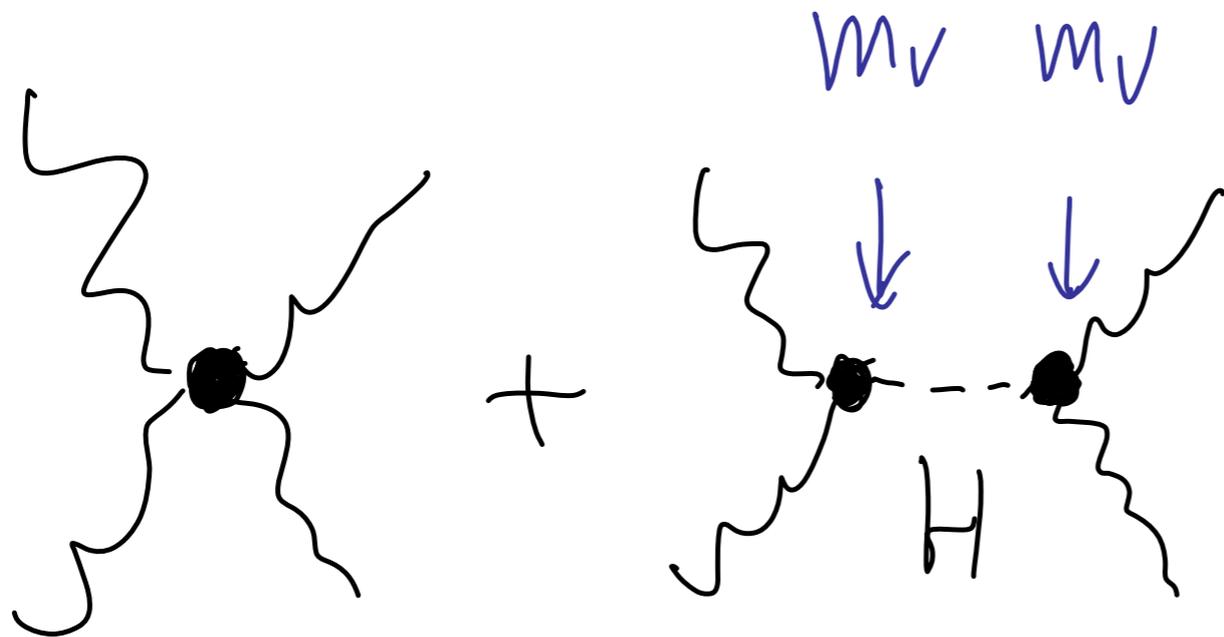
$$m \sim \frac{E^4 \times E^2}{E^2} \sim E^4$$

规范不变性
将之抵消

消除 E^4 的坏行为, 我们还有 E^2 的高能区行为

$$\left(\text{Diagram} \right) \sim \underline{M_V^2} E^2$$

↳ 要求存在一种粒子, 此粒子和规范玻色子作用要正比于 M_V \rightarrow Higgs boson



抵消掉 E^2 的坏行为