Quantum Mechanics 115B

Postulates of Quantum Mechanics

1) state of a QM system is represented by a wavefunction $\psi(x,t)$ or a ket $|\psi\rangle$ (p. 1, 118)

2) observables are represented by Hermitian operators, A, that act on kets (p. 97)

3) the only possible result of a measurement is an eigenvalue of the operator (p. 99)

$$A|\psi_n\rangle = a_n|\psi_n\rangle$$

Postulates of Quantum Mechanics

4) the probability of measuring a_n is

$$\mathcal{P}(a_n) = |\langle\psi_n|\psi\rangle|^2$$ (p. 107)

5) after a measurement yielding a_n the new state is a normalized projection (p. 99, 123)

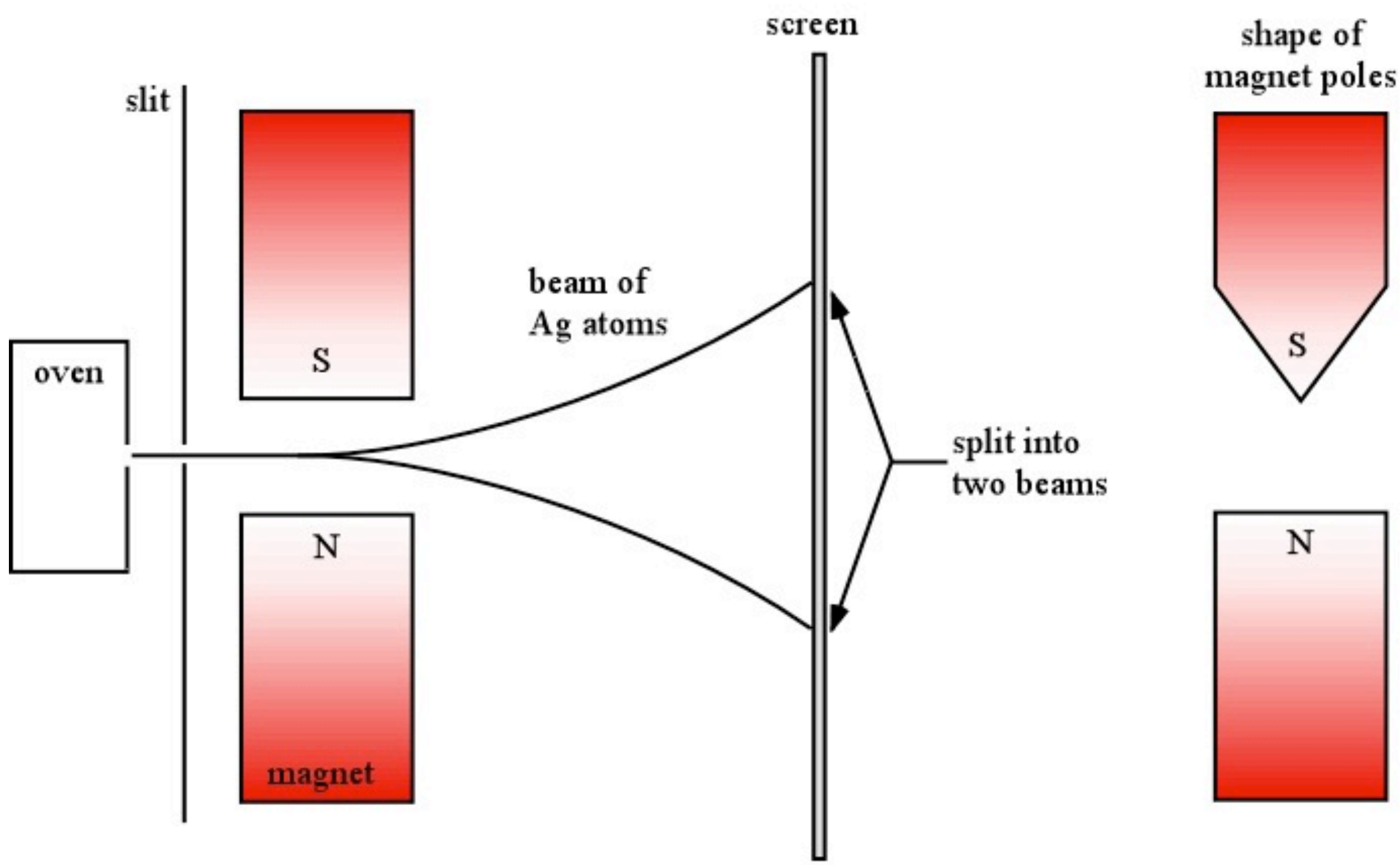
$$|\psi'\rangle = \frac{P_n|\psi\rangle}{\sqrt{\langle\psi|P_n|\psi\rangle}}$$

6) the time evolution of the state is given by the Schroedinger eq. (p. 1)

$$i\hbar\frac{d}{dt}|\psi(t)\rangle = H(t)|\psi(t)\rangle$$

Stern-Gerlach

neutral silver atoms



STERN-GERLACH EXPERIMENT

Stern-Gerlach

magnetic dipole moment

$$H = -\vec{\mu} \cdot \vec{B}$$

$$F_z = \frac{\partial}{\partial z}\left(\vec{\mu} \cdot \vec{B}\right) = \mu_z \frac{\partial B_z}{\partial z}$$

current loop

$$\mu = \frac{IA}{c}$$

$$\mu = \frac{qv\,\pi r^2}{c2\pi r} = \frac{qrv}{2c} = \frac{q}{2mc}L$$

Stern-Gerlach

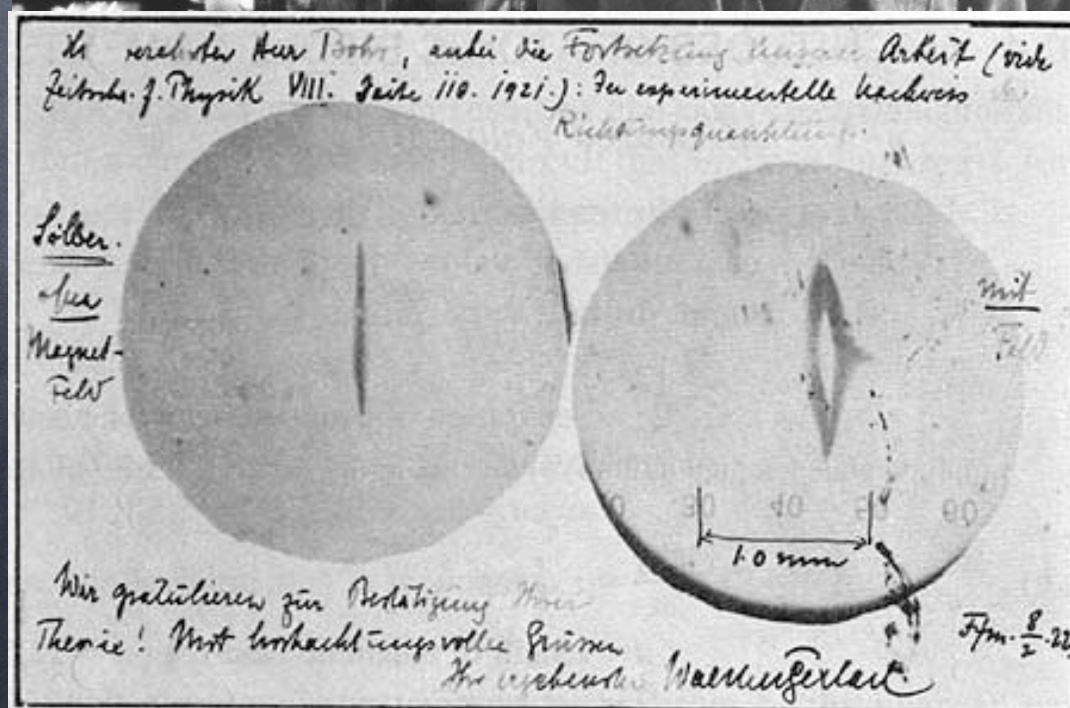
suppose electron has intrinsic angular momentum

$$\vec{\mu} = \gamma \vec{S}$$

$$F_z = \gamma S_z \frac{\partial B_z}{\partial z}$$

$$S_z = \pm \frac{\hbar}{2}$$

Stern-Gerlach



verehrter Herr Bohr, anbei die Fortsetzung Arbeit (vergl. Zeitschr. f. Physik VIII. Seite 110. 1921.): Den experimentellen Nachweis Richtungsquantelung.

Silber ohne Magnetfeld

mit Feld

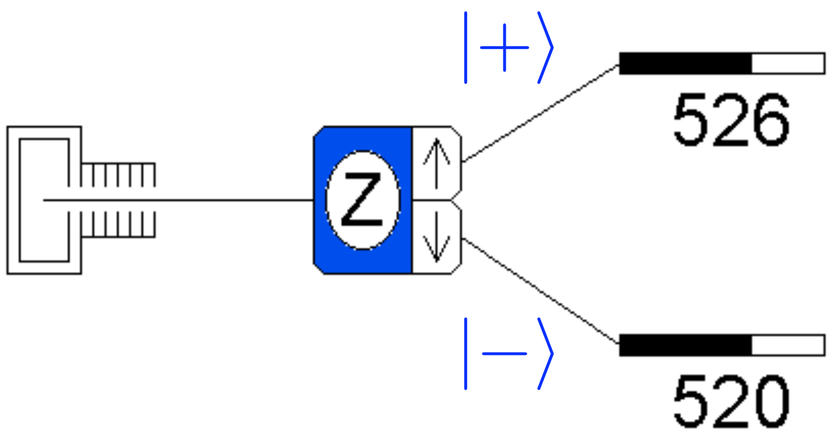
1.0 mm

Wir gratulieren zur Bestätigung Theorie! Mit hochachtungsvollem Gruss

Walther Gerlach

Ffm. 8/2.22

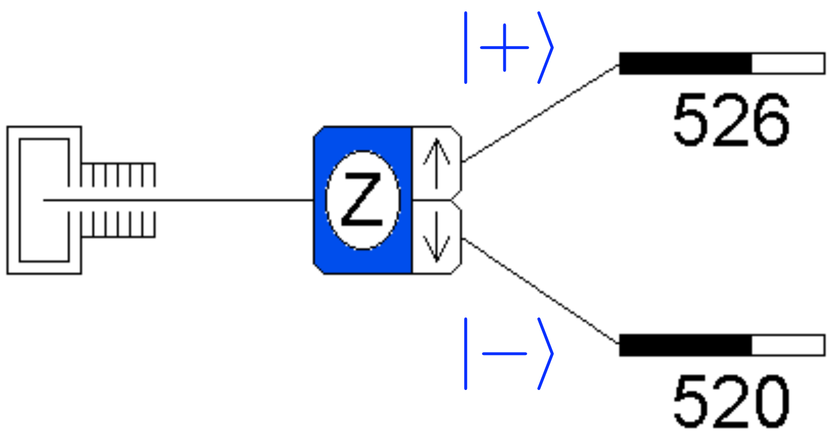
Stern-Gerlach



1) state of a QM system is represented by a wavefunction or a ket

$$|+\rangle, \ |\hbar/2\rangle, \ |S_z = \hbar/2\rangle, \ |+\hat{z}\rangle, \ |\uparrow\rangle$$

Stern-Gerlach



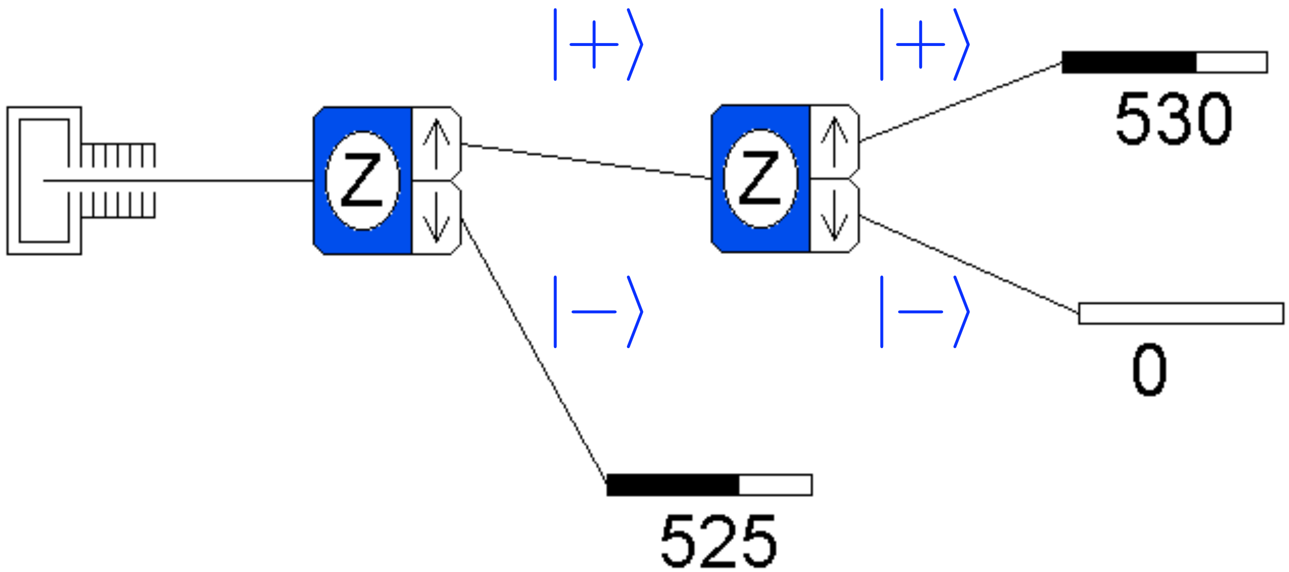
2) observables are Hermitian operators, they act on states

S_z

$|+\rangle$

$|-\rangle$

Stern-Gerlach



3) the only possible result of a measurement is an eigenvalue of the operator

$$
\begin{aligned}
S_z|+\rangle &= +\frac{\hbar}{2}|+\rangle \\
S_z|-\rangle &= -\frac{\hbar}{2}|-\rangle
\end{aligned}
$$

Eigenbasis: Normalization, Orthogonality, Completeness

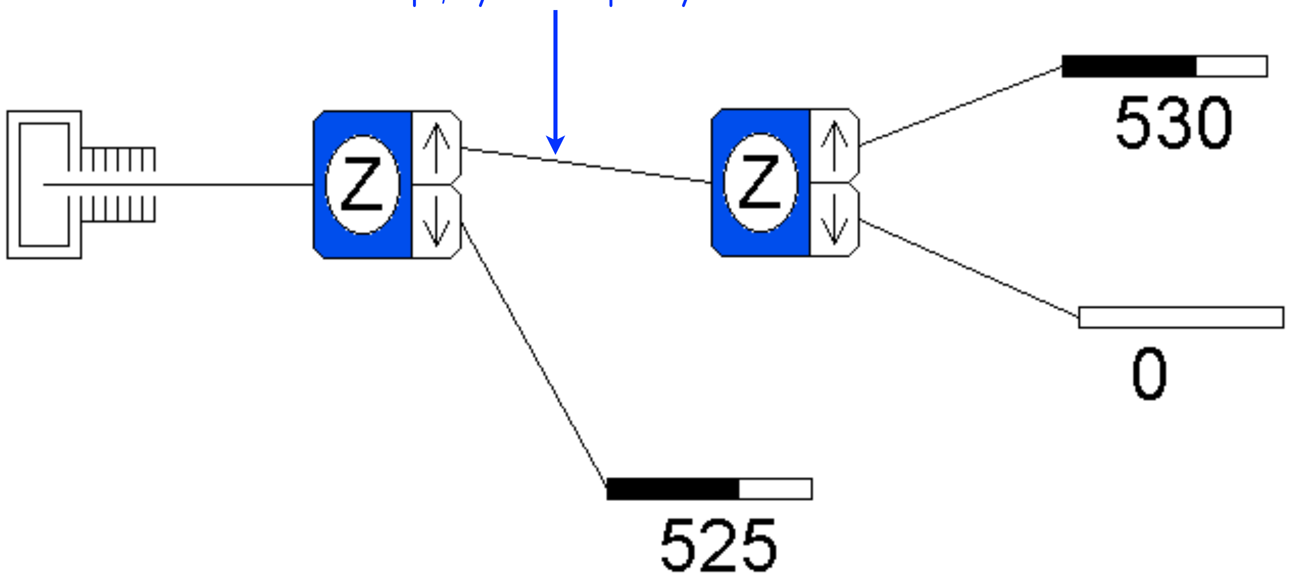
$$
\begin{aligned}
\langle +|+\rangle &= 1 \\
\langle -|-\rangle &= 1 \\
\langle +|-\rangle &= 0 \\
\langle -|+\rangle &= 0
\end{aligned}
$$

$$
\begin{aligned}
|\psi\rangle &= a|+\rangle + b|-\rangle \\
\langle\psi| &= a^*\langle +| + b^*\langle -|
\end{aligned}
$$

$$
\begin{aligned}
\langle\psi|\psi\rangle &= (a^*\langle +| + b^*\langle -|)(a|+\rangle + b|-\rangle) \\
&= |a|^2 + |b|^2 = 1
\end{aligned}
$$

Stern-Gerlach

$$|\psi\rangle = |+\rangle$$

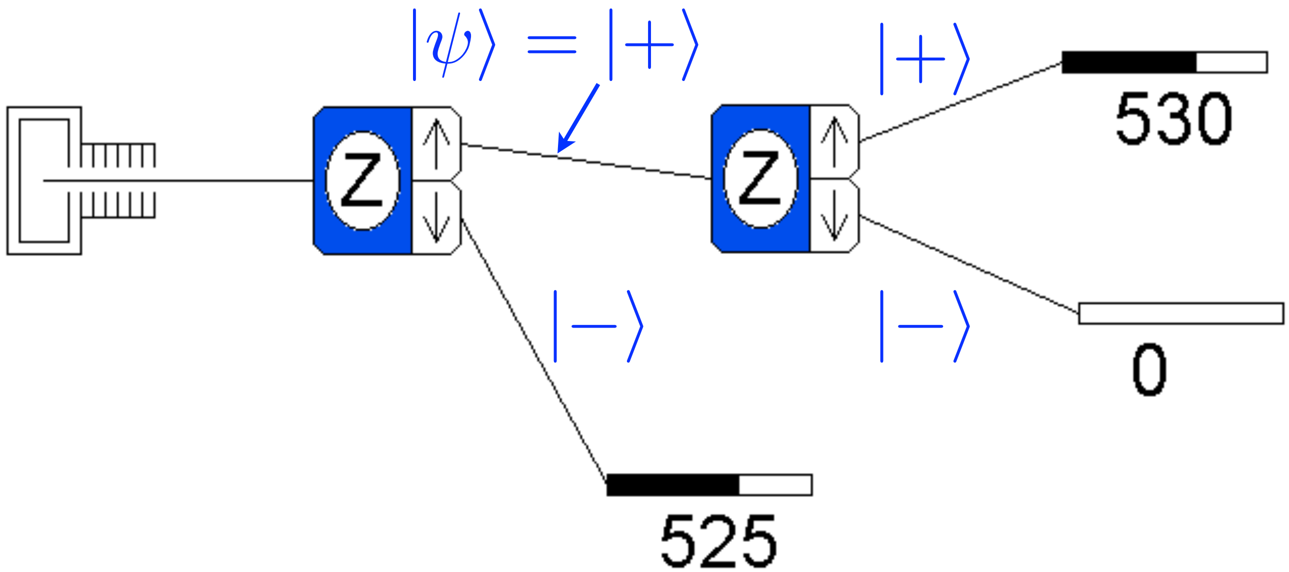


4) the probability of measuring + or – is

$$|\langle + | \psi \rangle|^2$$

$$|\langle - | \psi \rangle|^2$$

Stern-Gerlach



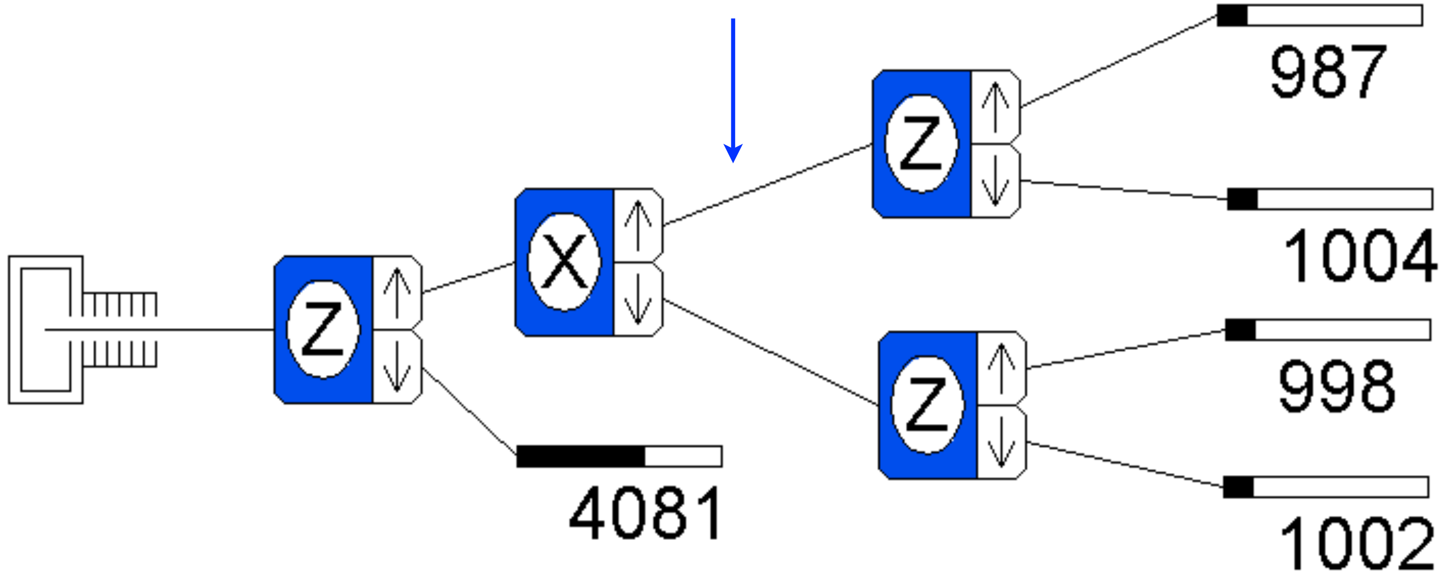
5) after a measurement yielding + the new state is a normalized projection

$$P_+(a|+\rangle + b|-\rangle) = a|+ >$$

$$|\psi'\rangle = \frac{P_+|\psi\rangle}{\sqrt{\langle\psi|P_+|\psi\rangle}} = \frac{a|+\rangle}{\sqrt{\langle\psi|a|+\rangle}} = |+\rangle$$

Stern-Gerlach

$$|+\rangle_x \quad = \quad a|+\rangle + b|-\rangle$$



Analysis

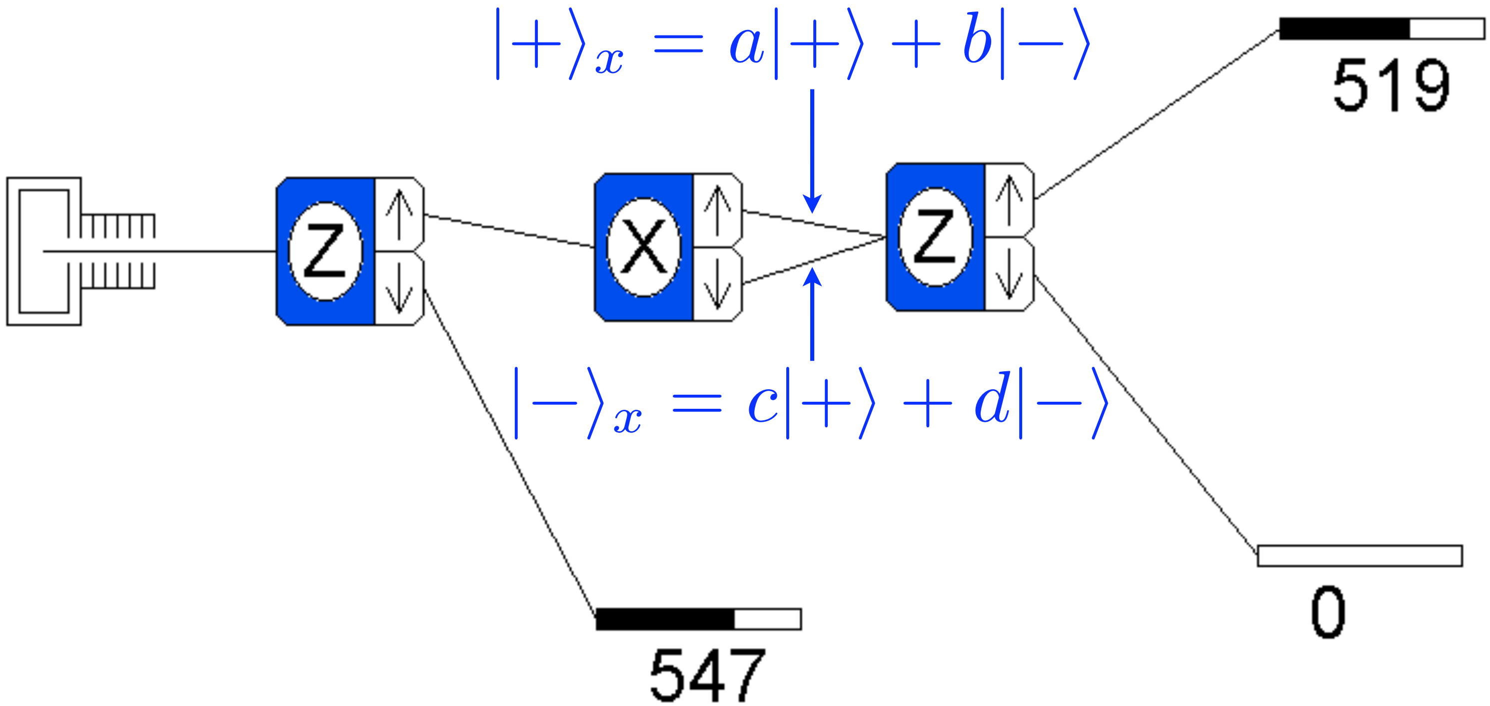
$$\begin{aligned}
\mathcal{P}_1(+) &= |{}_x\langle +|+\rangle|^2 = 1/2 \\
\mathcal{P}_1(-) &= |{}_x\langle -|+\rangle|^2 = 1/2 \\
\mathcal{P}_2(+) &= |{}_x\langle +|-\rangle|^2 = 1/2 \\
\mathcal{P}_2(-) &= |{}_x\langle -|-\rangle|^2 = 1/2
\end{aligned}$$

$$\begin{aligned}
|+\rangle_x &= a|+\rangle + b|-\rangle \\
|-\rangle_x &= c|+\rangle + d|-\rangle
\end{aligned}$$

$$\begin{aligned}
|{}_x\langle +|+\rangle|^2 &= |(a^*\langle +| + b^*\langle -|)\,|+\rangle|^2 \\
&= |a|^2 = 1/2
\end{aligned}$$

$$|a| = |b| = |c| = |d| = \frac{1}{\sqrt{2}}$$

Stern-Gerlach



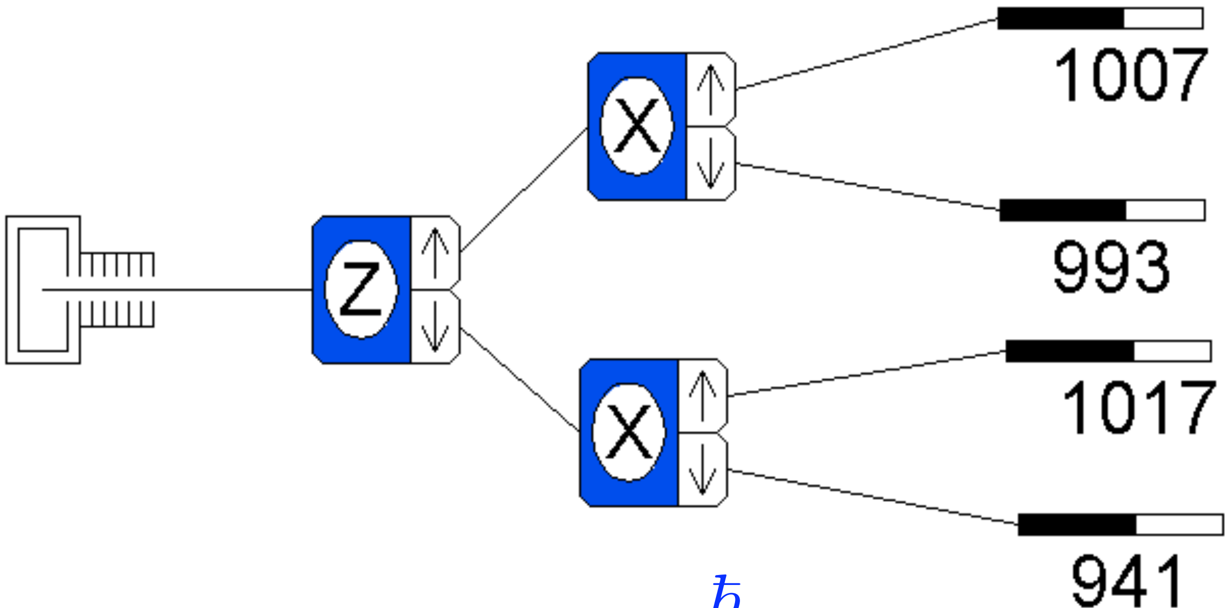
$b = -d$

destructive interference

Superposition

$$|+\rangle_x = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)$$

not the same as a mixture



$$S_x|+\rangle_x = +\frac{\hbar}{2}|+\rangle_x$$

always measure + for this state

Matrix Notation

$$|+\rangle \longrightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|-\rangle \longrightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|\psi\rangle \longrightarrow \begin{pmatrix} \langle +|\psi\rangle \\ \langle -|\psi\rangle \end{pmatrix}$$

$$|\psi\rangle = a|+\rangle + b|-\rangle \longrightarrow \begin{pmatrix} a \\ b \end{pmatrix}$$

Matrix Notation

$$|\psi\rangle = a|+\rangle + b|-\rangle \longrightarrow \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\langle\psi|\psi\rangle \longrightarrow (a^* \;\; b^*) \begin{pmatrix} a \\ b \end{pmatrix} = |a|^2 + |b|^2$$

$$S_z \longrightarrow \begin{pmatrix} \hbar/2 & 0 \\ 0 & -\hbar/2 \end{pmatrix}$$

$$\langle\psi|S_z|\psi\rangle \longrightarrow \quad (a^* \;\; b^*) \begin{pmatrix} \hbar/2 & 0 \\ 0 & -\hbar/2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

$$= |a|^2 \, (\hbar/2) + |b|^2 \, (-\hbar/2)$$

Pauli Matrix Notation

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S_x = \frac{\hbar}{2}\sigma^1, \; S_y = \frac{\hbar}{2}\sigma^2, \; S_z = \frac{\hbar}{2}\sigma^3,$$

$$S_x \left|+\right\rangle_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{\hbar}{2}\left|+\right\rangle_x$$

$$S_x \left|-\right\rangle_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = -\frac{\hbar}{2}\left|+\right\rangle_x$$

Matrix Notation

$$A_{ij} = \langle \psi_i | A | \psi_j \rangle$$

$$A \rightarrow \begin{pmatrix} A_{11} & A_{12} & A_{13} & \cdots \\ A_{21} & A_{22} & A_{23} & \cdots \\ A_{31} & A_{32} & A_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Matrix Projection

$$P_+ \longrightarrow \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$P_- \longrightarrow \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$P_+ P_+ = P_+$$

$$|\psi'\rangle = \frac{P_+|\psi\rangle}{\sqrt{\langle\psi|P_+|\psi\rangle}} = |+\rangle$$

Time Evolution

$$H = \omega\, S_z = \frac{\omega\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

6) the time evolution of the state is given by the Schroedinger eq.

$$i\hbar\frac{\partial}{\partial t}\psi(t) = H\psi(t)$$

$$\psi(0) = \begin{pmatrix} \cos(\alpha/2) \\ \sin(\alpha/2) \end{pmatrix}$$

$$\psi(t) = \begin{pmatrix} \cos(\alpha/2)e^{-i\omega t/2} \\ \sin(\alpha/2)e^{+i\omega t/2} \end{pmatrix}$$

$$\langle\psi|S_z|\psi\rangle = \cos\left(\frac{\alpha}{2}\right)^2\frac{\hbar}{2} - \sin\left(\frac{\alpha}{2}\right)^2\frac{\hbar}{2} = \frac{\hbar}{2}\cos(\alpha)$$