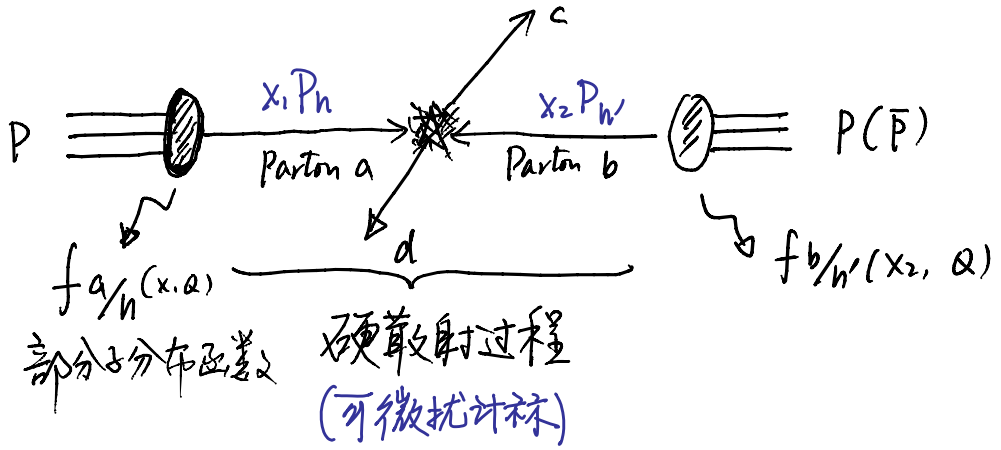
强子对撞机上的散射过程

$$p+p \longrightarrow c+d$$



$f_{a/h}(x,Q)$：在粒子h中找到动量为xP_h的部分子a的几率

$$\sigma(pp\to cd) = \int_0^1 dx_1 \int_0^{x_2} dx_2 \; f_{a/h}(x_1,Q^2)\, f_{b/h'}(x_2,Q^2)\, \hat{\sigma}(a+b\to c+d)$$

$P_a = x_1 P_h$, $P_b = x_2 P_{h'}$

$$+ \; (x_1 \leftrightarrow x_2)$$

a) x_1和x_2独立 $(x_1 \neq x_2)$ $\Longrightarrow$ $P_a + P_b = x_1 P_h + x_2 P_{h'} \neq 0$

实验室系并不是硬散射过程的质心系

b) 质心系能量是连续变化的

$$\hat{s} = (P_a + P_b)^2 = (x_1 P_h + x_2 P_{h'})^2 = x_1^2 \cancel{P_h^2} + x_2^2 \cancel{P_{h'}^2} + 2x_1x_2 P_h P_{h'}$$

入射质子对的不变质量

$$S = (P_h + P_{h'})^2 = \cancel{P_h^2} + \cancel{P_{h'}^2} + 2P_h P_{h'}$$

$$\Longrightarrow \hat{s} = x_1 x_2 S$$

好处：可同时扫描不同的质心系能量

坏处：无法确认质心系，特别是末态中有不可见粒子

c) 为计及不同部分子的贡献，我们需要对所有可能的部分子求和

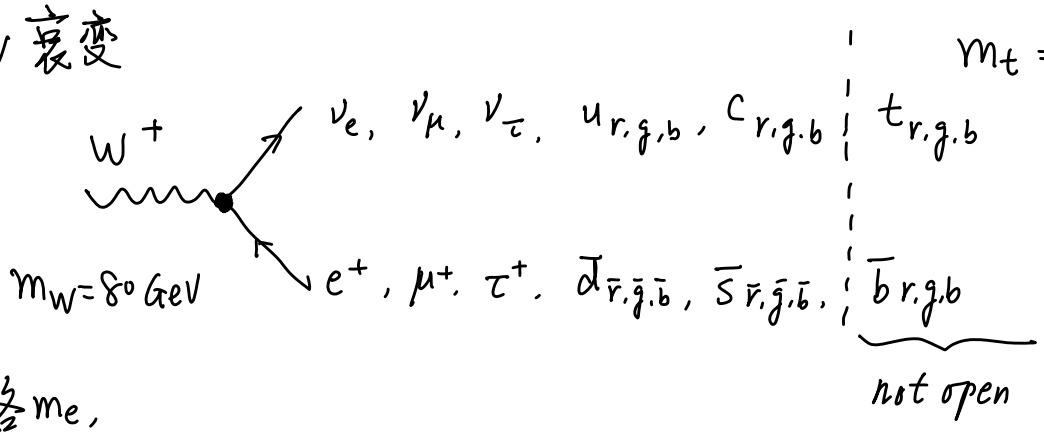
*) W和Z玻色子

1983年 W和Z玻色子首先在CERN的UA1实验($p\bar{p}$对撞机)中发现。

$\Longrightarrow$ 完美地验证了标准模型的弱电理论

a) W衰变

$m_t = 172\,\text{GeV}$



忽略 m_e,

$$\Gamma(W^+\to e^+\nu) = \frac{1}{2m_W}\frac{1}{8\pi}\underbrace{\frac{g^2}{2}m_W^2\cdot 2}_{|m|^2}\times\frac{1}{3} = \frac{\alpha_W}{12}m_W$$

$$\alpha_W = \frac{g^2}{4\pi} = \frac{e^2}{4\pi s_W^2}$$

因为 $m_{\mu,\tau,u,d,c,s} \ll m_W$,

所以 $\Gamma(W^+\to\nu e^+) = \Gamma(W^+\to\nu\mu^+) = \Gamma(W^+\to\nu\tau^+) = \frac{\alpha_W}{12}m_W$

$$\Gamma(W^+\to u\bar{d}) = \Gamma(W^+\to c\bar{s}) = \frac{\alpha_W}{12}m_W\times 3$$

$\Longrightarrow$ 总宽度 $\Gamma_{tot} = \frac{\alpha_W}{12}m_W\times(\underbrace{3}_{e,\mu,\tau}+\underbrace{2\times 3}_{u,c}) = \frac{3}{4}\alpha_W m_W$

$\alpha_W = \frac{1}{30}$, $m_W = 80.2\,\text{GeV} \Longrightarrow \Gamma^W_{tot} = 2.1\,\text{GeV}$

(满足NWA: $\Gamma \ll M$)

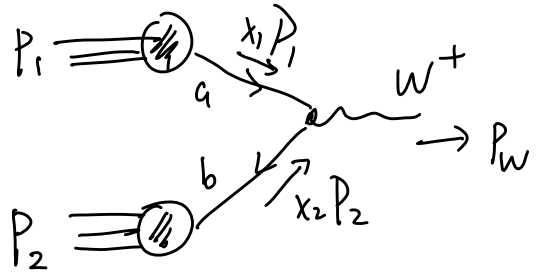
$$\tau_W = \frac{1}{\Gamma_W} = \frac{1}{2\,\text{GeV}} \approx 3\times 10^{-25}\,\text{s}$$

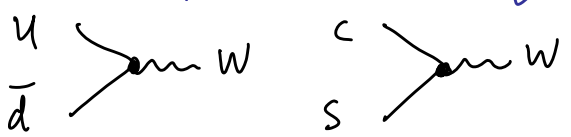
我们更关心的是Branching Ratio:

$$Br(W\to e^+\nu) = Br(W\to\mu\nu) = Br(W\to\tau\nu) = \frac{1}{9}$$

$$Br(W\to ud) = Br(W\to cs) = \frac{1}{3}$$

b) W产生



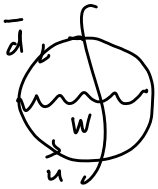


(假设 $V_{ud}=V_{cs}=1$)

$$\hat{\sigma}(u\bar{d}\to W^+) = \frac{1}{2\hat{s}}\int d\Pi_1 \, |m|^2$$

$$\int d\Pi_1 = \int \frac{d^3 P_W}{(2\pi)^3 \, 2E_W}(2\pi)^4 \delta(P_1+P_2-P_W) = 2\pi\,\delta(\hat{s}-m_W^2)$$

$$|m|^2 = \frac{g^2}{2} m_W^2 \times 2$$



所以,

$$\hat{\sigma}(u\bar{d}\to W^+) = \underbrace{\frac{1}{4}}_{\text{spin}} \times \underbrace{\left(\frac{1}{3}\times\frac{1}{3}\right)\cdot 3}_{\text{color}} \cdot \underbrace{\frac{1}{2m_W^2}}_{\frac{1}{2\hat{s}}=\frac{1}{2m_W^2}} \underbrace{(2\pi)\,\delta(\hat{s}-m_W^2)}_{d\Pi_1} \underbrace{\frac{g^2}{2} m_W^2 \times 2}_{|m|^2}$$

$$= \frac{\pi^2 \alpha_W}{3}\,\delta(\hat{s}-m_W^2)$$

$$\sigma(p\bar{p}\to W^+) = \int dx_1 f_{u/p}(x_1)\int dx_2 f_{\bar{d}/\bar{p}}(x_2)\,\hat{\sigma}\left(u(x_1P_1)\bar{d}(x_2P_2)\to W^+\right)$$

$$+\int dx_1 f_{\bar{d}/p}(x_1)\int dx_2 f_{u/\bar{p}}(x_2)\,\hat{\sigma}\left(u(x_2P_2)\bar{d}(x_1P_1)\to W^+\right)$$

在 $p\bar{p}$ 质心系中，部分子4动量为

$$p_1 = x_1 P_1 = (x_1 E, 0, 0, x_1 E)$$

$$p_2 = x_2 P_2 = (x_2 E, 0, 0, -x_2 E)$$

$$P_W = ((x_1+x_2)E, 0, 0, (x_1-x_2)E)$$

$$= (M\cosh Y, 0, 0, M\sinh Y)$$

$$\Rightarrow x_1 = \frac{M}{\sqrt{s}} e^{Y}, \quad x_2 = \frac{M}{\sqrt{s}} e^{-Y}$$

因为

$$\frac{\partial(x_1, x_2)}{\partial(M, Y)} = \begin{vmatrix} \frac{e^{Y}}{\sqrt{s}} & \frac{e^{-Y}}{\sqrt{s}} \\ \frac{Me^{Y}}{\sqrt{s}} & -\frac{Me^{-Y}}{\sqrt{s}} \end{vmatrix} = \frac{2M}{s}$$

所以

$$dx_1 dx_2 \delta(M^2 - m_W^2) = dM\, dY \frac{2M}{s} \delta(M^2 - m_W^2)$$

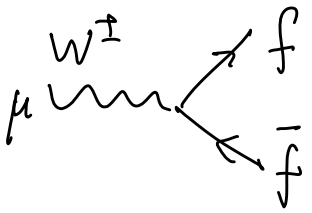
$$= dY \frac{1}{s}\Big|_{M=m_W}$$

$$\Rightarrow \frac{d\sigma}{dY}(p\bar{p} \to W^+) = \frac{\pi^2 \alpha_W}{3s}\left[f_{u/p}(x_1) f_{\bar{d}/\bar{p}}(x_2) + \cdots\right]$$

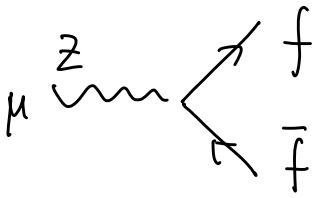
附注：W和Z衰变详细计算

$$\mathcal{L} = \frac{g}{2\sqrt{2}} W_\mu^+ \bar{\nu}\gamma^\mu(1-\gamma_5)e + \frac{g}{2\sqrt{2}} W_\mu^- \bar{e}\gamma^\mu(1-\gamma_5)\nu$$
$$+ \frac{g}{4c_\theta} Z_\mu \bar{\nu}\gamma^\mu(1-\gamma_5)\nu + \frac{g}{4c_\theta} Z_\mu \bar{e}\gamma^\mu\left[(-1+2s_\theta^2)(1-\gamma_5)+(2s_\theta^2)(1+\gamma_5)\right]e$$

Feynman rules:



$$i\frac{g}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)$$

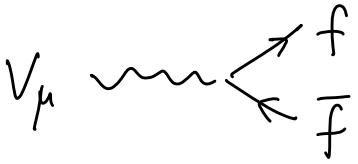


$$i\frac{g}{4c_\theta}\gamma^\mu(a'+b'\gamma_5)$$

其中 $a' = 2(T_3 - Qs_\theta^2) - 2Qs_\theta^2 = 2T_3 - 4Qs_\theta^2$

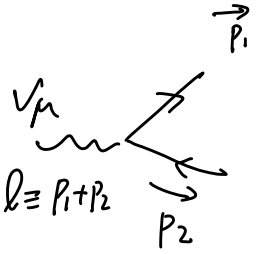
$b' = (-2T_3)$

下面我们采用如下一般形式



$$ig\gamma^\mu(a+b\gamma_5)$$

其衰变分宽度(在V的静止系内)是



$$d\Gamma = \frac{1}{2M}\overline{|m|^2}(2\pi)^4 d\Pi_2$$

$$\frac{d\Gamma}{d\Omega} = \frac{|\vec{p}_1|}{32\pi^2 M^2}\overline{|m|^2}$$

$$|\vec{p}_1| = \frac{M}{2} \Rightarrow \frac{d\Gamma}{d\Omega} = \frac{1}{64\pi^2 M}\overline{|m|^2}$$

$$\overline{|M|^2} = \underbrace{\frac{1}{3}}_{\substack{\text{V自旋}\\ \text{平均}}} \times \underbrace{1}_{\substack{V\\ \text{color}}} \times \sum_{\text{spin}} |M|^2$$

$$= g^2 \times \frac{1}{3} \times 1 \times \sum_{\text{spin}} \varepsilon_\mu \varepsilon_\nu^* \left[\bar{u}(p_1)\gamma^\mu(a+b\gamma_5)v(p_2)\right] \left[\bar{v}(p_2)(a-b\gamma_5)\gamma^\nu u(p_1)\right]$$

$$= g^2\left(\frac{1}{3}\right) \underbrace{\left(-g_{\mu\nu} + \frac{l_\mu l_\nu}{M^2}\right)}_{\text{sym.}} \underbrace{\mathrm{Tr}\left[\not{p}_1 \gamma^\mu (a+b\gamma_5)\not{p}_2(a-b\gamma_5)\gamma^\nu\right]}_{\text{仅需考虑对称项}}$$

$$= g^2 \frac{1}{3}\left(-g_{\mu\nu} + \frac{l_\mu l_\nu}{M^2}\right)\left\{(a^2+b^2)4\right\}\left\{p_1^\mu p_2^\nu - (p_1\cdot p_2)g^{\mu\nu} + p_1^\nu p_2^\mu\right\}$$

$$= g^2 \frac{1}{3}(4)(a^2+b^2)\left\{+2(p_1\cdot p_2) + \frac{1}{M^2}\left[2(l\cdot p_1)(l\cdot p_2) - (p_1\cdot p_2)(l\cdot l)\right]\right.$$

$\because\ l^2 = M^2,\quad 2p_1\cdot p_2 = M^2,\quad l\cdot p_1 = (p_1+p_2)\cdot p_1 = p_1\cdot p_2 = l\cdot p_2$

$\therefore\ [\ ] = 2(p_1\cdot p_2)^2 - (p_1\cdot p_2)l^2 = 2\left(\frac{M}{2}\right)^2 - \left(\frac{M^2}{2}\right)M^2 = 0$

$\Longrightarrow$ $l_\mu l_\nu$ 并不贡献，这是当规范玻色子和无质量费米子耦合时一般规律

故而，$\overline{|M|^2} = g^2 \frac{1}{3}(4)(a^2+b^2)M^2$

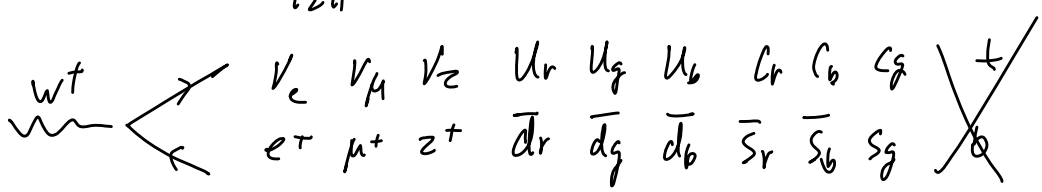
$$\frac{d\Gamma}{d\Omega} = \frac{1}{64\pi^2 M} g^2 \frac{1}{3} 4 (a^2+b^2) M^2 \qquad d\Omega = 4\pi$$

$$\Gamma_{V\to f\bar{f}'} = \frac{g^2 M}{12\pi}(a^2+b^2) = \frac{4\sqrt{2}G_\mu M_W^2 M}{12\pi}(a^2+b^2)$$

$$(g^2 = 4\sqrt{2}M_W^2 G_\mu)$$

(1) W decay : $a = b = \frac{1}{2\sqrt{2}}$

$$\Gamma_{W^+\to e^+\nu} = \frac{g^2 m_W}{12\pi}\left[\left(\frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{2\sqrt{2}}\right)^2\right] = \frac{\alpha_W}{12} m_W$$



(2) Z-decay:

$$\Gamma_Z \simeq 2.5\,\text{GeV}$$

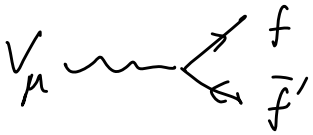
$Br(Z \to \nu_e \bar{\nu}_e) \sim 6.6\%$ $\times 3$ $(\nu_\mu \bar{\nu}_\mu, \nu_\tau \bar{\nu}_\tau)$

$Br(Z \to e^+ e^-) \sim 3.3\%$ $\times 3$ $(\mu^+\mu^-, \tau^+\tau^-)$

$Br(Z \to u\bar{u}) \sim 12\%$ $\times 2$ $(c\bar{c}$, not $t\bar{t})$

$Br(Z \to d\bar{d}) \sim 15\%$ $\times 3$ $(s\bar{s}, b\bar{b})$

作业：因为 $V_\mu \to f\bar{f}'$ 的衰变宽度正比于 (a^2+b^2)



$$ig\gamma^\mu(a+b\gamma_5) \equiv ig\gamma^\mu\left(c\frac{1-\gamma_5}{2}+d\frac{1+\gamma_5}{2}\right)$$

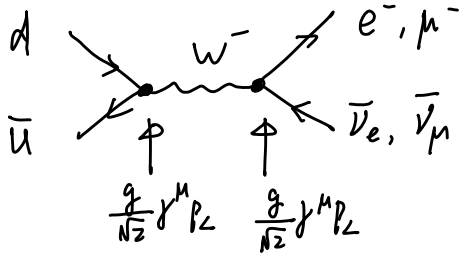
$$a=\frac{c+d}{2}, \quad b=\frac{c-d}{2}$$

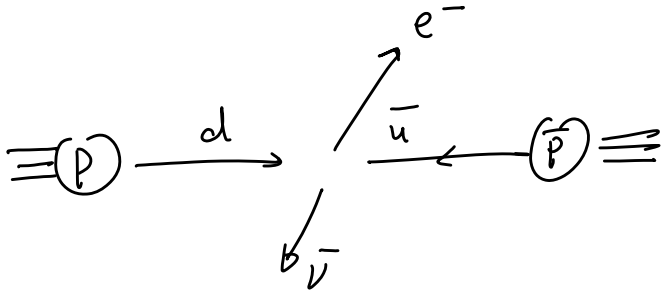
请写出 $W^\pm$ 和 Z 以及不同 f 末态的 a 和 b，
并推导 Z-boson 衰变总宽度和衰变分支比

V	f	f′	a	b	c	d
W^+	ν_e	e^+	$\frac{1}{2\sqrt{2}}$	$-\frac{1}{2\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0
⋮						

*) W玻色子的实验证据

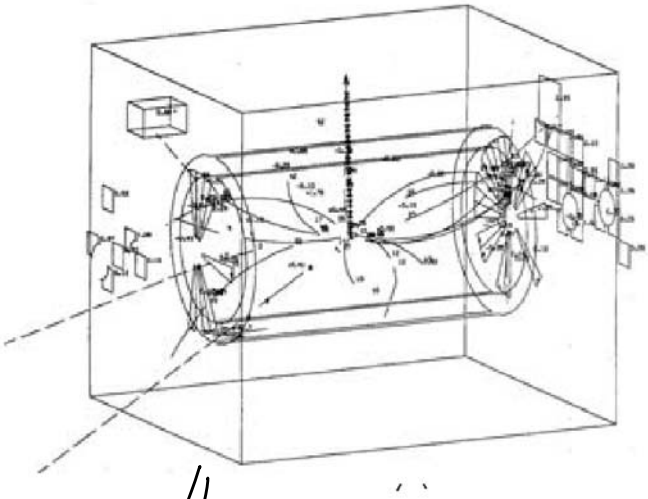
虽然W粒子主要衰变到夸克末态，即 $W \to q\bar{q}' \to jet + jet$，但强子对撞机上强子背景太大了，所以我们无法通过jet末态来探测W。历史上，人们首先通过W的轻子末态来确认W粒子存在。

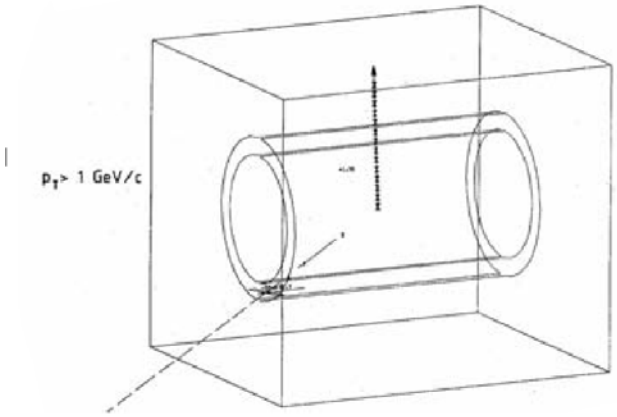




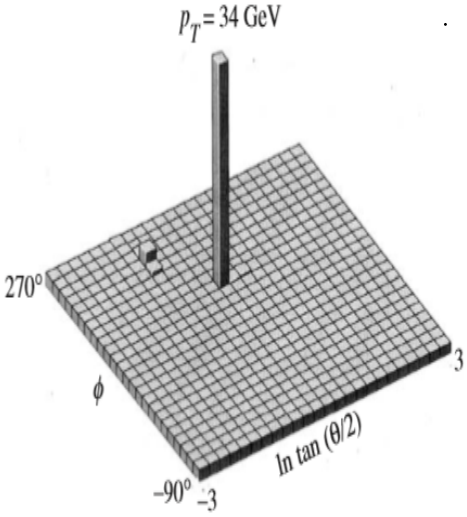
实验信号：一个大 P_T 的带电轻子和 $\not{p}_T$（来自中微子）

$P_T > 1\,GeV/c$



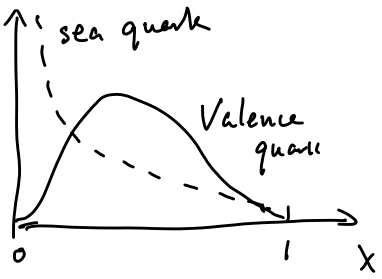


大量的 $P_T < 1GeV$ 的背景事例



*) 质子-反质子对撞机 (UA1)

部分子分布函数：



Note: (1)

	$p(uud)$	$\bar{p}(\bar{u}\bar{u}\bar{d})$
价夸克	u 和 d	$\bar{u}$, $\bar{d}$
海夸克	$\bar{u}, \bar{d}, c, \bar{c}, s, \bar{s}$ $b, \bar{b}, t, \bar{t}$	$u, d, c, \bar{c}, s, \bar{s}$ $b, \bar{b}, t, \bar{t}$

(2) $f_{\bar{d}/\bar{p}}(x_2) = f_{d/p}(x_2)$

(3) 从部分子分布函数可知，价夸克的x值比海夸克大，即价夸克的动量常常比海夸克动量大（统计意义）

$p(x_1)$	d	s	$\bar{u}$	$\bar{c}$
$\bar{p}(x_2)$	$\bar{u}$	$\bar{c}$	d	s

UA1 和 UA2 实验 @ CERN

$\sqrt{s} = 540\,\text{GeV}$

$\hat{s} = x_1 x_2 s = m_W^2$

$\Rightarrow \langle x \rangle_W \sim \frac{m_W}{\sqrt{s}} = 0.15$

$\Rightarrow$ 价夸克占主导地位

$$\sigma(p\bar{p} \to d\bar{u} \to W^- \to e^-\bar{\nu}_e)$$

$$= \int dx_1 dx_2 \Big\{ f_{d/p}(x_1) f_{\bar{u}/\bar{p}}(x_2) \hat{\sigma}(d(p_1)\bar{u}(p_2) \to W^- \to e^-\bar{\nu}_e)$$

主要贡献

$$+ f_{\bar{d}/p}(x_1) f_{u/\bar{p}}(x_2) \hat{\sigma}(d(p_2)\bar{u}(p_1) \to W^- \to e^-\bar{\nu}_e)$$

$$+ f_{s/p}(x_1) f_{\bar{c}/\bar{p}}(x_2) \hat{\sigma}(s(p_1)\bar{c}(p_2) \to W^- \to e^-\bar{\nu}_e)$$

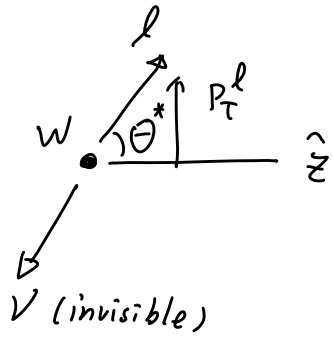
$$+ f_{\bar{c}/p}(x_1) f_{s/\bar{p}}(x_2) \hat{\sigma}(s(p_2)\bar{c}(p_1) \to W^- \to e^-\bar{\nu}_e) \Big\}$$

次要贡献

*) M_W 测量

a) Jacobian peak technique

在W静止系中



$$\frac{d\sigma}{dp_T^\ell} = \frac{d\sigma}{d\cos\theta} \times \frac{d\cos\theta}{dp_T^\ell}$$

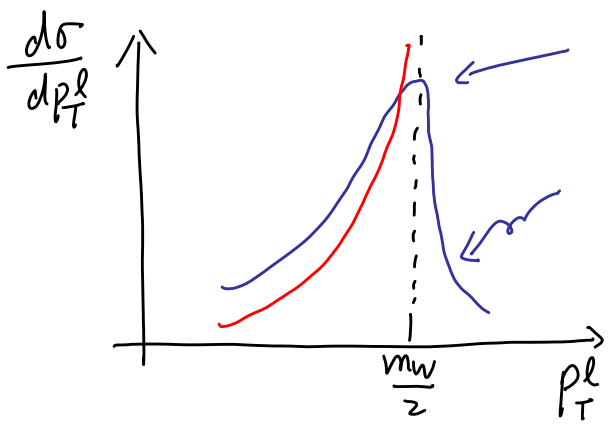
$$p_T^\ell = \left(\frac{M_W}{2}\right)\sin\theta \Rightarrow \frac{d\cos\theta}{dp_T^\ell} = \frac{1}{\frac{dp_T^\ell}{d\cos\theta}} = \frac{-2}{M_W}\frac{\sin\theta}{\cos\theta}$$

$$\left(dp_T = \frac{m_W}{2}\cos\theta\, d\theta = \frac{m_W}{2}\frac{\cos\theta}{\sin\theta}\sin\theta\, d\theta = -\frac{m_W}{2}\frac{\cos\theta}{\sin\theta}\, d\cos\theta\right)$$

$$\cos\theta = \sqrt{1-\left(\frac{2p_T^\ell}{m_W}\right)^2} = \frac{2}{m_W}\sqrt{\left(\frac{m_W}{2}\right)^2-\left(p_T^\ell\right)^2}$$

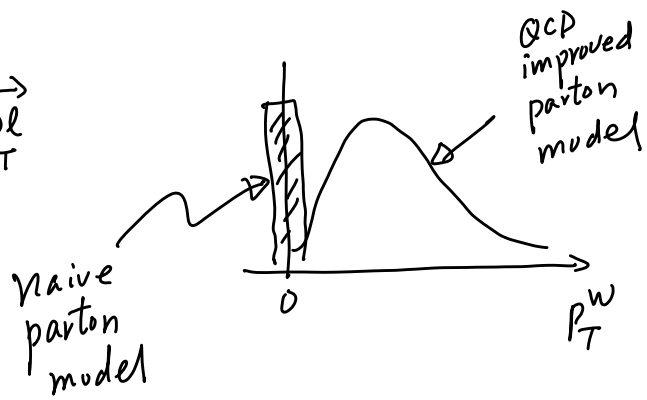
所以，
$$\frac{d\cos\theta}{dp_T} = \frac{2p_T^\ell}{m_W}\frac{1}{\sqrt{\left(\frac{m_W}{2}\right)^2-\left(p_T^\ell\right)^2}}$$
(Jacobian 变换因子)

$$\Rightarrow \frac{d\sigma}{dp_T^\ell} = \underbrace{\frac{d\sigma}{d\cos\theta}}_{\text{无发散行为}} \times \frac{2p_T^\ell}{m_W}\frac{1}{\sqrt{\left(\frac{m_W}{2}\right)^2-\left(p_T^\ell\right)^2}}$$



$\frac{d\sigma}{dp_T}$ peaks at $p_T^\ell = \frac{m_W}{2}$

Non-zero p_T^W for Γ_W spread the curve out



2) Transverse mass (M_T)

在强子对撞机上，我们无法探测中微子，也无法得到其四动量 P^μ.
从而传统的利用不变质量谱方法来测量共振态的方法不再适用。
但我们可以得到中微子的横向动量

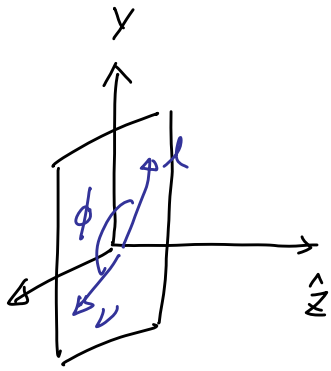
$$\vec{P}_T(W) = \vec{P}_T(\ell) + \vec{P}_T(\nu) \Rightarrow \vec{P}_T(\nu) = -\vec{P}_T(\ell)$$

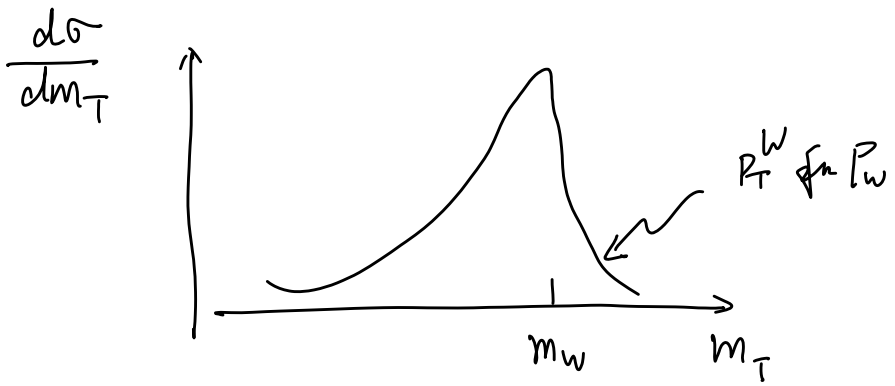
$\hookrightarrow 0$ in parton model

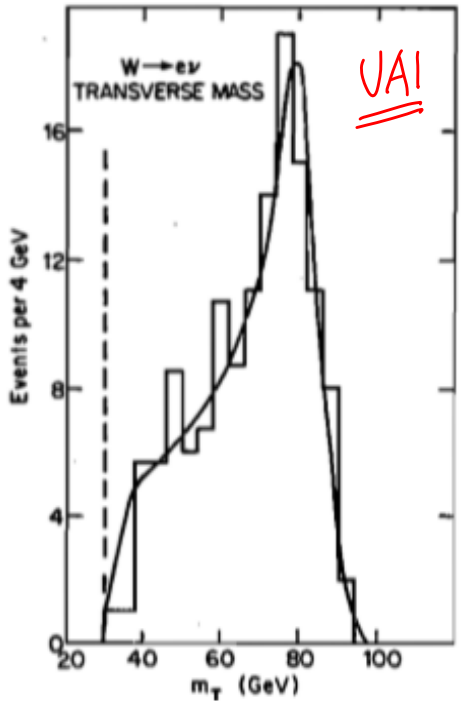
定义横向质量 (transverse mass) 如下

$$M_T^2 = 2\underbrace{P_T(\ell)}_{|\vec{P}_T(\ell)|}\underbrace{P_T(\nu)}_{\not{E}_T}\left[1-\cos\phi_{\ell\nu}\right]$$

separation angle in azimuthal angle







作业：使用 CALCHEP 重复 $\frac{d\sigma}{dM_T}$ 微分分布

$p\bar{p}$ 对撞 $E_p = E_{\bar{p}} = 270\,\text{GeV}$

$$\frac{d\hat{\sigma}}{dm_T^2} = \frac{1}{4\pi}\left(\frac{G_F m_W^2}{\sqrt{2}}\right)^2 \frac{1}{(s-m_W^2)^2+(m_W\Gamma_W)^2}\,\frac{2-\frac{m_T^2}{\hat{s}}}{\sqrt{1-\frac{m_T^2}{\hat{s}}}}$$

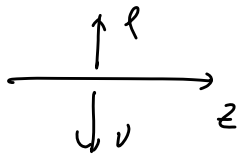
M_T的物理意义

定义M_T的动机如下:

如果 l 和 ν 都只有横向动量而无沿 beam 方向动量

$$\eta_l = \eta_\nu = 0, \quad P_z(l) = P_z(\nu) = 0$$

$$\Rightarrow P_T(l) = \not{E}_T = \frac{m_W}{2}$$



那么 $m_T = m_W$

一般情况下, 单粒子的运动学状态可表述如下

$$E = m_\perp \cosh y, \quad P_z = m_\perp \sinh y, \quad P_x, P_y$$

$$\cosh y = \frac{1}{2}(e^{-y} + e^{y}) \quad \sinh y = \frac{1}{2}(e^{y} - e^{-y})$$

$$m_\perp^2 = m^2 + P_x^2 + P_y^2 = E^2 - P_z^2 = m^2 + P_\perp^2$$

当 $m = 0$ 时 $m_\perp = P_\perp$

$$M_W^2 = (P_l + P_\nu)^2 = P_l^2 + P_\nu^2 + 2P_l \cdot P_\nu$$

$$= 2\left[E_l \cdot E_\nu - P_z(l)P_z(\nu) - P_x(l)P_x(\nu) - P_y(l)P_y(\nu)\right]$$

$$= 2P_T(l)P_T(\nu)\left[\cosh y_l \cosh y_\nu - \sinh y_l \sinh y_\nu - \cos\phi_{l\nu}\right]$$

$$= 2P_T(l)P_T(\nu)\left[\cosh(y_l - y_\nu) - \cos\phi_{l\nu}\right]$$

$$\geq 2P_T(l)P_T(\nu)\left[1 - \cos\phi_{l\nu}\right]$$

for $y_l = y_l$

- 对于每个事例($\vec{P}(l)$ 和 $\vec{\not{E}}_T$), 我们都可得到一个 m_T

  此 m_T 数值给出 m_W 的下限 ($m_T \leq m_W$)

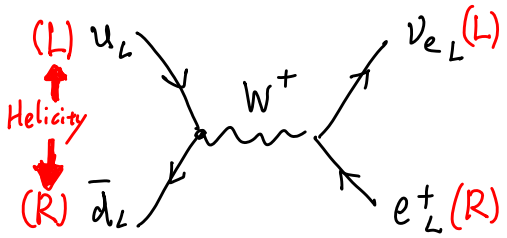
- $P_T(l) = \frac{m_W}{2}$ 时 $m_T = m_W \Rightarrow \frac{d\sigma}{dm_T}$ peaks at $m_T = m_W$。

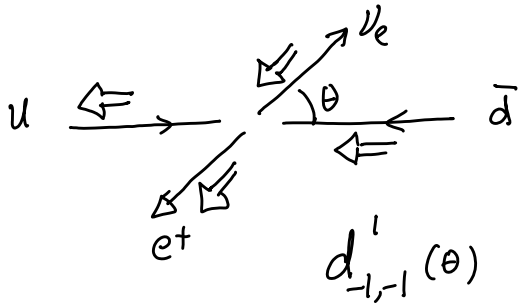
*) W自旋

W粒子的自旋将初态粒子和末态轻子关联起来(自旋关联效应)

$\Rightarrow$ 末态轻子的出射方向依赖于初态粒子

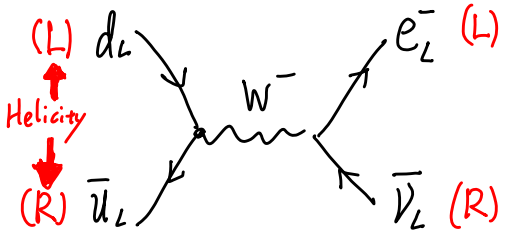
因为弱带电流是纯左手型相互作用顶点，所以初态和末态粒子都是左手型（正粒子的螺旋度=手征性，反粒子 Helicity = −手征性）

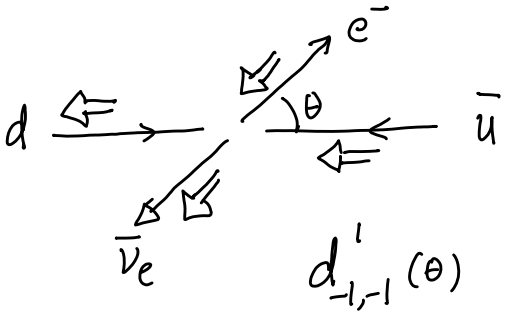




在忽略初末态夸克和轻子质量的情况下，散射振幅的角度依赖关系为

$$\mathcal{M} \sim 1+\cos\theta_\nu = 1-\cos\theta_{e^+} \quad (\theta_e = \pi-\theta_\nu)$$



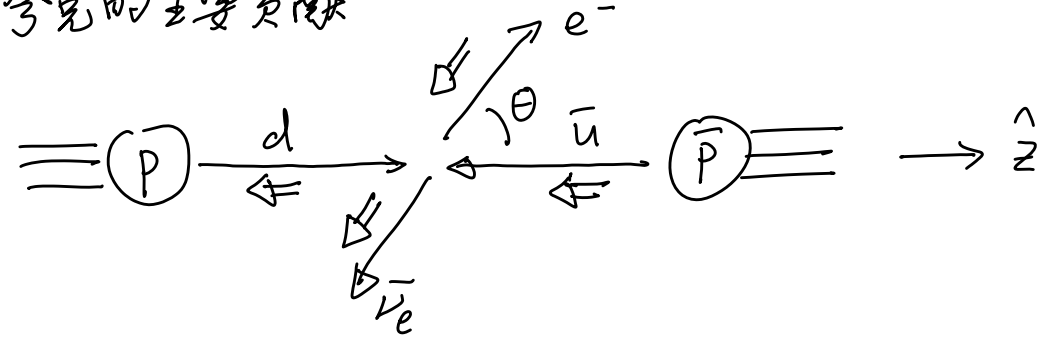


$$\mathcal{M} \sim 1+\cos\theta_{e^-}$$

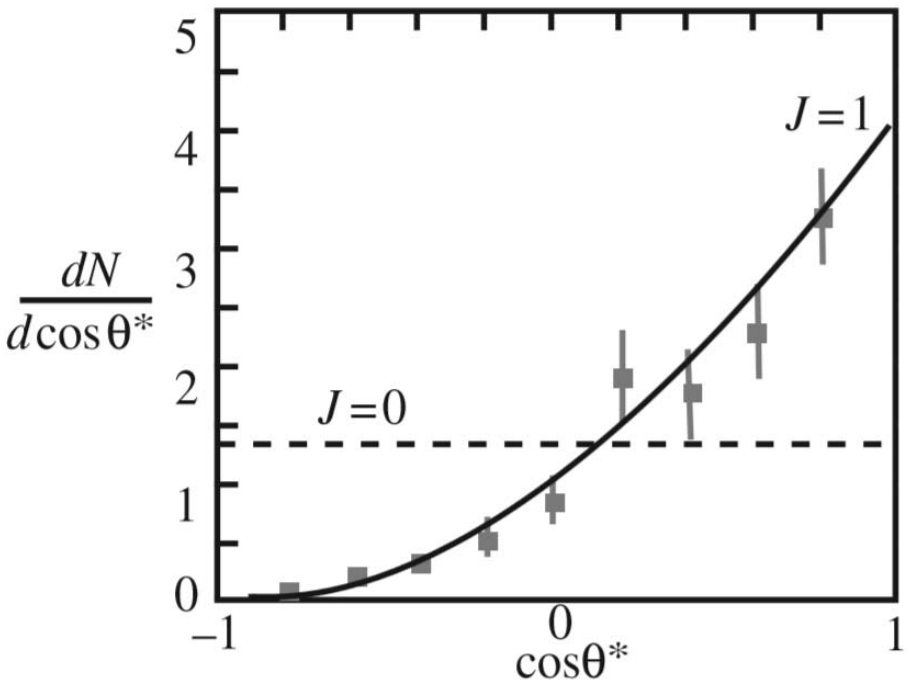
我们看到末态 e^+/e^- 相对于入射夸克有非平衡的角度依赖关系。

但如何在 $p\bar{p}$ 对撞机验证这个角度关系？

来自价夸克的主要贡献



选取质子的入射方向为 $+\hat{z}$ 方向，

$$\theta_{e^-d} \longrightarrow \theta_{e^-p}$$

验证W自旋为1，但无法区分(V-A)和(V+A)作用