C T E Q

Introduction to Collider Physics

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C T E Q

Due to the limit of time, I can only talk about an example:

QCD and
Global Analysis of
Parton Distribution Functions

I will also briefly comment on some aspects of collider phenomenology related to electroweak interactions.

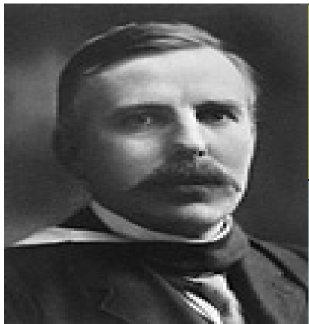
The Goal of this series of lectures

In this series of lectures, I would like to convey the following ideas.
(1) How to construct theoretical model to explain experimental data.
Example: From SLAC-MIT experimental data to
the birth of naive parton model.
(2) How to complete a consistent model with symmetry principles.
Example: Construct QCD theory (from an SU(3) non-abelian
local gauge symmetry) and the QCD improved parton model.
(3) How to test a theoretical model against experimental data.
Example: Check predictions of QCD improved parton model
via global analysis of Deep-inelastic scattering (DSI), Drell-Yan pair,
and jet data at various lepton-hadron and hadron-hadron colliders.
(4) I will also extend the above methodology to discussing the
phenomenology of electroweak interactions.

Contents

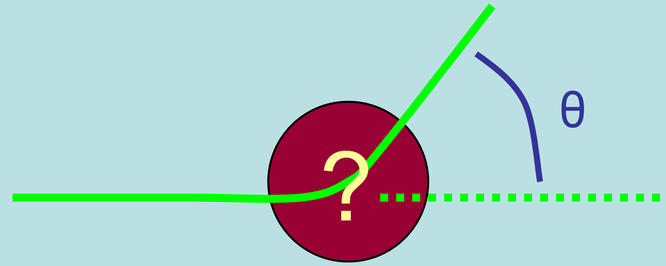
- QCD and its success
- A NLO calculation of pQCD
- Collider phenomenology and Global analysis of PDFs

QCD and Its Success



Rutherford Scattering

Rutherford taught us the most important lesson:
use a scattering process to learn about the structure of matter



This story is well known:

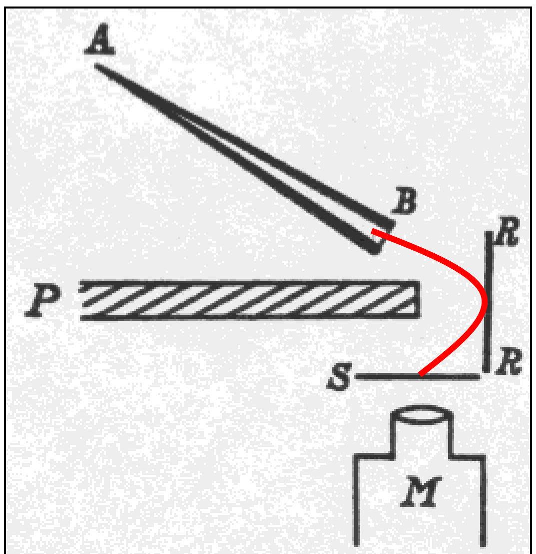
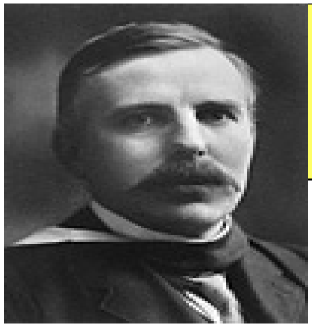


FIG. 1.

H. Geiger and E. Marsden observed that α-particles were sometimes scattered through very large angles.

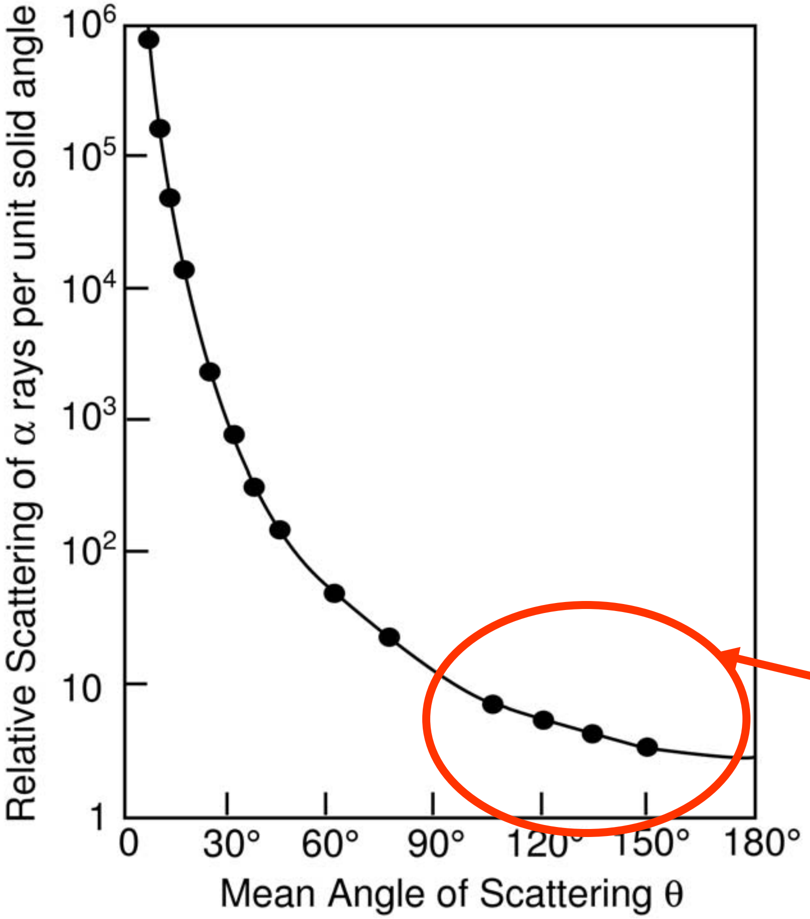
Rutherford interpreted these results as due to the coulomb scattering of the α-particles with the atomic nucleus:

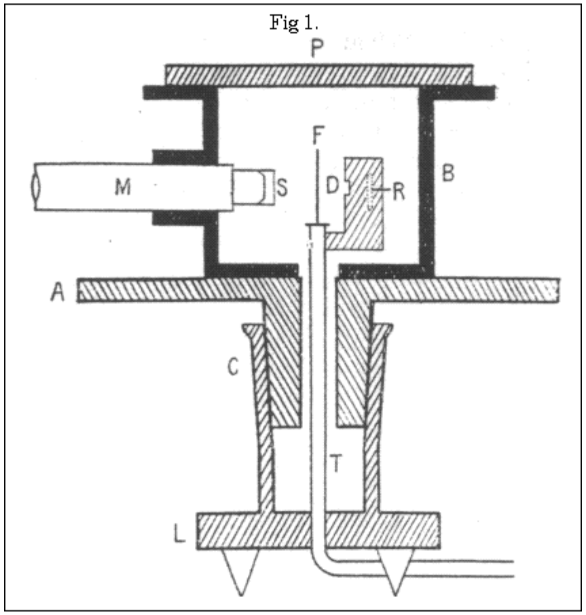
$$\sigma(\theta) = \frac{z^2 Z^2 e^4}{16 E^2} \frac{1}{\sin^4 \frac{1}{2}\theta}$$



Rutherford Scattering

In a subsequent paper Geiger/Marsden precisely verified Rutherford theory





Discovery of atomic nucleus

→ N. Bohr Old Quantum theory…

Developments…

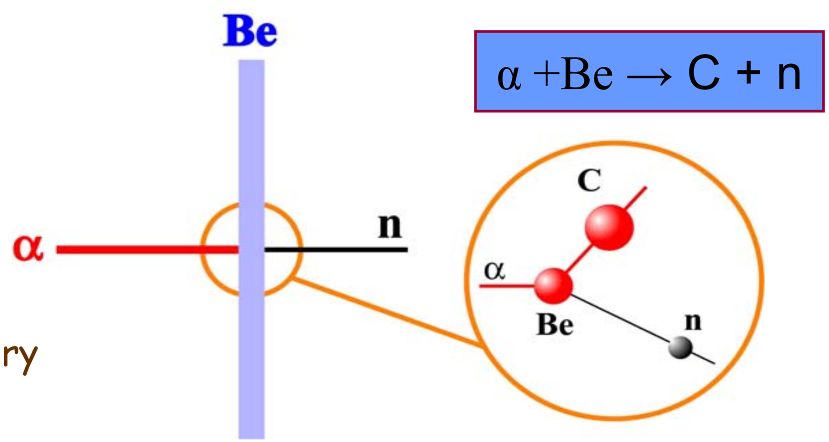
-Quantum mechanics rapidly developed in the years 1924-27

-The nucleus composition remained a mistery (e.g. N_7^{14}) till…

Discovery of neutron

(Chadwick 1932)

Instrumental to the Fermi's beta decay (n→pev) theory



Main information concerning geometric detailes of nuclear structure (mirror nuclei, fast neutron capture, binding energies etc) could be summed up in:

$R = r_0 \times A^{1/3}$ fm with $r_0 = 1.45$ fm

$\rho_m = 0.08$ nucl/fm^3 and $\rho_c = (Z/A) \times 0.08$ (prot. charges)/fm^3

The nucleus form factor

Stimulated by accelerators technology advances and fully muture QED various theoreticians (Rose (48), Elton(50)) started to calculate cross sections for elastic electron-Nucleus scattering

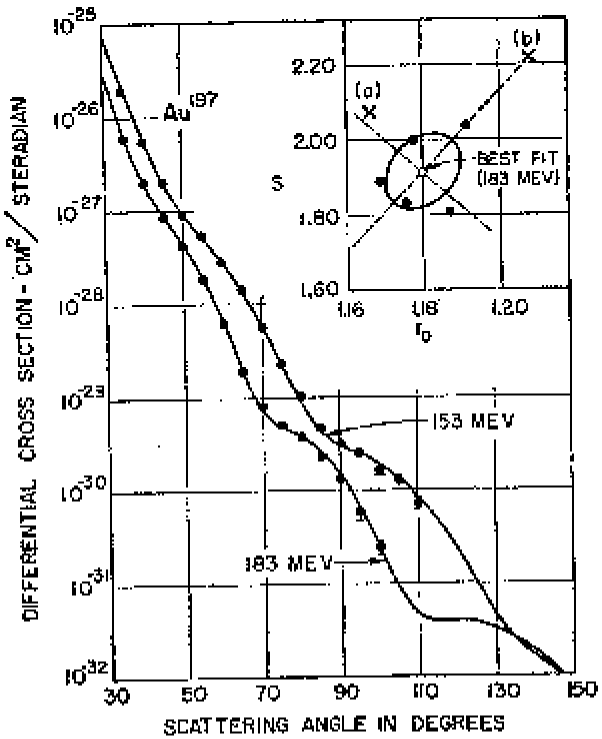
Interference between the scattered wavelets arising from the different parts of the same,finite,nucleus

$$\sigma_M(\theta) = \left(\frac{Ze^2}{2E}\right)^2 \frac{\cos^2\frac{1}{2}\theta}{\sin^4\frac{1}{2}\theta}$$ Mott

$$\sigma_s(\theta) = \left(\frac{Ze^2}{2E}\right)^2 \frac{\cos^2\frac{1}{2}\theta}{\sin^4\frac{1}{2}\theta} \left| \int_{\substack{\text{nuclear}\\ \text{volume}}} \rho(r) e^{i\mathbf{q}\cdot\mathbf{r}} d\tau \right|^2$$

$$\sigma_s(\theta) = \left(\frac{Ze^2}{2E}\right)^2 \frac{\cos^2\frac{1}{2}\theta}{\sin^4\frac{1}{2}\theta} \left[\int_0^\infty \rho(r) \frac{\sin qr}{qr} 4\pi r^2 dr \right]^2.$$

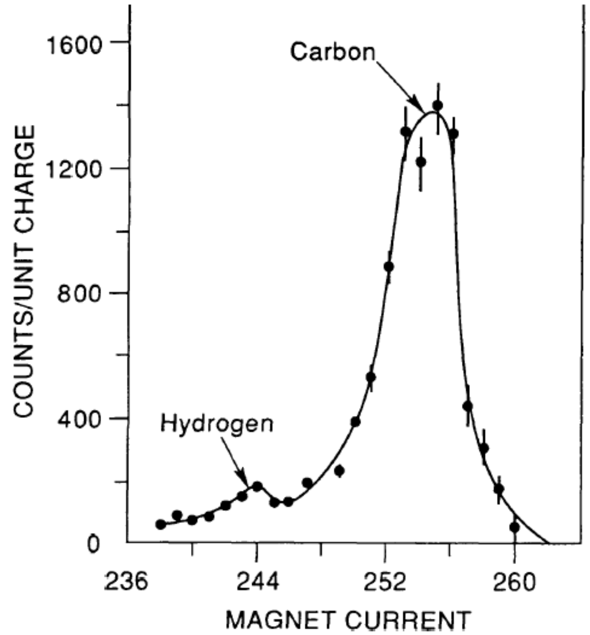
$$F = \frac{4\pi}{q} \int_0^\infty \rho(r) \sin(qr) r dr$$ Nucleus form factor

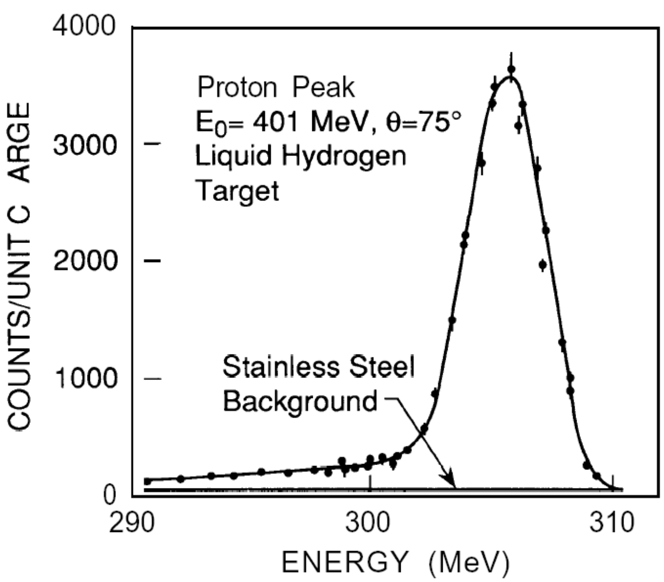


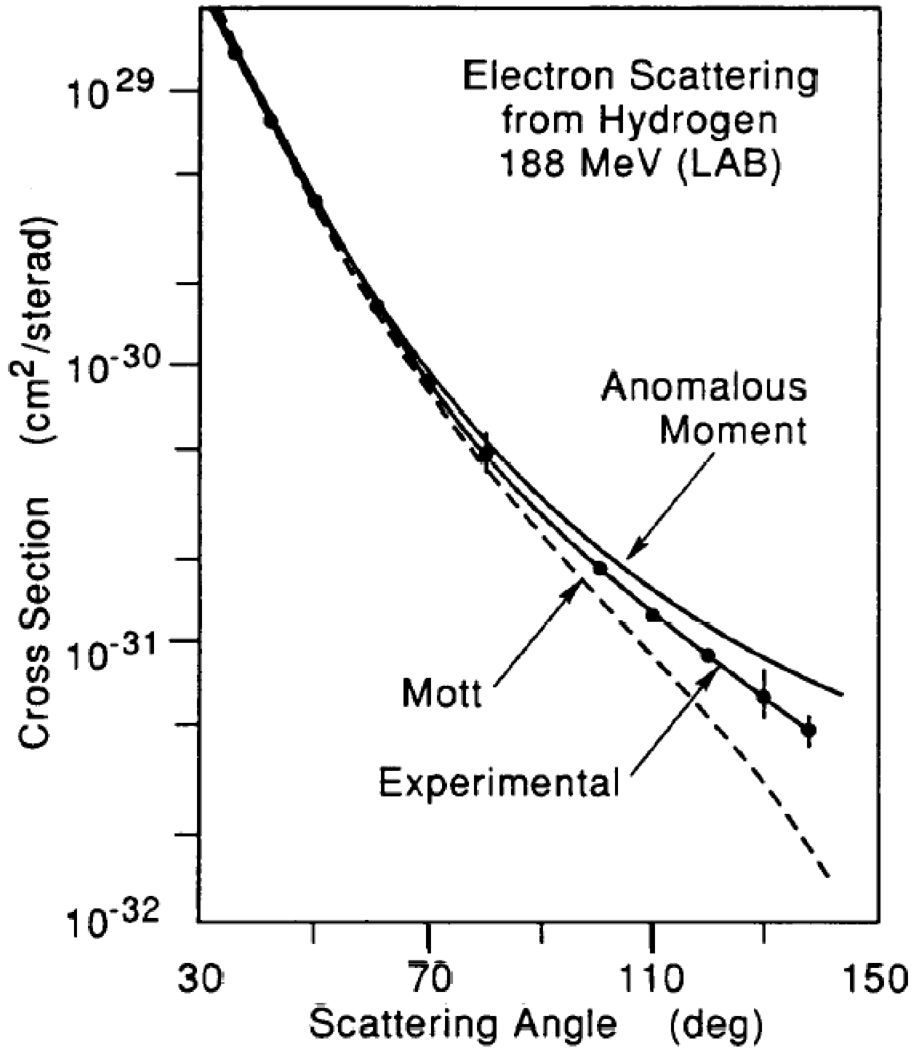
Phase-shift analysis

R. Hofstadter: e-p elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_0^2 \sin^4 \theta/2} \cdot \cos^2 \theta/2 \cdot \frac{E'}{E_0} \left[\frac{G_E^2 + \tau G_M^2}{1 + \tau} + 2\tau G_M^2 \tan^2 \theta/2 \right]$$







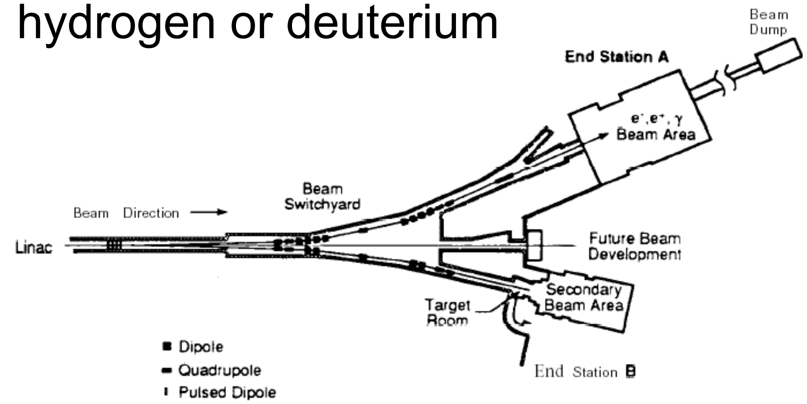
The SLAC-MIT Experiment

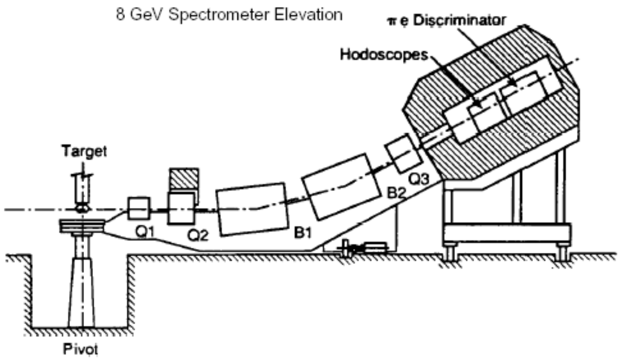
1990 Nobel Prize

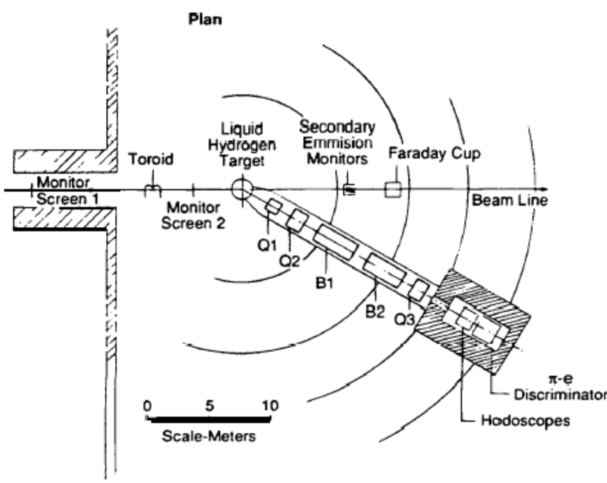
Under the leadership of Taylor, Friedman, Kendall

~ 1969

Electron scattered by
liquid hydrogen or deuterium



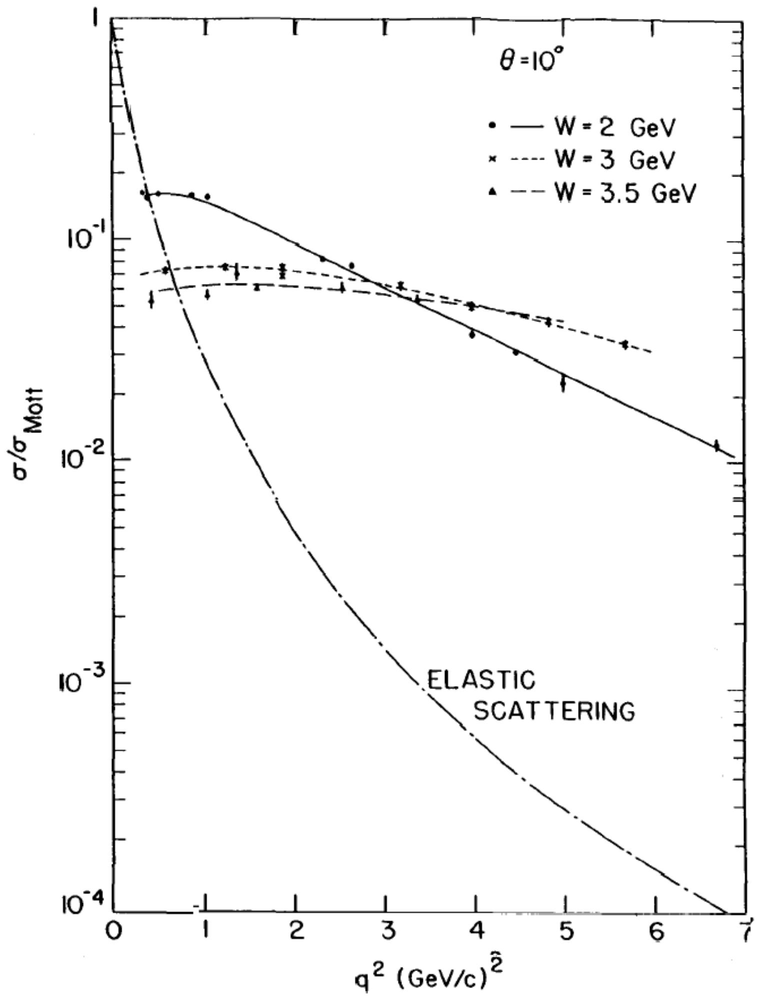


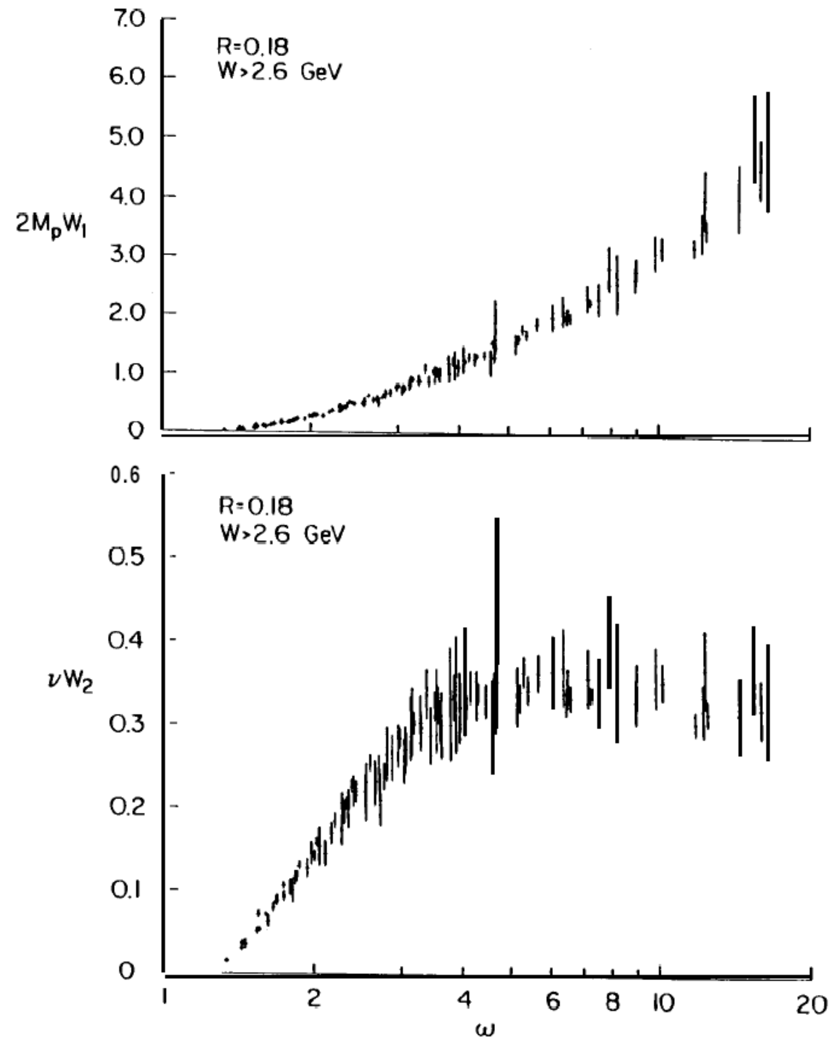


First SLAC-MIT results

Two unexpected results…

Deep-inelastic scattering (DIS)





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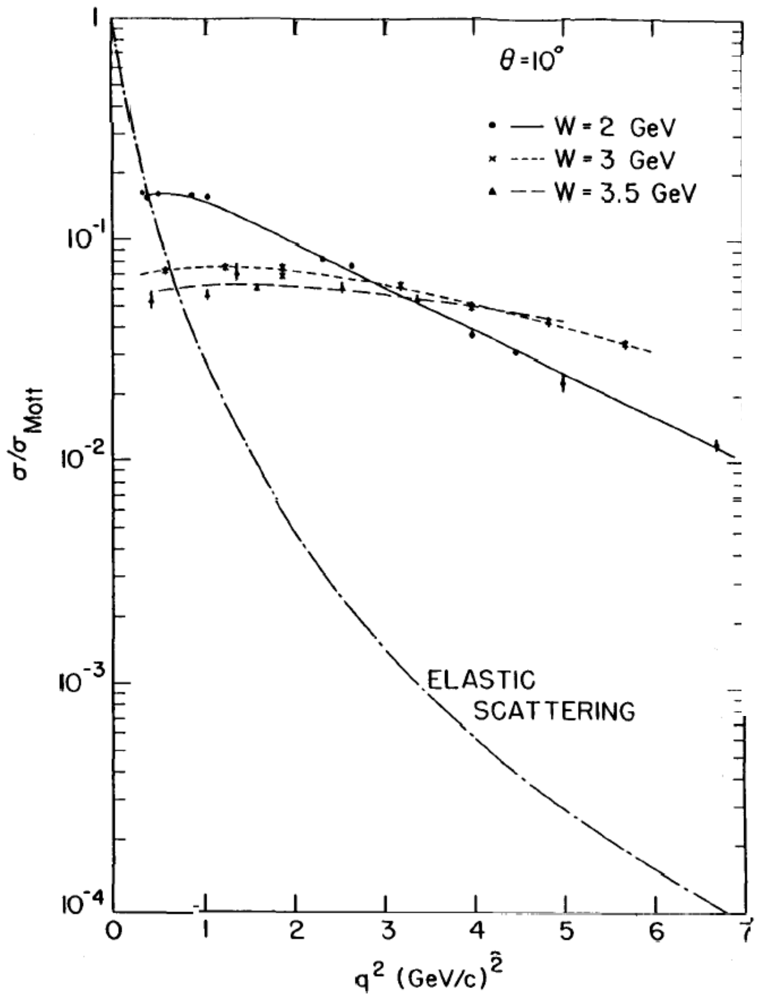
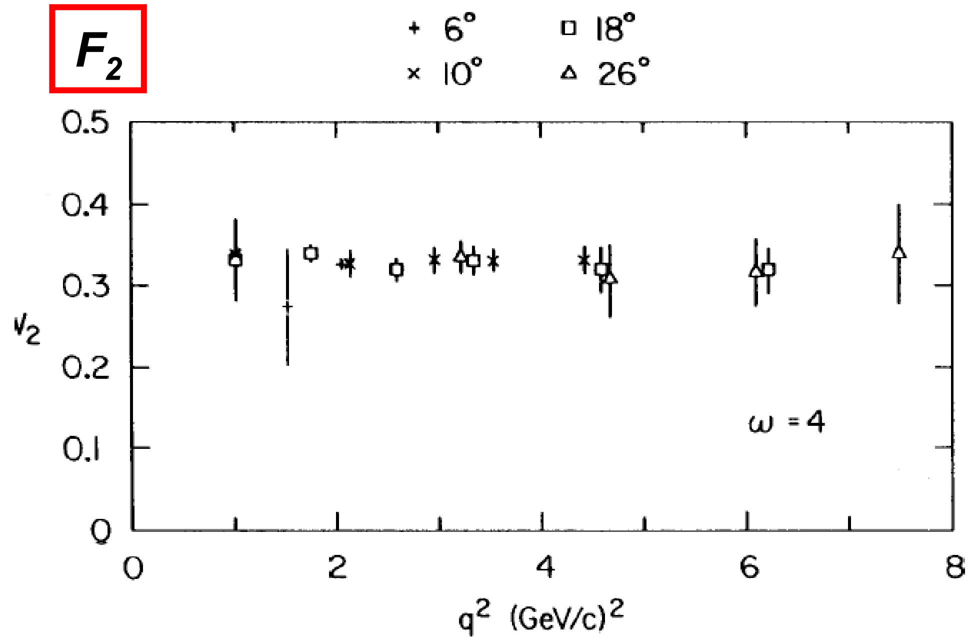
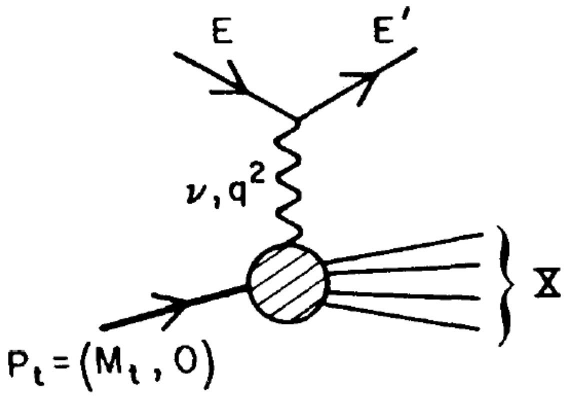


Scaling behavior



$\omega = 1/x$

Deep inelastic scattering (DIS) and structure Fs



Kinematic variables

$$\nu = \frac{P \cdot q}{\sqrt{P \cdot P}} = E_1 - E_2$$

$$x = \frac{-q^2}{2P \cdot q} = \frac{Q^2}{2M\nu}$$

$$Q^2 = 2E_1E_2(1 - \cos\theta)$$

$$\frac{d^2\sigma}{d\Omega\, dE'}(E, E', \theta) = \sigma_M \left[W_2(\nu, q^2) + 2W_1(\nu, q^2)\tan^2(\theta/2)\right]$$

$$2MW_1(\nu, q^2) = F_1(\omega)$$
$$\nu W_2(\nu, q^2) = F_2(\omega)$$

Bjorken scaling (1969)
(Predicted prior to data)

$\omega = 1/x$

Quantum Chromodynamics

Fields: Quarks ψ^{color}_{flavor} and Gluon $G^{color}(A{\cdot}T, g)$.

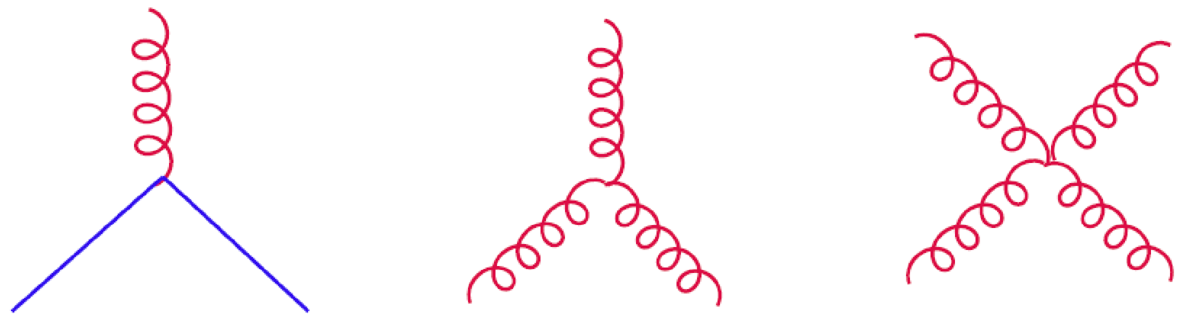
Basic Lagrangian:

$$\mathcal{L} = \bar{\psi}(i \not\partial - g \not A \cdot t - m)\psi - \frac{1}{4} G(A \cdot T, g) \cdot G(A \cdot T, g)$$

- g: gauge Coupling Strength
- m_i: quark masses
- t & T: color SU(3) matrices in the fundamental and adjoint representations.

  Group factors: $C_F = \frac{4}{3}$; $T_F = \frac{1}{2}$; $C_A = 3$

Interaction Vertices:



Why does QCD play such a crucial role in High Energy Phenomenology?

- The parton picture language provides the foundation on which all modern particle theories are formulated, and all experimental results are interpreted.
- The validity of the parton picture is based empirically on an overwhelming amount of experimental evidence collected in the last 30-40 years, and theoretically on the Factorization Theorems of PQCD.

How could the *simple* (almost non-interacting) *parton picture* possibly hold in QCD — a strongly interacting quantum gauge field theory?

Answer: 3 unique features of QCD:

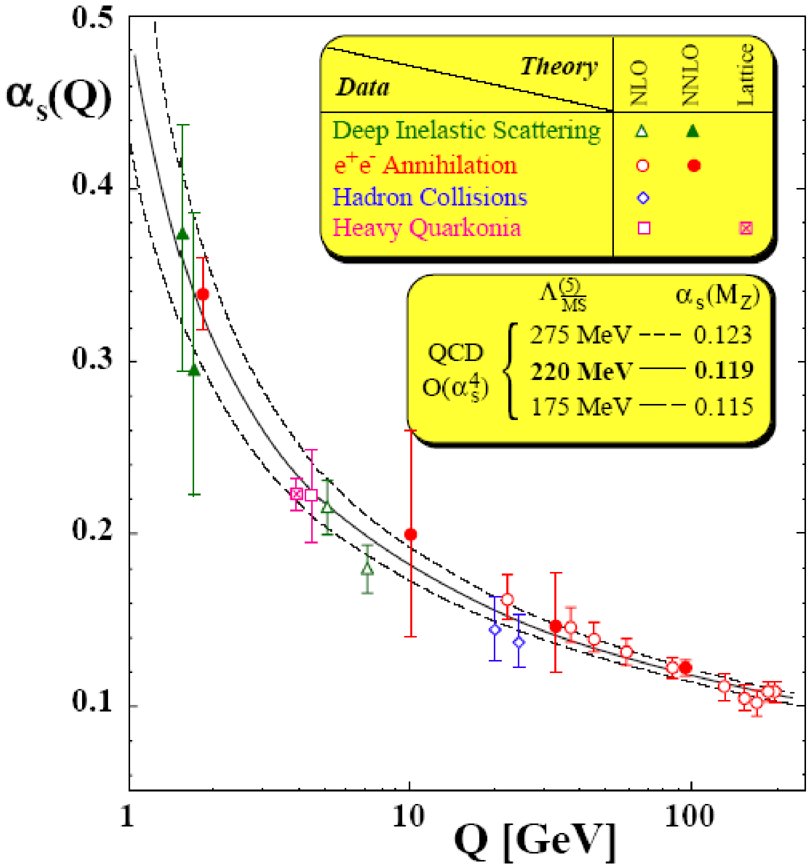
1. Asymptotic Freedom:
   A strongly interacting theory at long-distance can become weakly interacting at short-distance.
2. Infra-red Safety:
   There are classes of *infra-red safe* quantities which are independent of long-distance physics, hence are calculable in PQCD.
3. Factorization:
   There are an even wider class of physical quantities which can be *factorized* into a long-distance piece (not calculable, but *universal*) and short-distance piece (process-dependent, but infra-red safe, hence *calculable*).

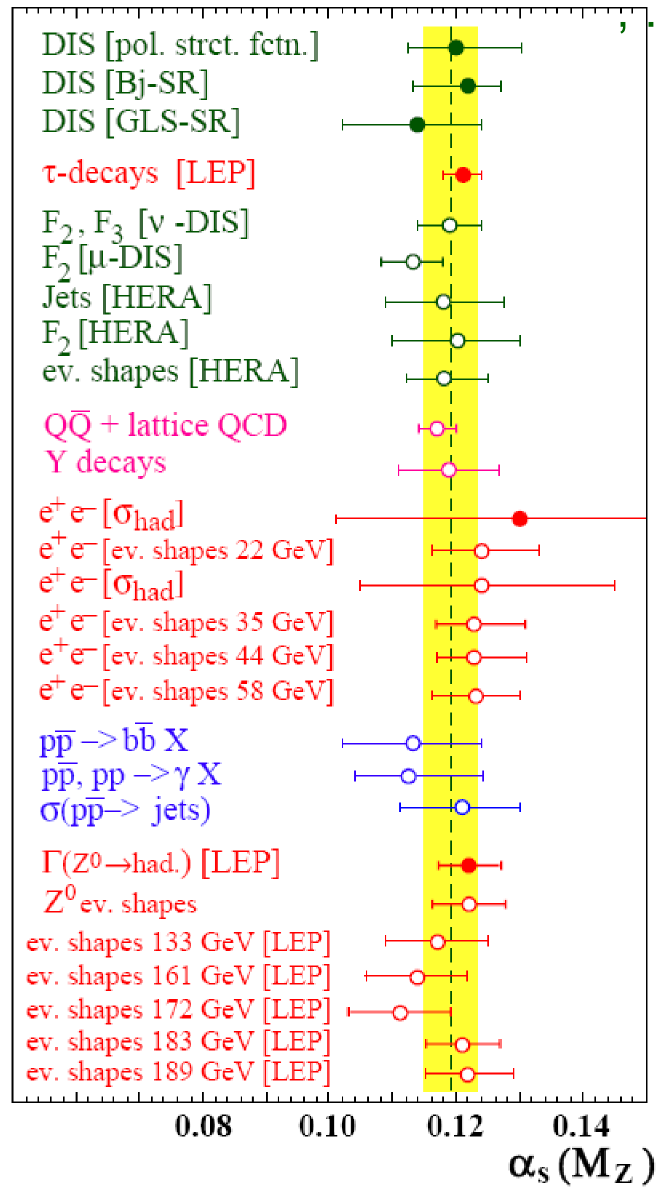
Key concepts: Ultra-violet divergences, bare Green fns, renormalization,
RGE, anomalous dimensions,
renormalized G.Fs
; … etc.

Asymptotic Freedom

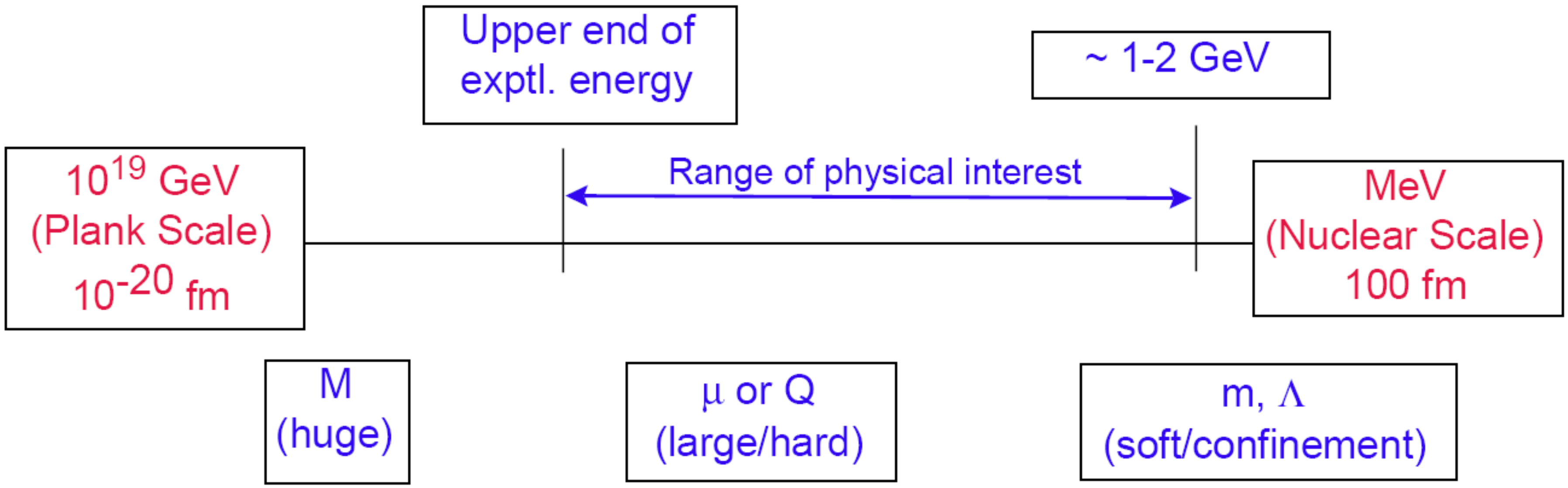
Universal (running) coupling:

$$\alpha_s(Q) = \frac{g^2}{4\pi} = \frac{b}{\ln(Q/\Lambda)}(1 + \ldots)$$





The importance of ***Scales*** -- Renormalization and Factorization



What to do with the long-distance physics associated with colinear/soft singularities in PQCD?

1st strategy:

Identify physical observables which are insensitive to the singularities! (Infra-red-safe (IRS) quantities)

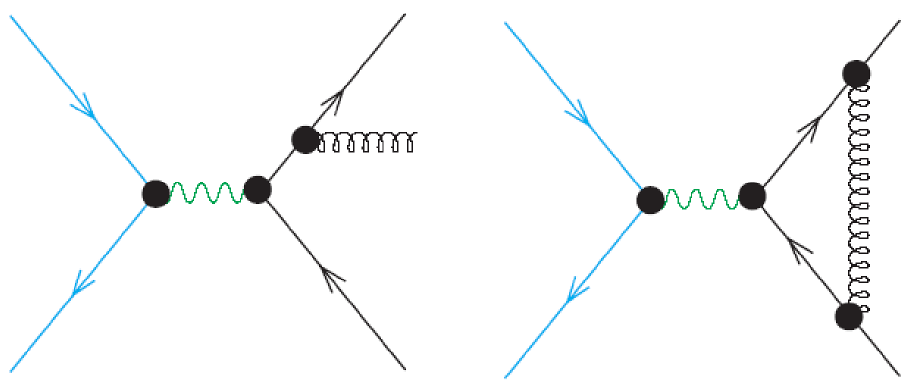
Total Hadronic Cross-section (*inclusive*):

$$\sigma_{tot}(s) = \sigma_0(s)[1 + \alpha_s(s)\, c_1 + \ldots]$$

$Block - Nordsieck\ Thm \ \rightarrow\ c_{1,2,\ldots}\ are\ finite,\ i.e.\ IRS$ *(unitarity)*

Order α_s:

Cancellation of the colinear/soft singularities between real and virtual diagrams



Infra-Red-Safe observables:

Total hadronic Cross-section $\sigma_{tot}/\sigma_{\mu+\mu-}$

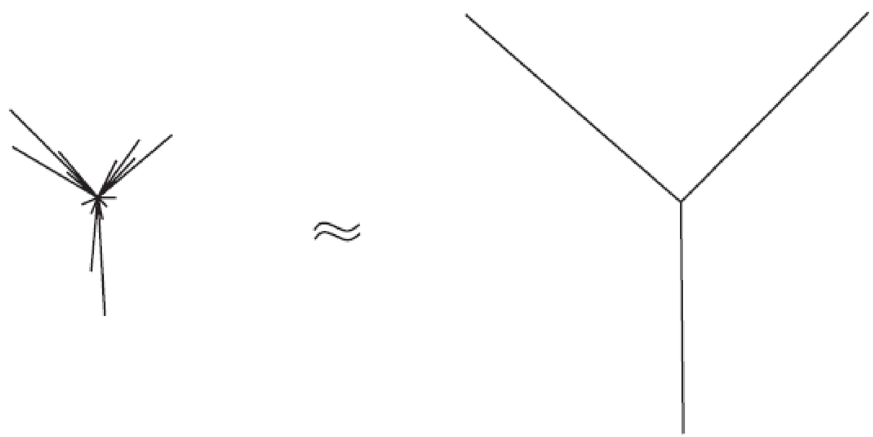
Sterman-Weinberg jet cross-sections and their modern variations (*Jade-*, *Durham-*, ... *algorithms*);

Jet shape observables: Thrust, ... ;

energy-energy correlation ; ....

Essential feature of a general IRS physical quantity:

the observable must be such that it is insensitive to whether n or n+1 particles contributed -- if the n+1 particles has n-particle kinematics



e.g. a IRS "jet algorithm"

σ and R in e^+e^- Collisions

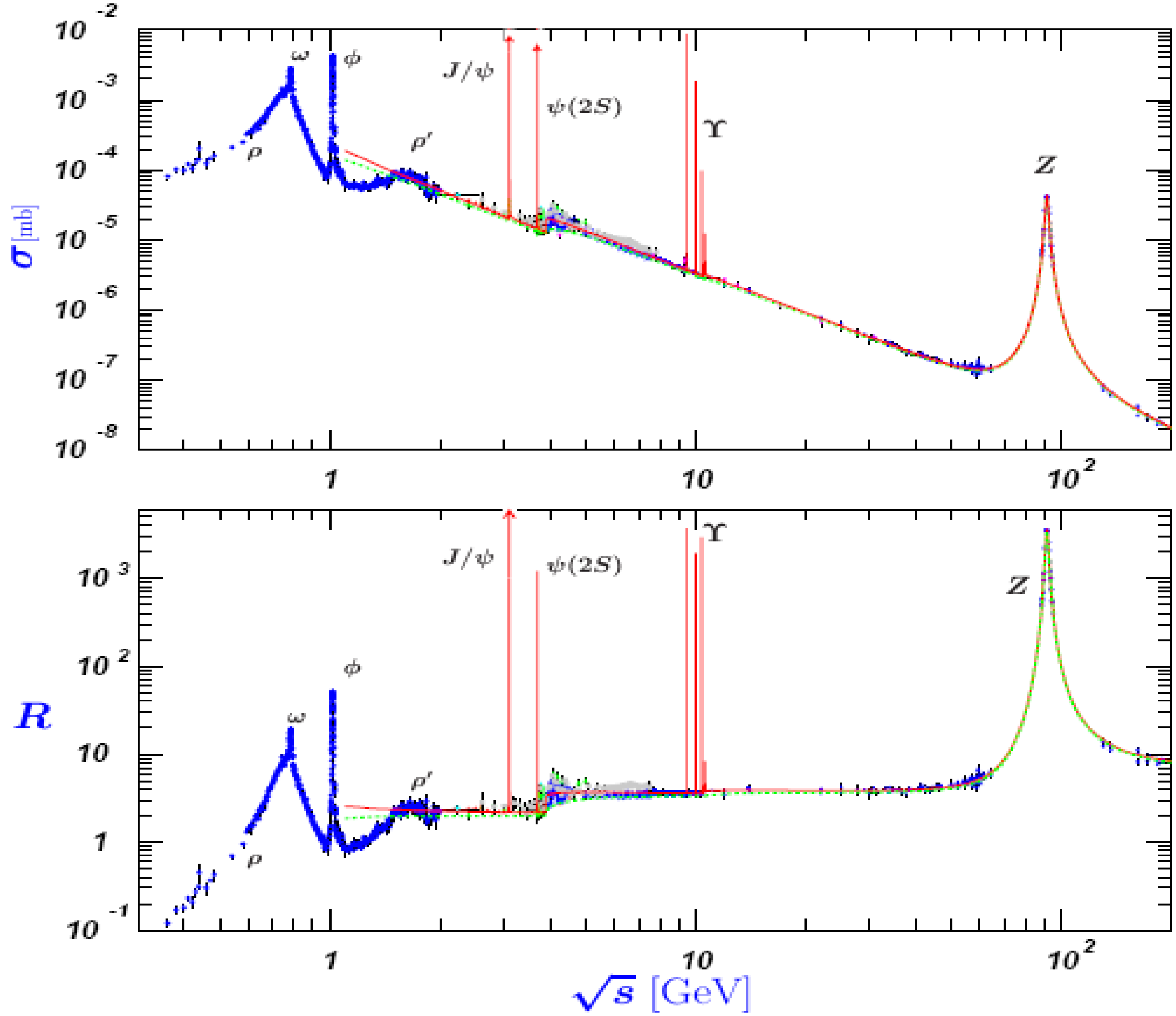
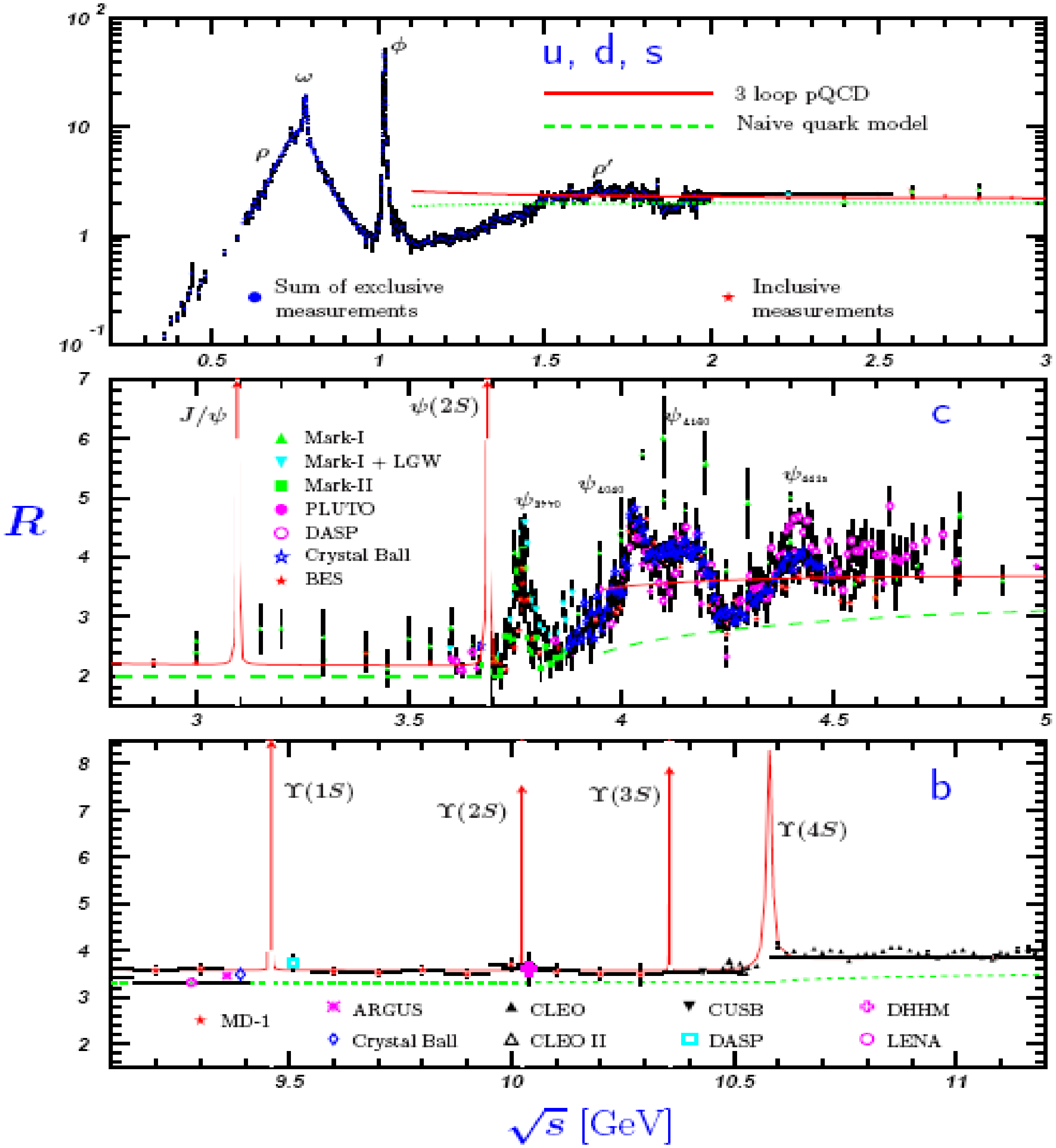


Figure 40.6: World data on the total cross section of $e^+e^- \to hadrons$ and the ratio $R(s) = \sigma(e^+e^- \to hadrons, s)/\sigma(e^+e^- \to \mu^+\mu^-, s)$. $\sigma(e^+e^- \to hadrons, s)$ is the experimental cross section corrected for initial state radiation and electron-positron vertex loops, $\sigma(e^+e^- \to \mu^+\mu^-, s) = 4\pi\alpha^2(s)/3s$. Data errors are total below 2 GeV and statistical above 2 GeV. The curves are an educative guide: the broken one (green) is a naive quark-parton model prediction and the solid one (red) is 3-loop pQCD prediction (see "Quantum Chromodynamics" section of this *Review*, Eq. (9.12) or, for more details, K. G. Chetyrkin *et al.*, Nucl. Phys. B586, 56 (2000) (Erratum *ibid.* B634, 413 (2002)). Breit-Wigner parameterizations of J/ψ, $\psi(2S)$, and $\Upsilon(nS), n = 1, 2, 3, 4$ are also shown. The full list of references to the original data and the details of the R ratio extraction from them can be found in [arXiv:hep-ph/0312114]. Corresponding computer-readable data files are available at `http://pdg.ihep.su/xsect/contents.html`. (Courtesy of the COMPAS(Protvino) and HEPDATA(Durham) Groups, August 2005. Corrections by P. Janot (CERN) and M. Schmitt (Northwestern U.)) See full-color version on color pages at end of book.

R in Light-Flavor, Charm, and Beauty Threshold Regions

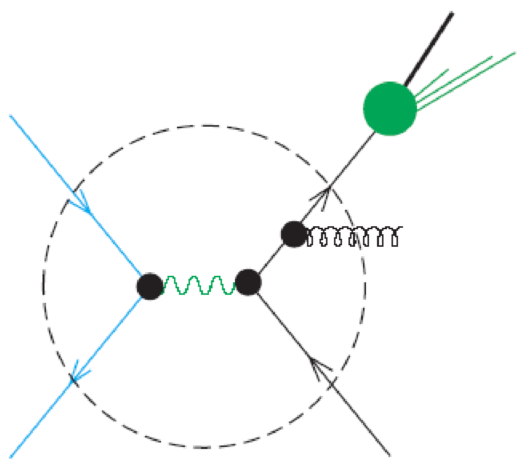


The 2nd strategy:

Factorization ⟶ QCD Parton model

Factorize the physical observable into a calculable ***IRS*** part and a non-calculable but ***universal*** piece.

Example: One particle inclusive cross-section



Fragmentation function:
Long-distance physics;
Universal.

Hard scattering:
Short distance physics;
IRS, perturbatively cal.

$$\sigma(s,z) = \int_z^1 \frac{d\varsigma}{\varsigma}\, \hat{\sigma}^a\left(\frac{s}{\mu}, \frac{z}{\varsigma}, \alpha_s(\mu)\right) \cdot D_a(\varsigma, \mu)$$

“Renormalization” and “Factorization”

UV renormalization			Collinear/soft factorization	
A:	Bare Green Func.	$G_0(\alpha_0, m_0, ..)$	Partonic X-sect	F_a
B:	Ren. constants	$Z_i(\mu)$	Pert. parton dist.	$f_a^b(\mu)$
C:	Ren. Green Fun.	$G_R = G_0/Z$	Hard X-sect	$\hat{F} = F\,/\,f$
D:	Anomalous dim.	$\gamma = \frac{\mu}{Z}\frac{d}{d\mu}Z$	Splitting fun.	$P = \frac{\mu}{f}\frac{d}{d\mu}f$
E:	Phys. para. α, m	$\alpha_0 Z_i \ldots$	Had. parton dist. f_A	resummed
F:	Phys sc. amp.	$\alpha(\mu)\, G_R(m, \mu)$	Hadronic S.F.'s F_A	$f_A(\mu) \times \hat{F}(\mu)$

Some common features:

A : divergent; but, independent of “scheme” and scale μ;

B : divergent; scale and scheme dependent;
universal; absorbs all ultra-violet/soft/collinear divergences;

C & D : finite; scheme-dependent;
D controls the μ dependence of E & F;

E : physical parameters to be obtained from experiment;

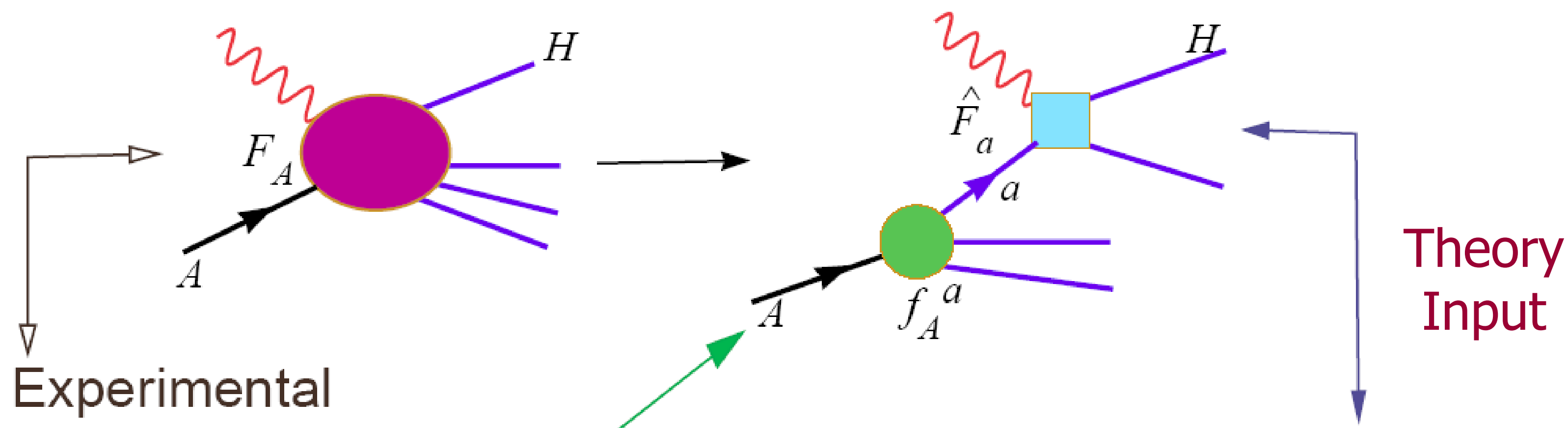
F : Theoretical “prediction”; μ-indep. to all orders,
but μ-dep. at finite order n; $\mu\frac{d}{d\mu} \sim \mathcal{O}(\alpha^{n+1})$

Note: “Renormalization” is factorization (of UV divergences);
“factorization” is renormalization (of soft/collinear div.)

Lepton-hadron Sc.

Master Equation for QCD Parton Model
– the Factorization Theorem

$$F_A^\lambda(x,\frac{m}{Q},\frac{M}{Q}) = \sum_a f_A^a(x,\frac{m}{\mu}) \otimes \widehat{F}_a^\lambda(x,\frac{Q}{\mu},\frac{M}{Q}) + \mathcal{O}((\frac{\Lambda}{Q})^2)$$

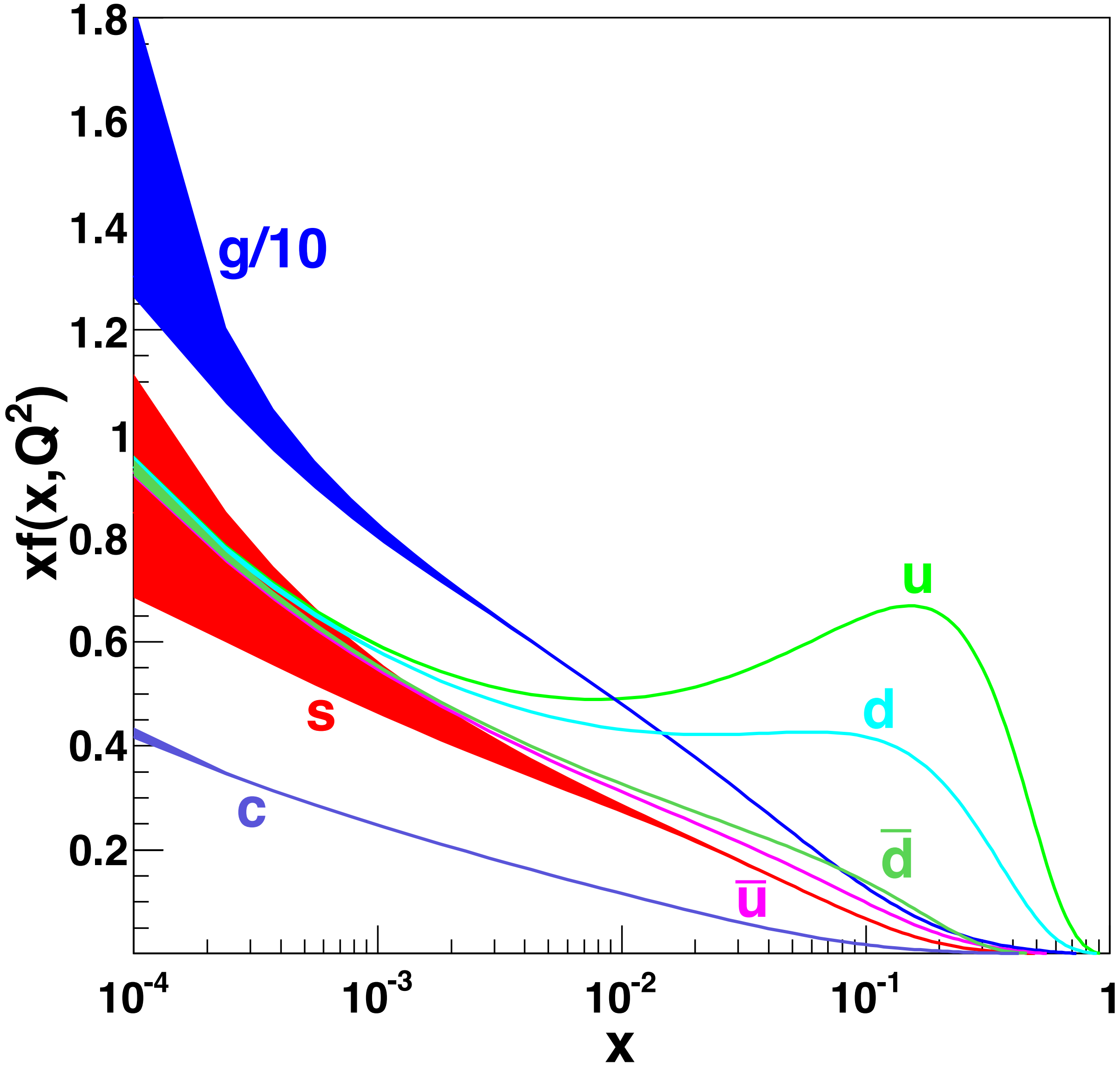


Hard Cross-section
perturbative, calculable
(may contain $\alpha_s^n \mathrm{Log}^n(M/Q)$)

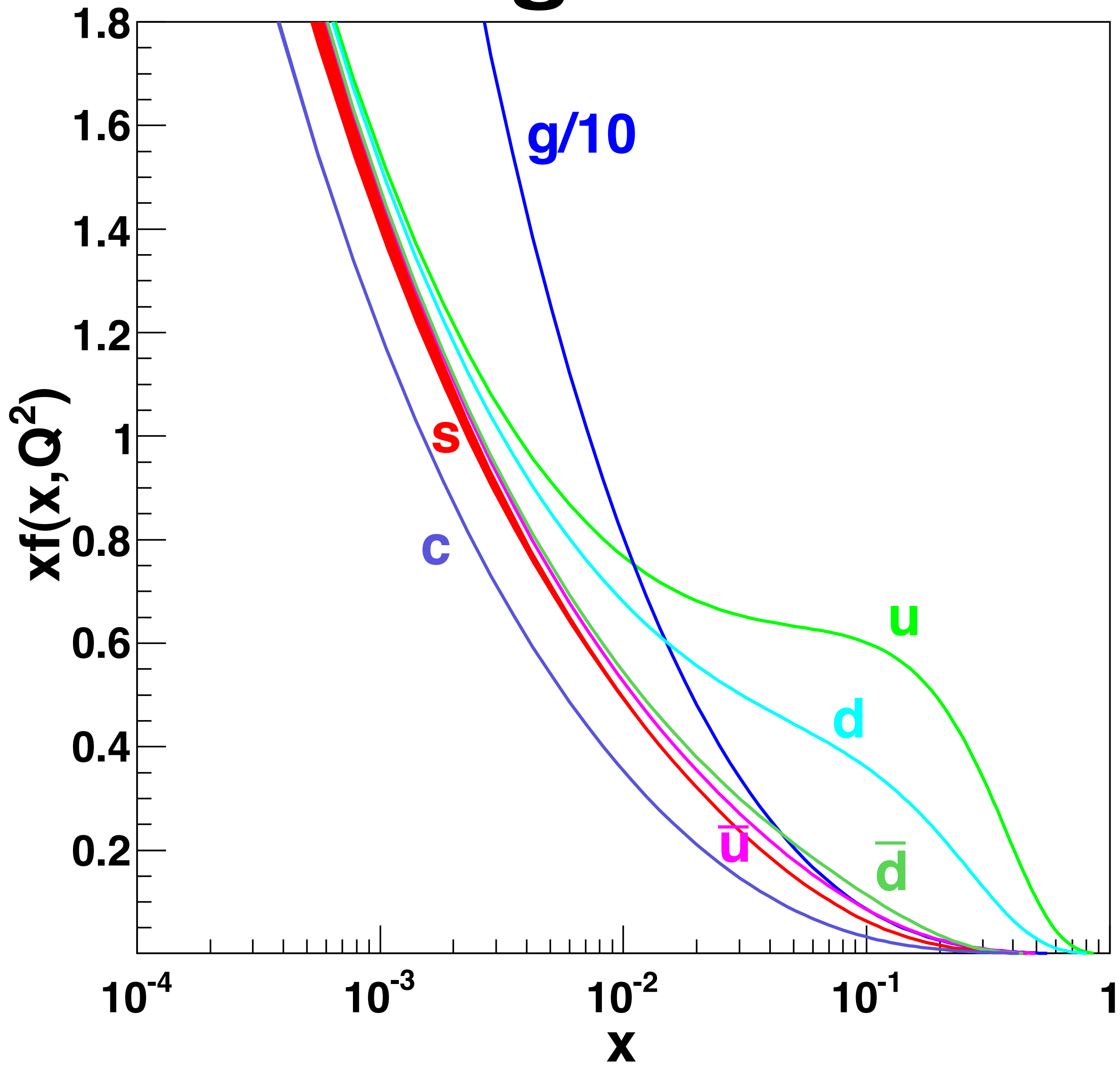
universal Parton Dist. Fn.
Non-Perturbative Parametrization at Q_0
GLAP Evolution to Q
extracted by global analysis

CT10 PDF plots

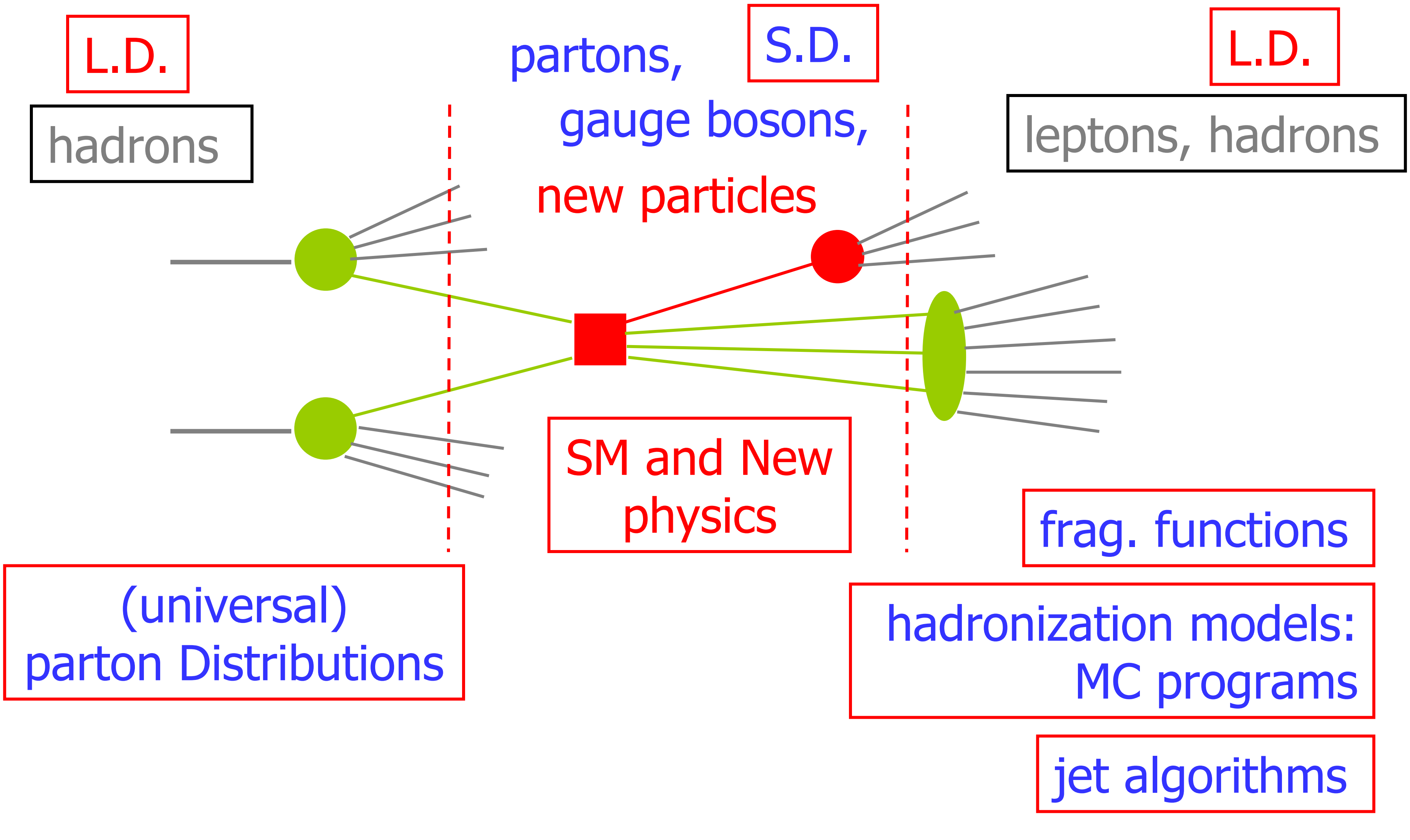
Low Scale



High Scale



Hadron Collider Physics



Deep Inelastic Scattering (DIS) in Lepton-Hadron Collisions

Probing the Parton Structure of the Nucleon with Leptons

Deep Inelastic Scattering in Lepton-Hadron Collisions
—Probing the Parton Structure of the Nucleon with Leptons

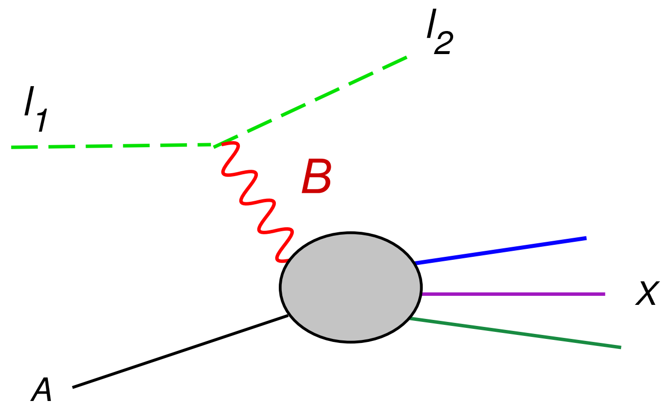
- Basic Formalism
  (indep. of strong dynamics and parton picture)
- Experimental Development
  - Fixed target experiments
  - HERA experiments
- Parton Model and QCD
  - Parton Picture of Feynman-Bjorken
  - Asymptotic freedom, factorization and QCD
- Phenomenology
  - QCD parameters
  - Parton distribution functions
  - Other interesting topics

Basic Formalism

(leading order in EW coupling)

Lepton-hadron scattering process

$$\ell_1(\ell_1) + N(P) \longrightarrow \ell_2(\ell_2) + X(P_X)$$



(B = g, W, Z)

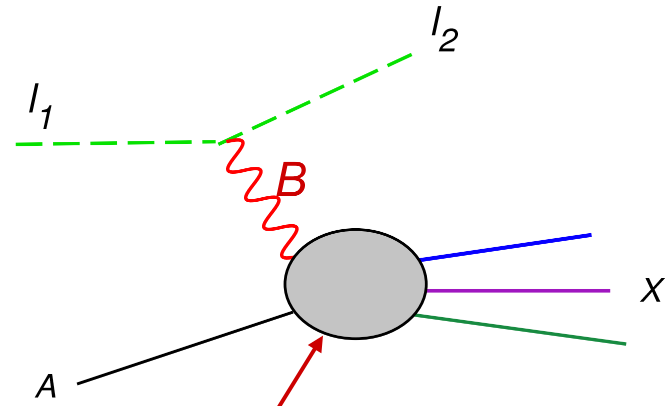
Effective fermion-boson electro-weak interaction Lagrangian:

$$\mathcal{L}_{\rm int}^{\rm EW} = -g_B \left[j_\mu^{(\ell)}(x) + J_\mu^{(h)}(x) \right] \; V_B^\mu(x)$$

EW SU(2)xU(1) gauge coupling constants

B	γ	$W^\pm$	Z
g_B	$-e$	$\frac{g}{2\sqrt{2}}$	$\frac{g}{2\cos\theta_W}$

Scattering Amplitudes



$$\mathcal{M} = J^{*}_{\mu}(P,q)\frac{g_B^2 G^{\mu}{}_{\nu}}{Q^2 + M_B^2} j^{\nu}(q,\ell)$$

Spin 1 projection tensor $G^{\mu}{}_{\nu} = g^{\mu}{}_{\nu} - q^{\mu}q_{\nu}/M_B^2.$

Lepton current amplitude (known):

$$j^{\mu}(q,\ell) = \langle \ell_2 | j^{\mu} | \ell_1 \rangle = \overline{u}(\ell_2)\gamma^{\mu}[g_R(1+\gamma^5) + g_L(1-\gamma^5)]u(\ell_1)$$

Hadron current amplitude (unknown):

$$J^{*}_{\mu}(P,q) = \langle P_X | J^{\dagger}_{\mu} | P \rangle$$

Object of study:

- Parton structure of the nucleon; (short distance)
- QCD dynamics at the confinement scale (long dis.)

Basic Formalism: Cross section

Cross section

(amplitude)^2 phase space / flux

$$d\sigma = \frac{G_1\, G_2}{2\Delta(s, m_{\ell_1}^2, M^2)}\; 4\pi Q^2\, L_\nu^\mu\, W_\mu^\nu\, d\Gamma$$

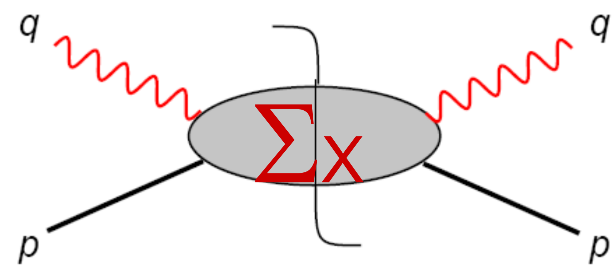
$$G_i = g_{B_i}^2/(Q^2 + M_{B_i}^2)$$

Lepton tensor (known):

$$L^\mu{}_\nu = \frac{1}{Q^2}\overline{\sum_{\text{spin}}}\langle \ell_1|j_\nu^\dagger|\ell_2\rangle\langle\ell_2|j^\mu|\ell_1\rangle$$

Hadron tensor (unknown):

$$W^\mu{}_\nu = \frac{1}{4\pi}\overline{\sum_{\text{spin}}}(2\pi)^4\delta^4(P+q-P_X)\langle P|J^\mu|P_X\rangle\langle P_X|J_\nu^\dagger|P\rangle$$



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Basic Formalism: Structure Functions

Expansion of $W^{\mu}{}_{\nu}$ in terms of *independent* components

$$W^{\mu}{}_{\nu} = -g^{\mu}{}_{\nu}W_1 + \frac{P^{\mu}P_{\nu}}{M^2}W_2 - i\frac{\epsilon^{Pq\mu}{}_{\nu}}{2M^2}W_3 + \\ +\frac{q^{\mu}q_{\nu}}{M^2}W_4 + \frac{P^{\mu}q_{\nu}+q^{\mu}P_{\nu}}{2M^2}W_5 + \frac{P^{\mu}q_{\nu}-q^{\mu}P_{\nu}}{2M^2}W_6$$

Cross section in terms of the structure functions

$$\frac{d\sigma}{dE_2\, d\cos\theta} = \frac{2E_2^2}{\pi M}\frac{G_1\, G_2}{n_\ell}\left\{g_{+\ell}^2\left[2W_1\sin^2\frac{\theta}{2} + W_2\cos^2\frac{\theta}{2}\right] \pm g_{-\ell}^2\left[\frac{E_1+E_2}{M}W_3\sin^2\frac{\theta}{2}\right]\right\}$$

Charged Current (CC) processes (neutrino beams):
W-exchange (diagonal); left-handed coupling only; ….

Neutral Current (NC) processes (e,μ scat.)---low energy:
(fixed tgt): γ-exchange (diagonal); vector coupling only; …

Neutral Current (NC) processes (e,μ scat.)---high energy
(hera): γ & Z exchanges: G_1^2, G_1G_2, G_2^2 terms; ….

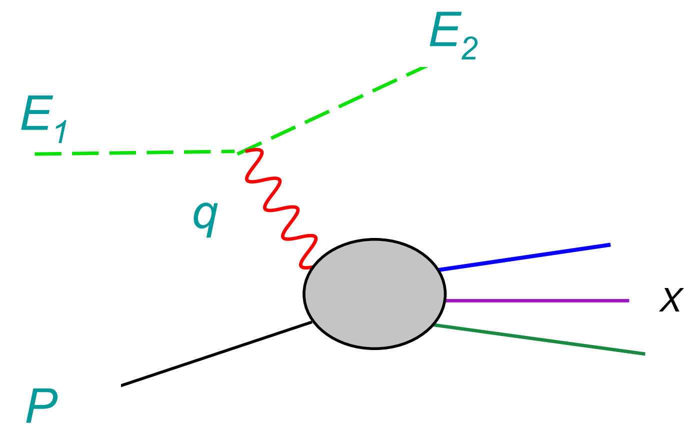
Basic Formalism: Scaling structure functions

Kinematic variables

$$\nu = \frac{P \cdot q}{\sqrt{P \cdot P}} = E_1 - E_2$$

$$x = \frac{-q^2}{2P \cdot q} = \frac{Q^2}{2M\nu}$$

$$y = \frac{P \cdot q}{P \cdot \ell_1} = \frac{\nu}{E_1}$$



Scaling (dimensionless) structure functions

$$\begin{aligned} F_1(x,Q) &= W_1 \\ F_2(x,Q) &= \frac{\nu}{M} W_2 \\ F_3(x,Q) &= \frac{\nu}{M} W_3 \end{aligned}$$

Scaling form of cross section formula:

$$(\; g^2_{\pm\ell} = g^2_{L\ell} \pm g^2_{R\ell} \;)$$

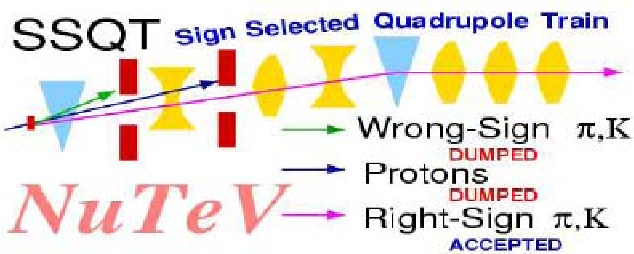
$$\frac{d\sigma}{dx dy} = \frac{2ME_1}{\pi} \frac{G_1 G_2}{n_\ell} \left\{ g^2_{+\ell} \left[xF_1 y^2 + F_2 \left[(1-y) - \left(\frac{Mxy}{2E_1} \right) \right] \right] \pm g^2_{-\ell} \left[xF_3 y (1 - \frac{y}{2}) \right] \right\}$$

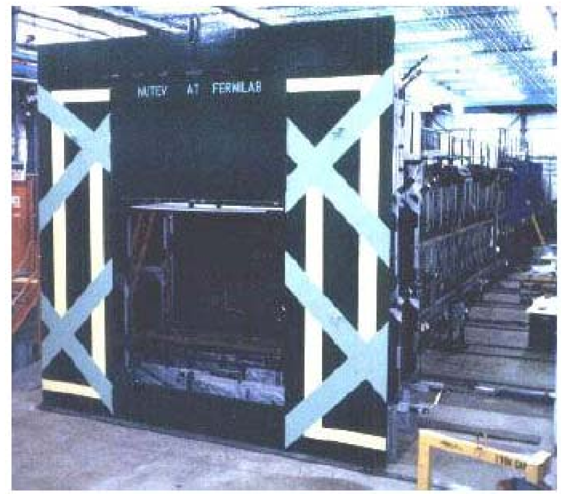
n_ℓ is the number of polarization states of the incoming lepton.

The highest energy (anti-) neutrino DIS experiment

The NuTeV experiment at FNAL

ν-N DIS, sign-selected beam $<E_\nu>$~120 GeV
and continuous test beam calibration

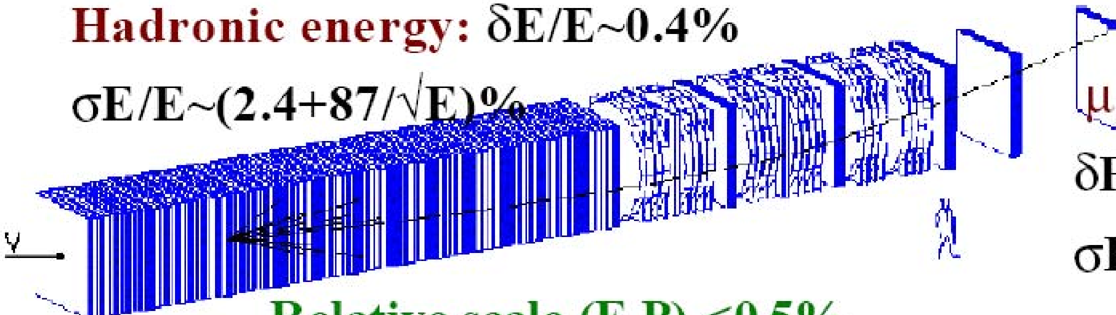




Data taken during 1996-97

Hadronic energy: $\delta E/E$~0.4%

$\sigma E/E$~$(2.4+87/\sqrt{E})$%



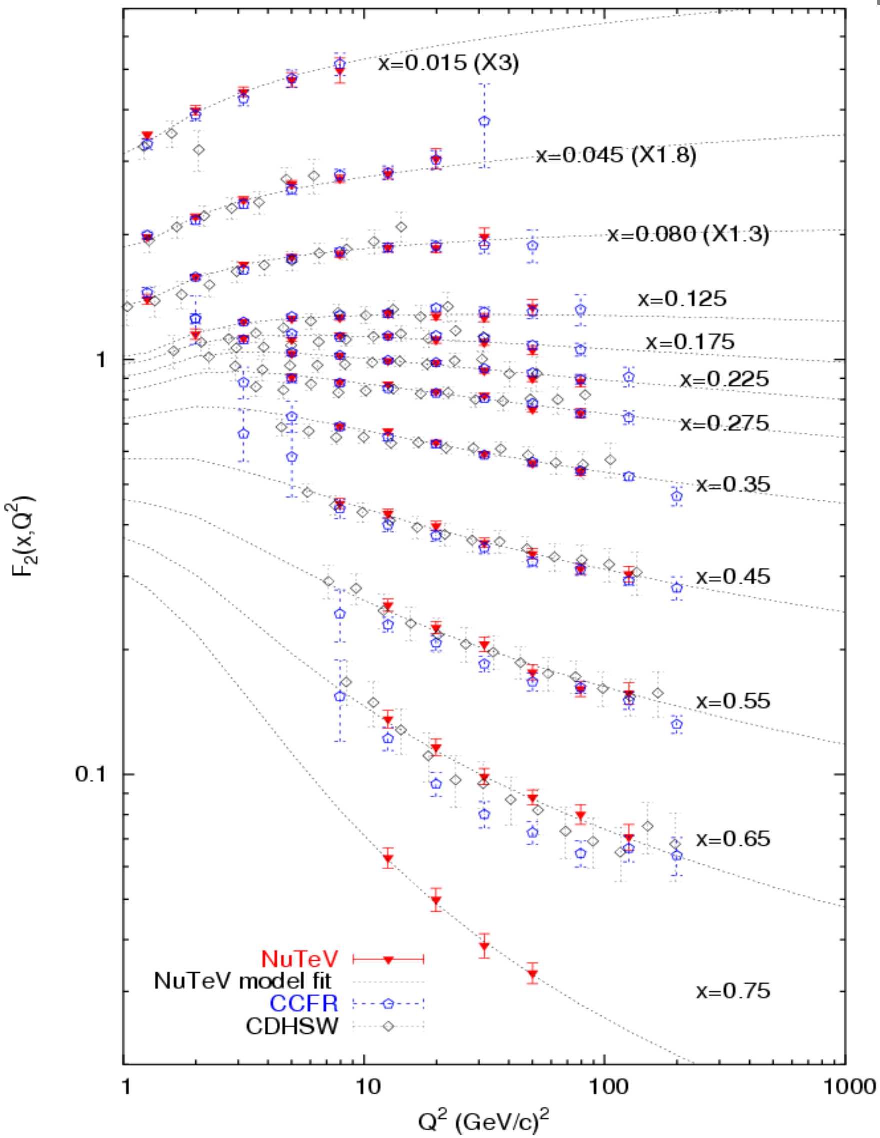
μ momentum

$\delta P/P$~1%

$\sigma P/P$~11%

Relative scale (E,P) <0.5%

F_2 Measurement



- **Isoscalar ν-Fe F_2**
- **NuTeV F_2 is compared with CCFR and CDHSW results**
  - **the line is a fit to NuTeV data**
- **All systematic uncertainties are included**
- **All data sets agree for *x<0.4*.**
- **At *x>0.4* NuTeV agrees with CDHSW**
- **At *x>0.4* NuTeV is systematically above CCFR**

The HERA Collider

The first and only ep collider in the world

Located in Hamburg

$e^{\pm}$ → 27.5 GeV

p ← 920 GeV

$\sqrt{s}$ = 318 GeV

Equivalent to fixed target experiment with 50 TeV $e^{\pm}$

The Collider Experiments

H1 Detector

Complete 4π detector with

Tracking
- **Si-μVTX**
- **Central drift chamber**

Liquid Ar calorimeter

$\Delta E/E = 12\%/\sqrt{E[GeV]}$ (e.m.)

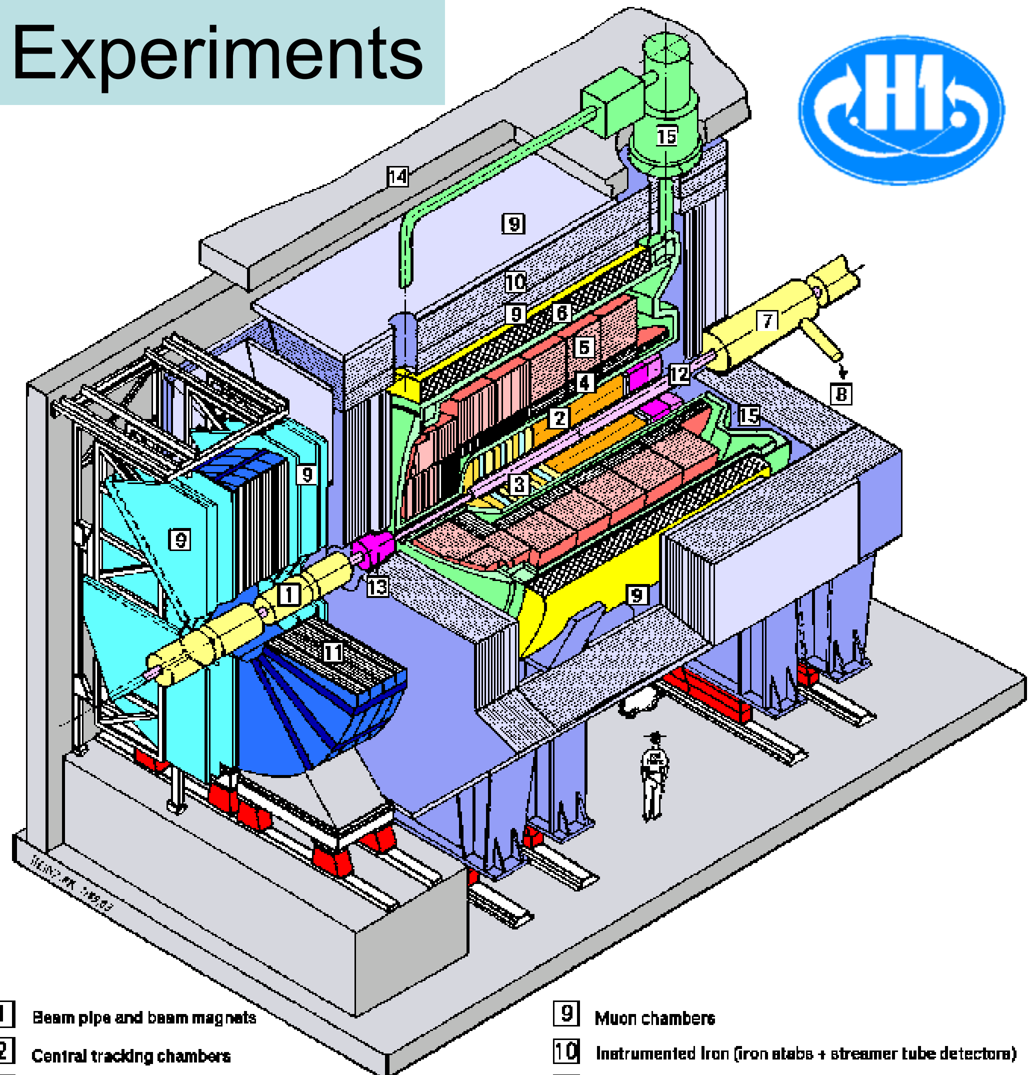
$\Delta E/E = 50\%/\sqrt{E[GeV]}$ (had)

Rear Pb-scintillator calorimeter

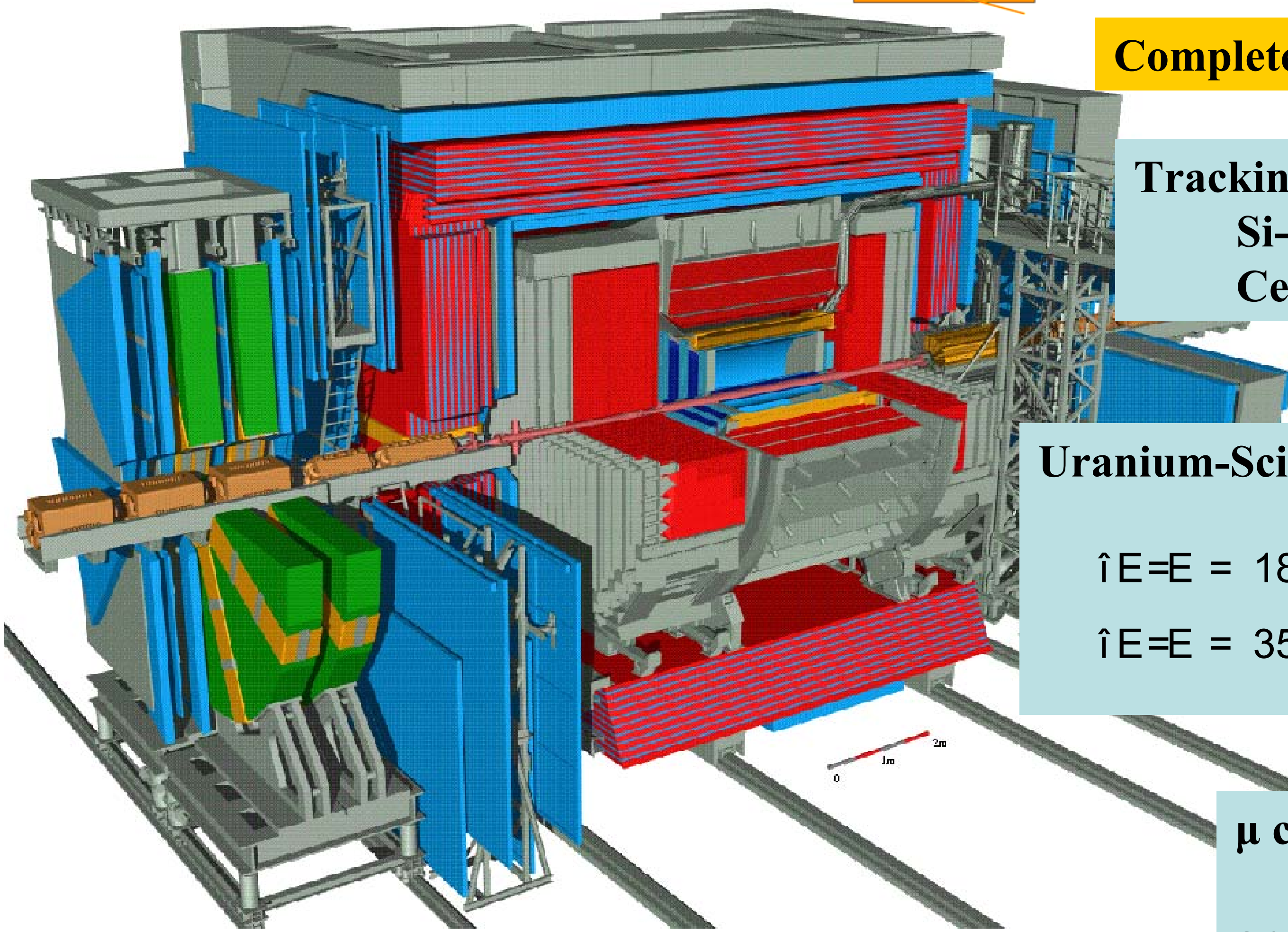
$\Delta E/E = 7.5\%/\sqrt{E[GeV]}$ (e.m.)

μ chambers

and much more…



ZEUS Detector



Complete 4π detector with

Tracking
Si-μVTX
Central drift chamber

Uranium-Scintillator calorimeter

$\sigma_E/E = 18\%/\sqrt{E[GeV]}$ (e.m.)

$\sigma_E/E = 35\%/\sqrt{E[GeV]}$ (had) ←

μ chambers

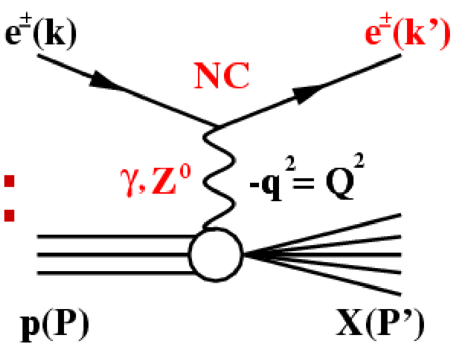
and much more…

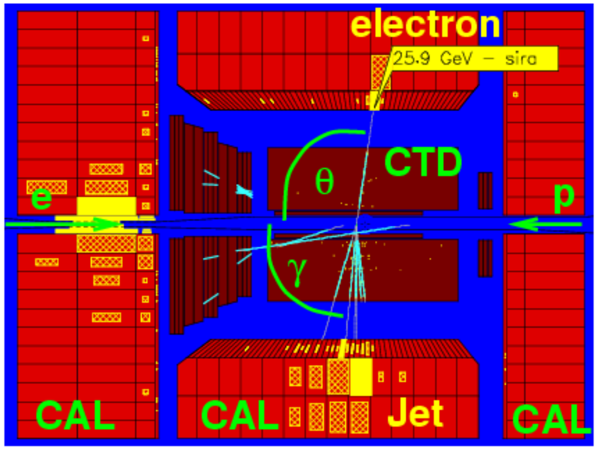
Both detectors asymmetric

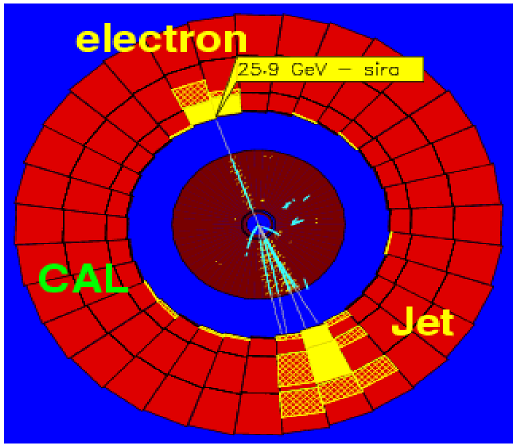
NC and CC incl. Processes measured at HERA

$$\text{NC}: \quad e^{\pm} + p \rightarrow e^{\pm} + X, \qquad \text{CC}: \quad e^{\pm} + p \rightarrow \bar{\nu}_e(\nu_e) + X$$

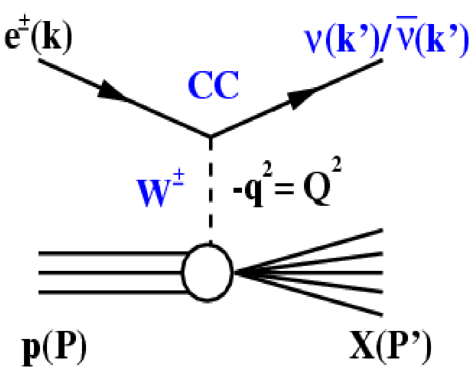
NC:

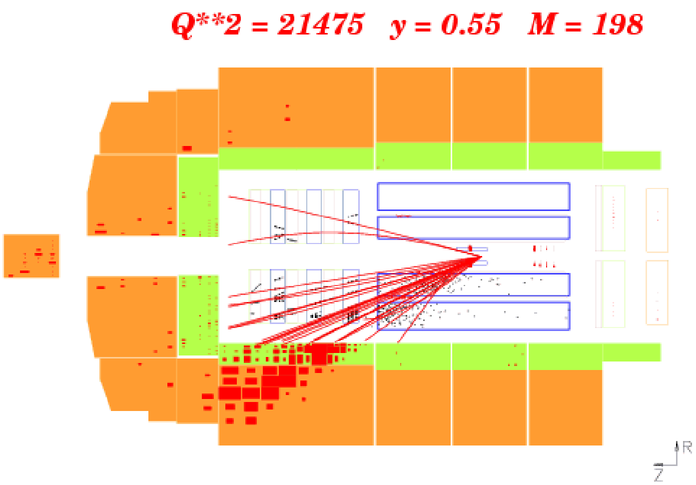


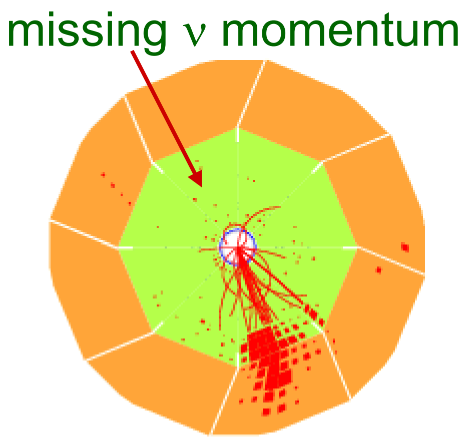




CC:

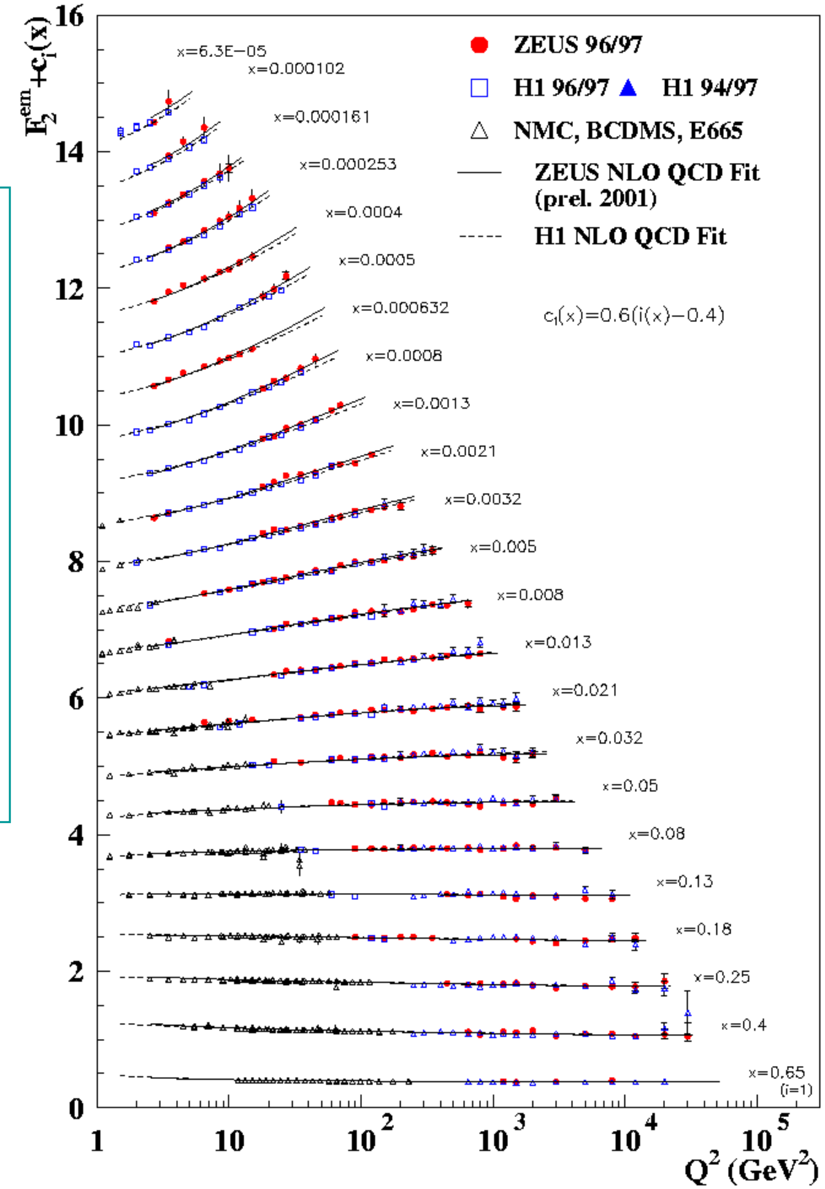






Measurement of $F^{\gamma}_2(x,Q^2)$

- For $Q^2 \ll M_Z^2 \rightarrow xF_3$ negligible;
- F_L only important at high y;
- Both F_L and xF_3 ~ calculable in QCD
- Correct for higher order QED radiation
- Extract $F_2(x,Q^2)$ from measurement of $\frac{d^2\hat{\sigma}^{ep}}{dx\,dQ^2}$

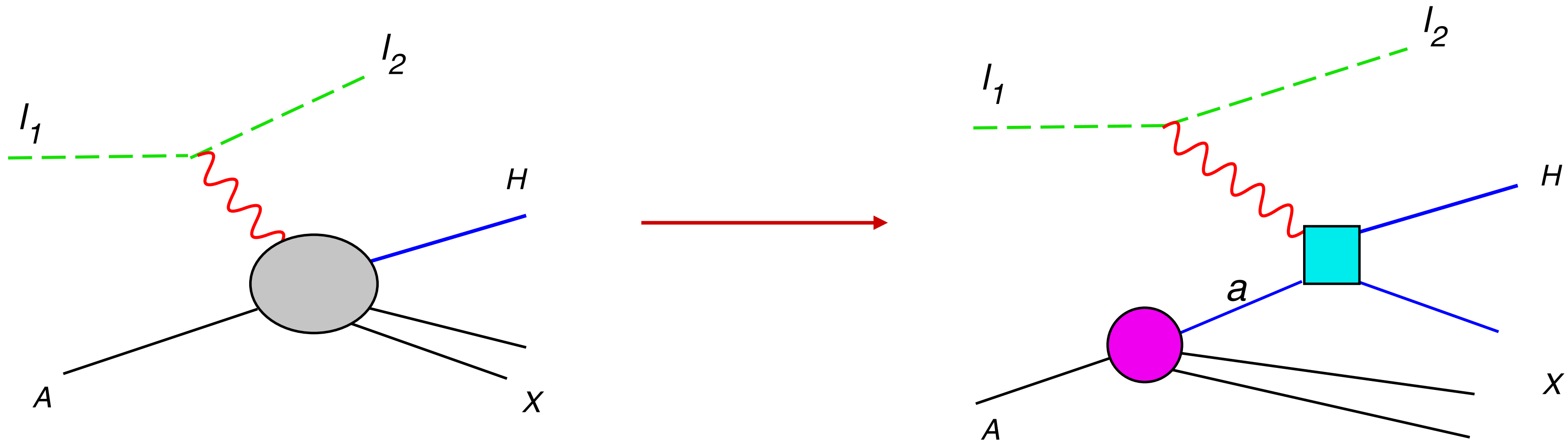


These are difficult measurements:
nevertheless precision level has reached: errors of 2-3%

Physical Interpretations of DIS Structure Function measurements

- The Parton Model (Feynman-Bjorken)
- Theoretical basis of the parton picture and the QCD improved parton model

High energy (Bjorken) limit:
(large Q^2 and ν, for a fixed x value)

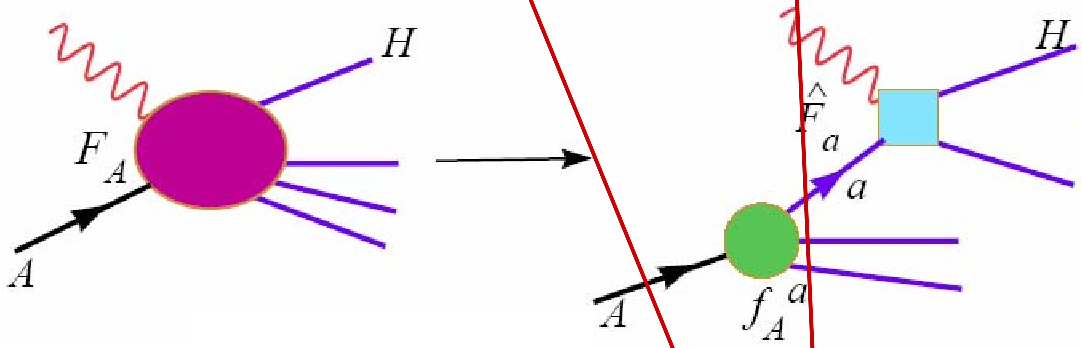


QCD and DIS

Master Equation for QCD Parton Model

– the Factorization Theorem

$$F_A^\lambda(x,\frac{m}{Q},\frac{M}{Q}) = \sum_a f_A^a(x,\frac{m}{\mu}) \otimes \hat{F}_a^\lambda(x,\frac{Q}{\mu},\frac{M}{Q}) + \mathcal{O}((\frac{\Lambda}{Q})^2)$$



A physical observable is independent of μ, i.e., renormalization group invariant.

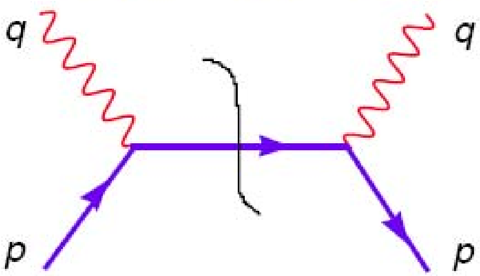
μ is the factorization scale. Usually choose μ = Q: that is how $f(x,Q)$ acquires Q-dep.

Parton Model results on Structure Functions

$$F_\lambda(x,Q^2) \sim \int_0^1 \frac{d\xi}{\xi} \sum_a \; f_A^a(\xi)\; \widehat{F}_\lambda^a(x/\xi, Q^2) + \mathcal{O}(\frac{m}{Q}).$$

where $\widehat{F}_\lambda^a(z,Q^2)$ is the "partonic structure function" for DIS on the parton target a.

The Feynman diagram contributing to this elementary quantity and the result of a straightforward calculation are (for electro-magnetic coupling case):



$$\begin{aligned}
\widehat{F}_T^a(x/\xi, Q^2) &= Q_a^2\; \delta(x/\xi \Leftrightarrow 1) \\
\widehat{F}_L^a(x/\xi, Q^2) &= 0 \\
\widehat{F}_{PV}^a(x/\xi, Q^2) &= 0
\end{aligned}$$

$\Longrightarrow$ the simple <u>scaling</u> parton model results:

$$\begin{aligned}
F_T(x,Q^2) &= \textstyle\sum_a \; Q_a^2 \; f_A^a(x) && \text{(Bj.} \Leftrightarrow \text{Feynman)} \\
F_L(x,Q^2) &= 0 && \text{(Callan} \Leftrightarrow \text{Gross)}
\end{aligned}$$

Structure functions: Quark Parton Model

Quark parton model (QPM) NC SFs for proton target:

$$[F_2^{\gamma}, F_2^{\gamma Z}, F_2^{Z}] = x \sum_q [e_q^2, 2e_q v_q, v_q^2 + a_q^2]\{q + \overline{q}\}$$

$$[xF_3^{\gamma Z}, xF_3^{Z}] = 2x \sum_q [e_q a_q, v_q a_q]\{q - \overline{q}\} = 2x \sum_{q=u,d} [e_q a_q, v_q a_q] q_v$$

QPM CC SFs for proton targets:

$$xF_{2,W+}^{CC} = x\{\overline{u} + \overline{c} + d + s\}, \qquad xF_{3,W+}^{CC} = x\{d + s - (\overline{u} + \overline{c})\}$$

$$xF_{2,W-}^{CC} = x\{u + c + (\overline{d} + \overline{s})\}, \qquad xF_{3,W-}^{CC} = x\{u + c - (\overline{d} + \overline{s})\}$$

For neutron targets, invoke (flavor) isospin symmetry:

$u \Leftrightarrow d$ and $\overline{u} \Leftrightarrow \overline{d}$

continued

Consequences on CC Cross sections (parton model level):

$$\frac{d\sigma^{\nu/\bar{\nu}}}{dxdy} \propto W \cdot L \propto F_{\nu/\bar{\nu}} \left(\frac{1 \pm \cosh \psi}{2} \right)^2$$

$$\cosh \psi = \frac{2-y}{y} \quad \longrightarrow \quad \frac{1 \pm \cosh \psi}{2} \propto \begin{cases} 1 \\ 1-y \end{cases}$$

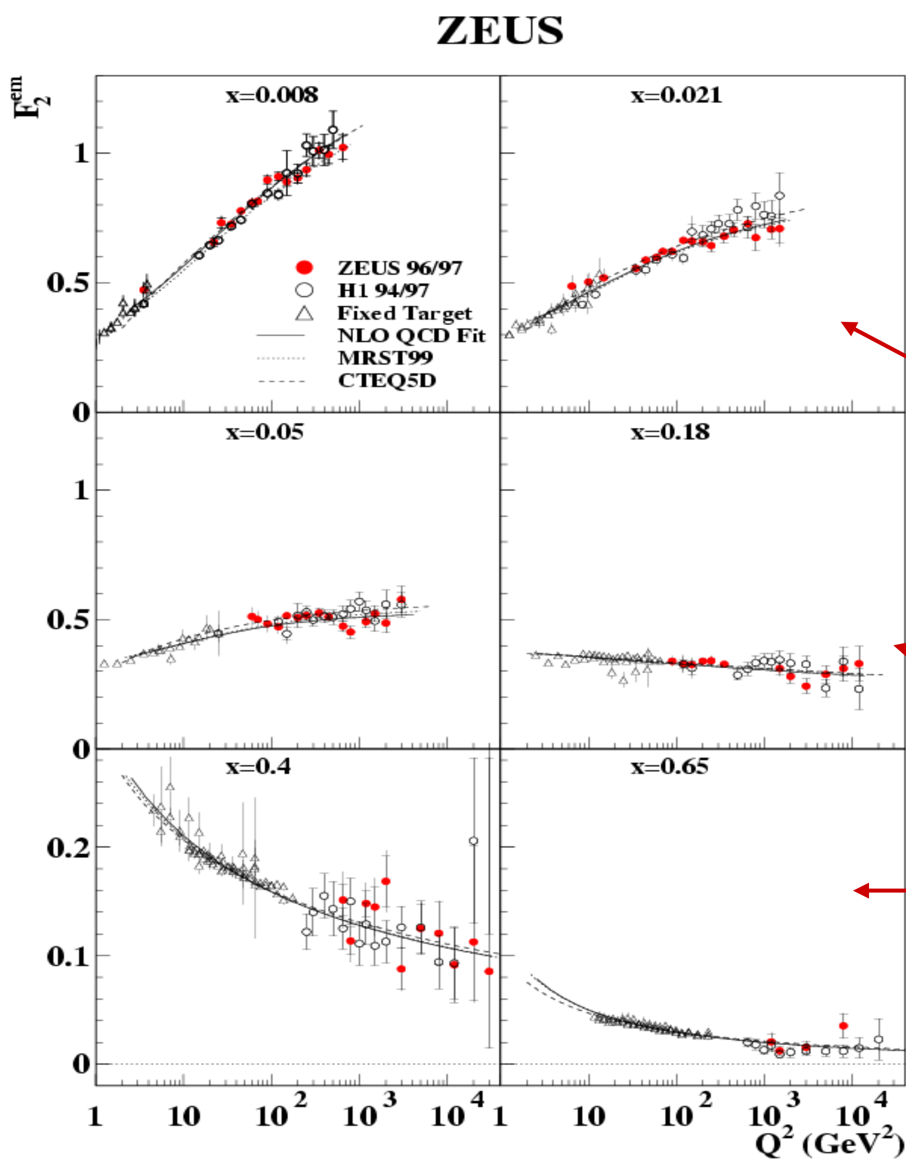
$$\longrightarrow \quad \frac{\sigma^{\bar{\nu}}}{\sigma^{\nu}} = \int dy \frac{d\sigma^{\bar{\nu}}}{dy} \Big/ \int dy \frac{d\sigma^{\nu}}{dy} \approx \frac{1}{3}$$

These qualitative features were verified in early (bubble chamber) high energy neutrino scattering experiments.

Gargamelle (CERN)

Refined measurements reveal QCD corrections to the approximate naïve parton model results. These are embodies in all modern “QCD fits” and “global analyses”.

F_2 : "Scaling violation" — Q-dependence inherent in QCD



Renormalization group equation governs the scale dependence of parton distributions and hard cross sections. (DGLAP)

Rise with increasing Q at small-x

Flat behavior at medium x

decrease with increasing Q at high x

QCD evolution

Evolution performed in terms of (1/2/3) non-singlet, singlet and gluon densities:

$$\frac{\partial}{\partial \ln \mu_F^2} q_{NS}^{\pm} = P_{NS}^{\pm} \otimes q_{NS}^{\pm}$$

$$\frac{\partial}{\partial \ln \mu_F^2} \begin{Bmatrix} \Sigma \\ g \end{Bmatrix} = \begin{Bmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{Bmatrix} \otimes \begin{Bmatrix} \Sigma \\ g \end{Bmatrix} = \mathrm{P} \otimes \mathrm{q}$$

Where

$$P(x) = a_S P^{(0)}(x) + a_S^2 \left[P^{(1)}(x) - \beta_0 \ln \frac{\mu_F^2}{\mu_R^2} P^{(0)}(x) \right] \qquad \text{with} \qquad a_S = \frac{\alpha_S(\mu_R^2)}{4\pi}$$

$$\frac{da_S}{d \ln \mu_R^2} = \beta(a_S) = \sum_{l=0}^{\infty} a_S^{l+2} \beta_l \cong a_S^2 \beta_0 + a_S^3 \beta_1 \qquad \text{where} \quad \beta_0 = 11 - \frac{2}{3} N_F$$

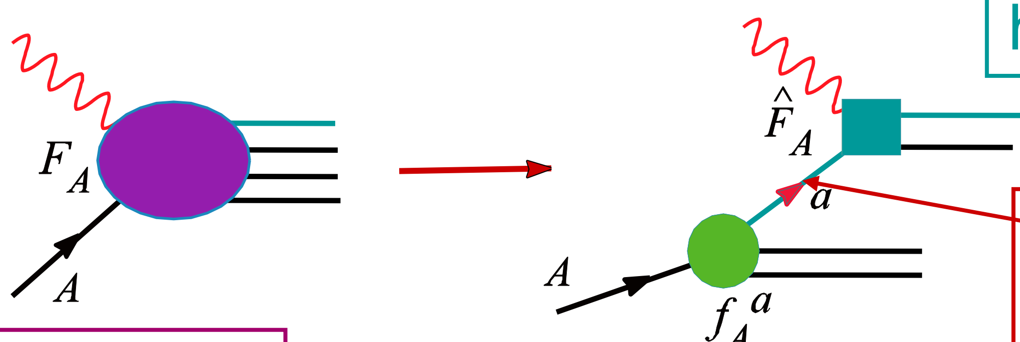
$$\text{and} \quad \beta_1 = 102 - \frac{38}{3} N_F$$

Parton Distribution Functions (PDF):
most significant physical results derived from DIS
(with help from other hard scattering processes)

A common misconception:

Parton distribution functions $\not\approx$ "Structure functions"

These are the hard Xsecs.



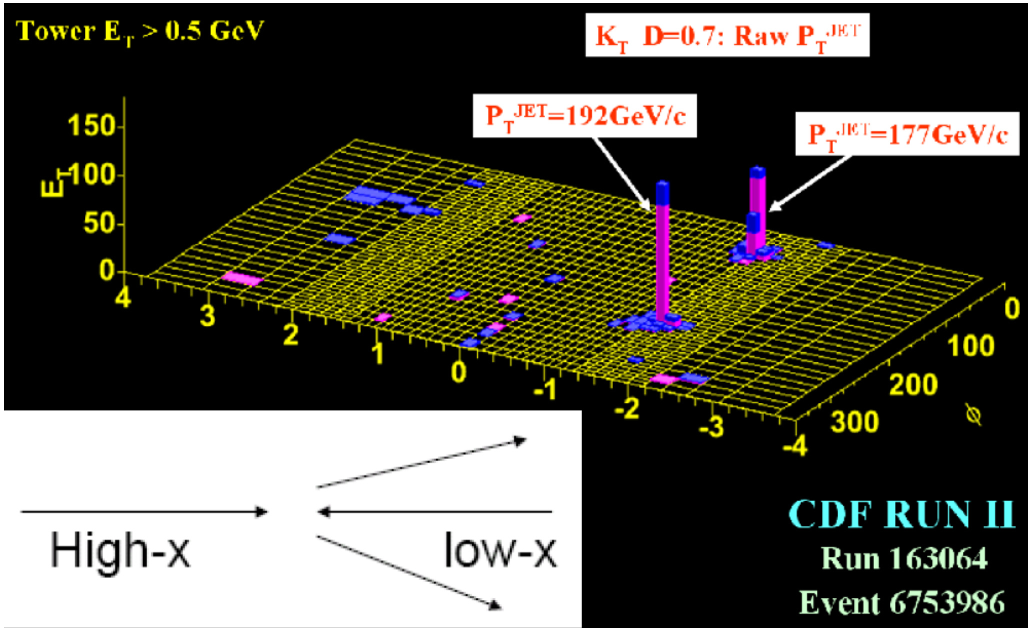
There is a convo-lution integral and a summation over partons here!

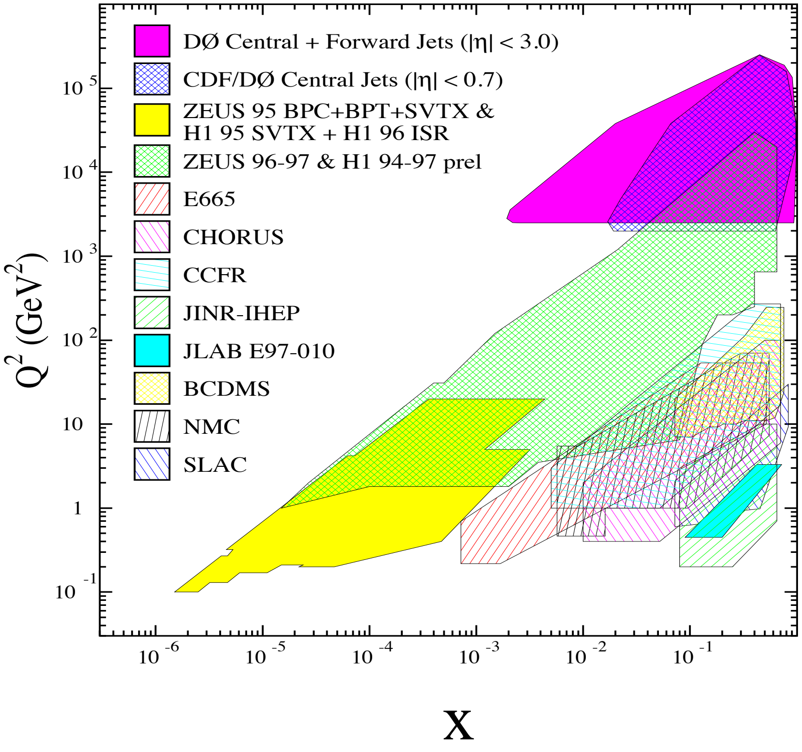
These are the (process-dep) S.F.s

These are the (universal) PDFs

Forward Jet Measurement

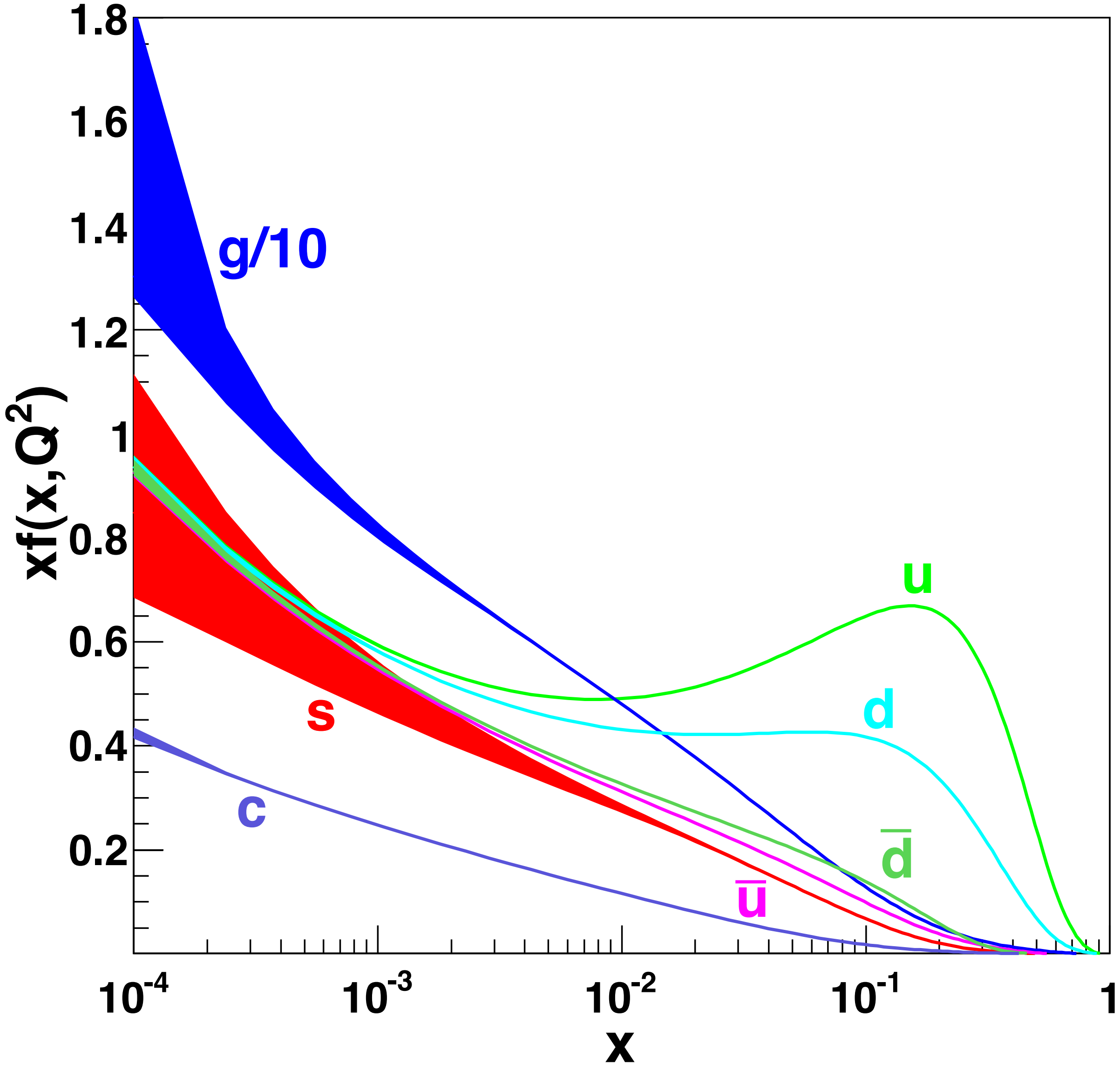
- Forward jets probe high-x at lower Q^2 ($= -q^2$) than central jets
  - Q^2 evolution given by DGLAP
  - Essential to distinguish PDF and possible new physics at higher Q^2
- Also, extend the sensitivity to lower x



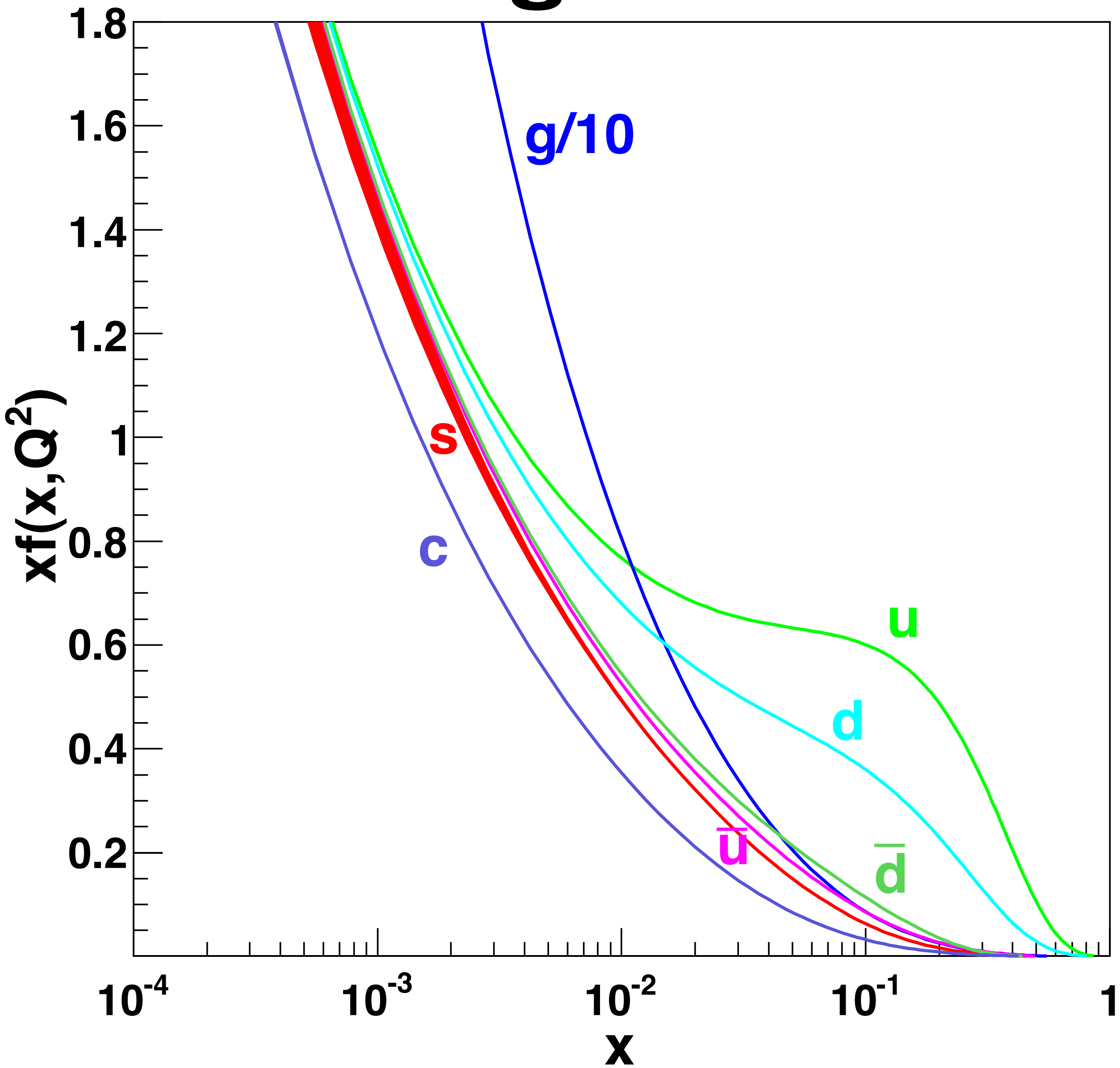


CT10 PDF plots

Low Scale



High Scale

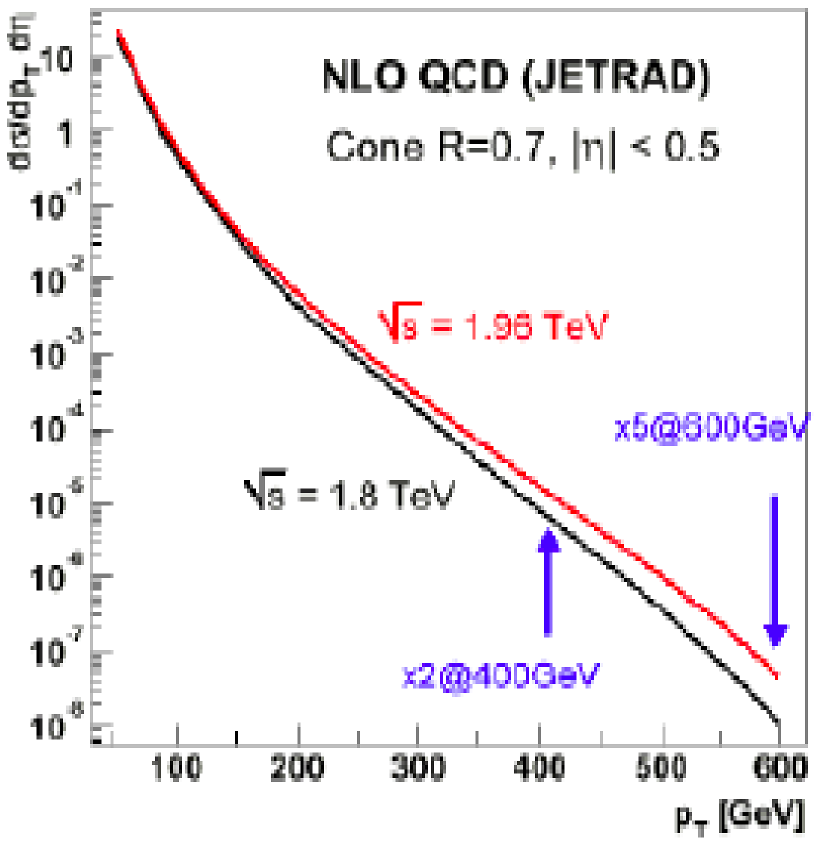


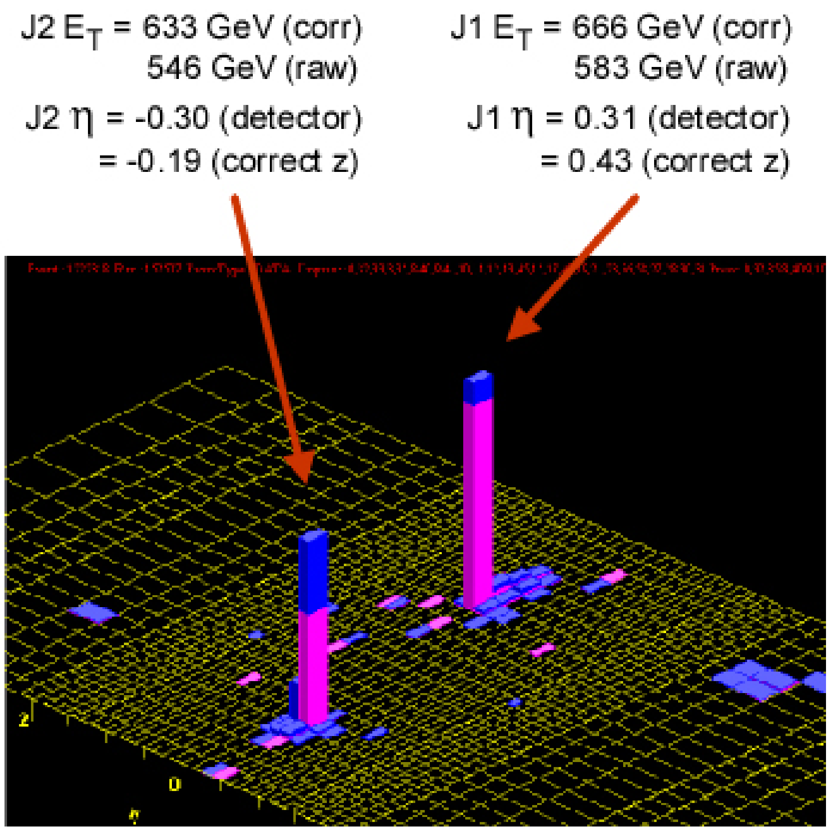
QCD in Hadron Collisions

Jets

Inclusive Jet Production

- Nowhere is the increase in center-of-mass energy more appreciated





CDF Run 2 Preliminary

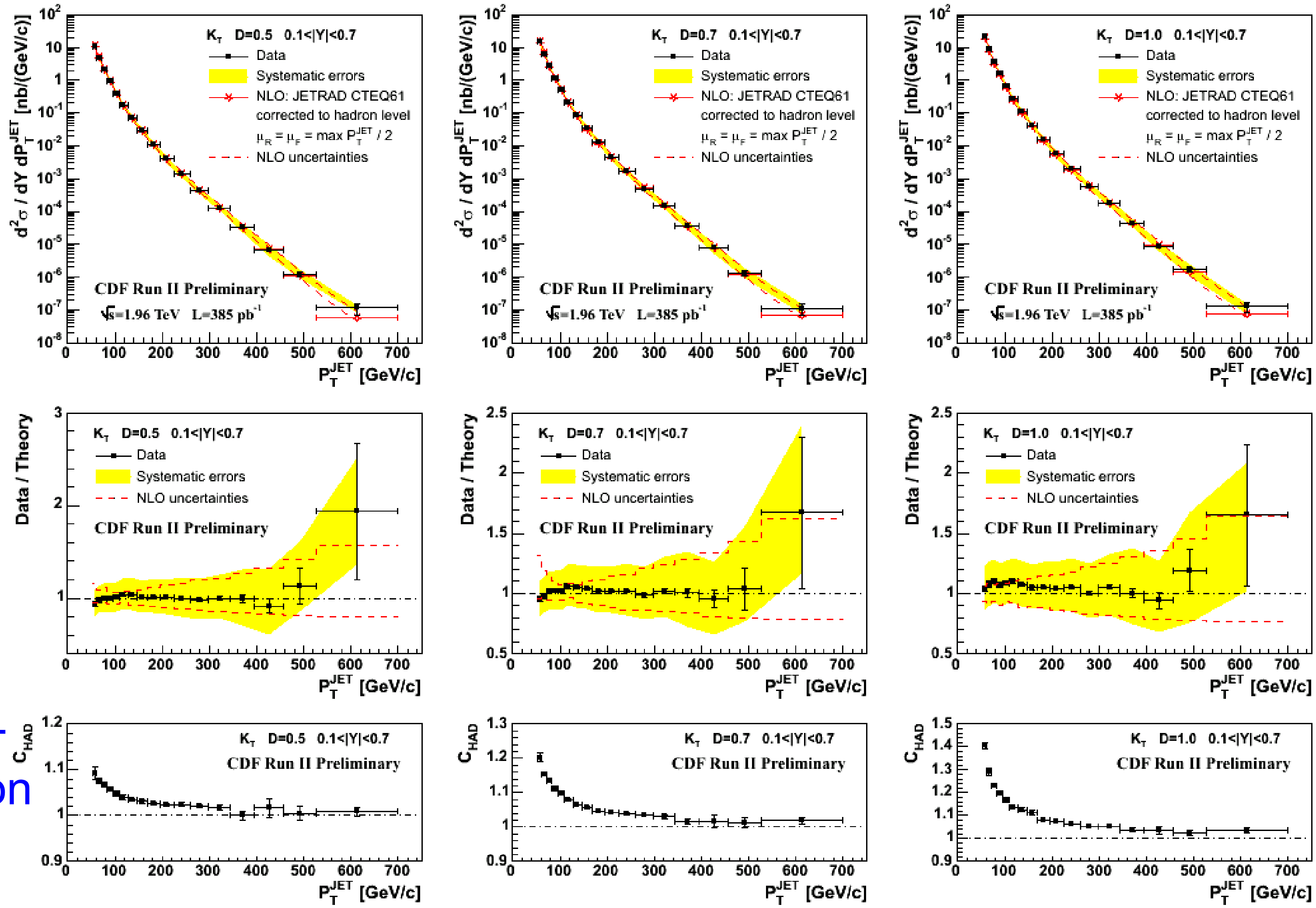
CDF: k_T jet cross section results

$$d_{ij} = \min(P^2_{T,i}, P^2_{T,j}) \frac{\Delta R^2}{D^2}$$

$$d_i = (P_{T,i})^2$$

k_T algorithm seems to work well at a hadron collider

underlying + hadronization correction



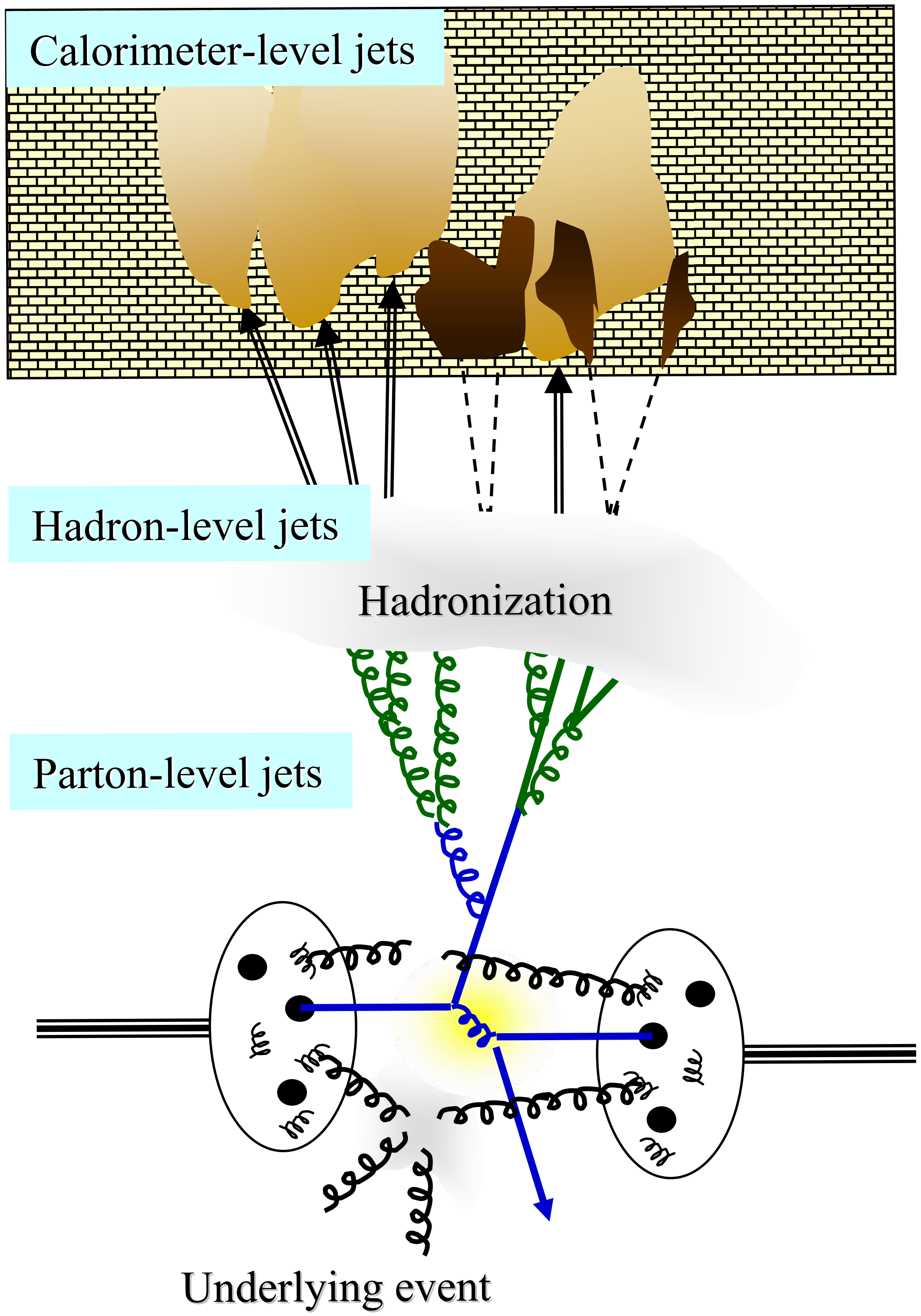
Jet
in experimental data

Outlines

- Motivation
- Jet function
- Resummation
- Jet energy profile
- Jet mass distribution
- Summary

arXiv: 1107.4535 [hep-ph]
1206.1344

Jet Production



- Jets are collimated spray of hadrons originating from quarks/gluons coming from the hard scattering

  (Jets are experimental signatures of quarks and gluons)

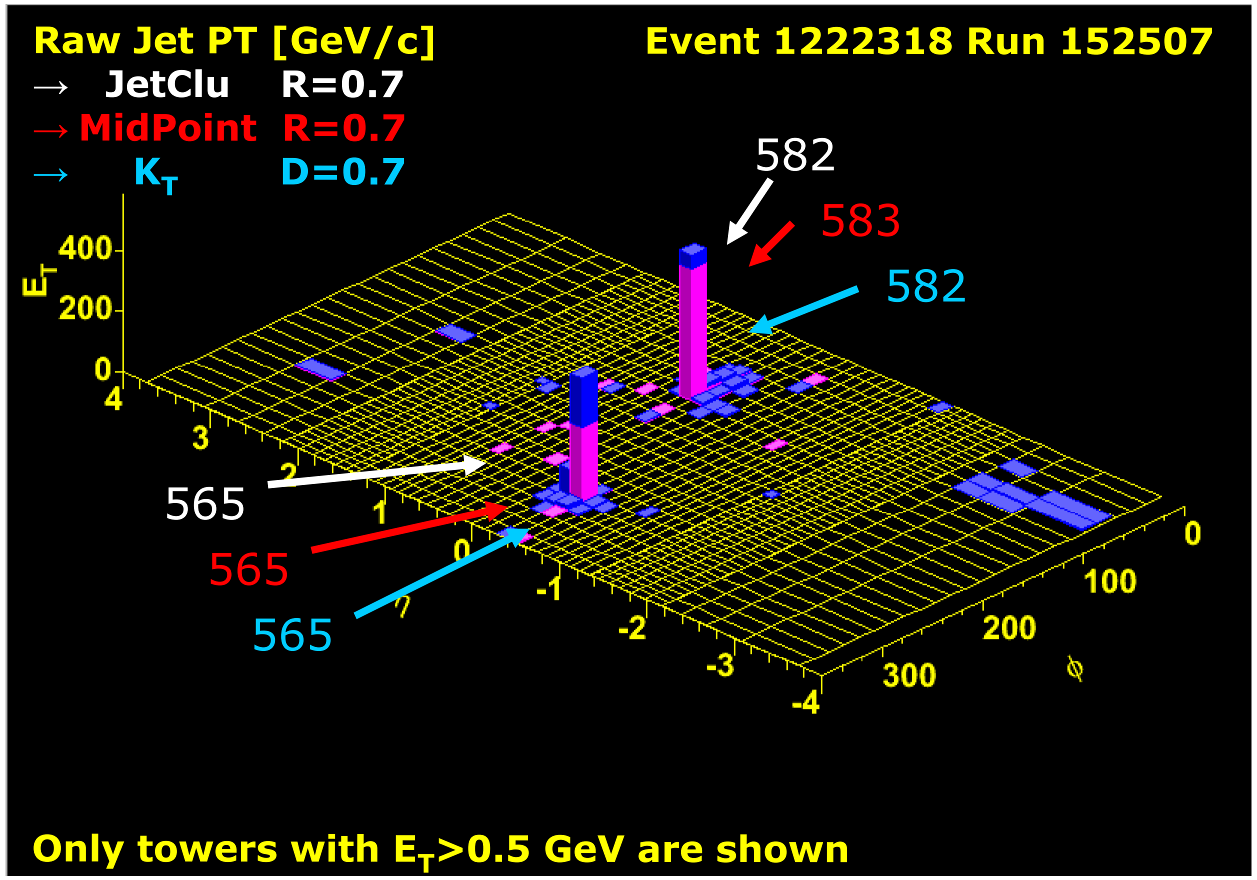
- Unlike photons, leptons etc, jets have to be defined by an algorithm for quantitative studies

- Need a well-defined algorithm that gives close relationship between calorimeter-level jets, hadron-level jets, and parton-level jets

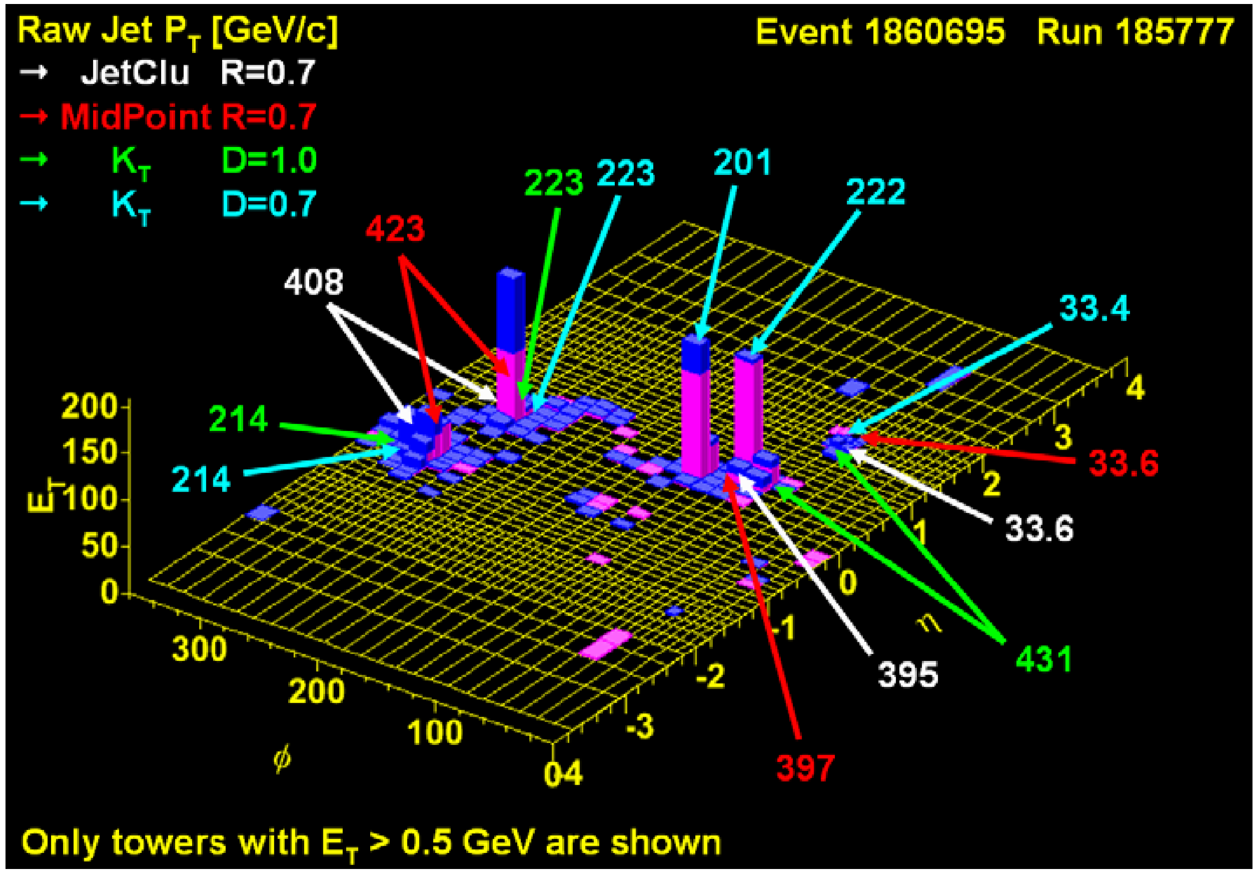
Jet Clustering Algorithms

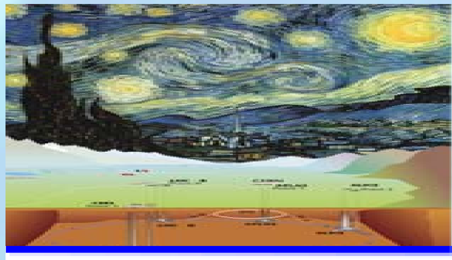
- Algorithms should be well-defined so that they map the experimental measurements with theoretical calculations as close as possible.
- Different algorithms with different parameters provide different sets of resulting jets.

"Simple" event
(all algorithms give essentially the same results)

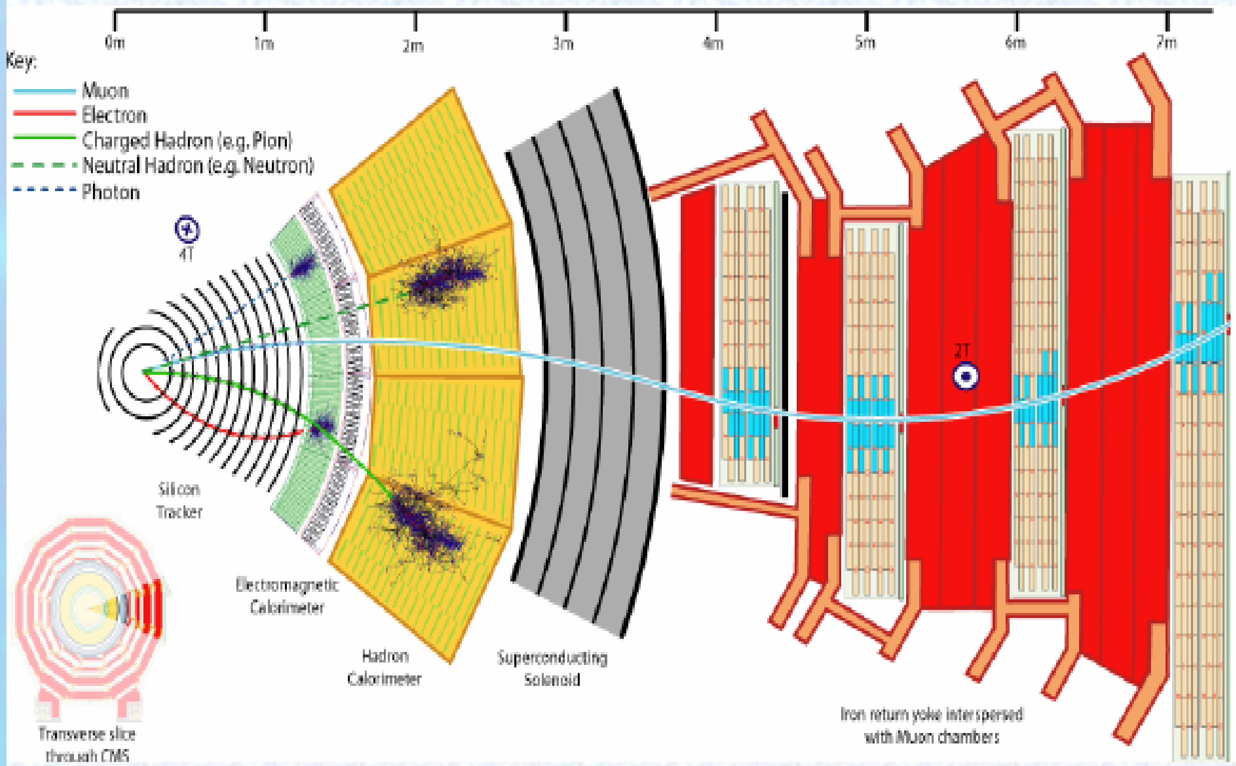


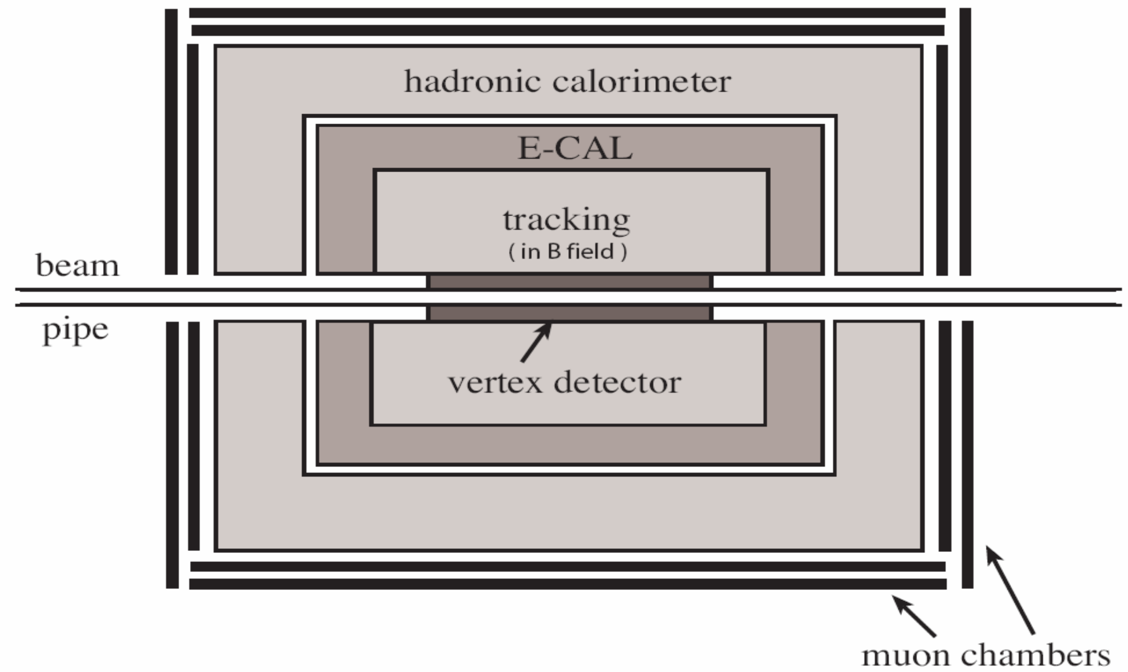
"Complicated" event
(Resulting jets depend on jet algorithms)





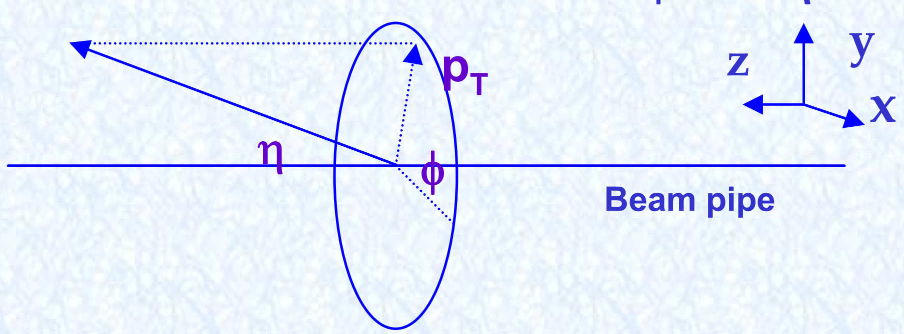
Objects at the LHC





- **objects**
  - Photons: no tack, energy in ECAL, no energy in HCAL
  - Electrons: tack, energy in ECAL, no energy in HCAL
  - Muons: tack, tack in the muon chamber
  - Jets: tracks and energy in the calorimeter
  - Missing transverse enegy (MET) : inferred from the conservation of momentum in a plane perpendicular to the beam direction

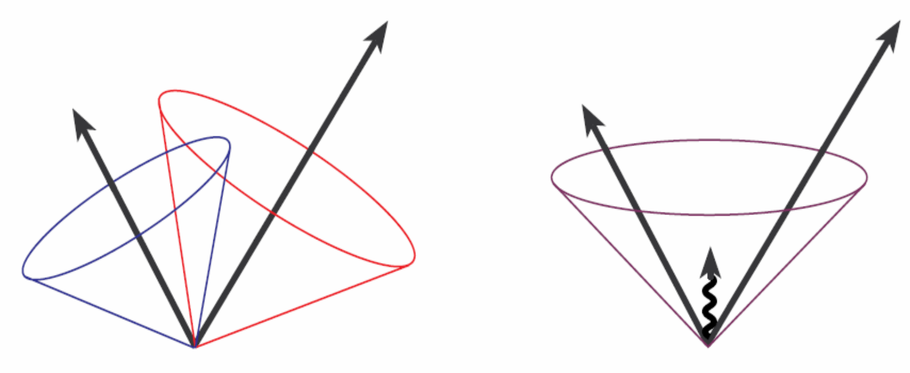
- **Typical variables**
  - Transverse momentum: p_T
  - Azimuth angle: ϕ
  - Pseudorapidity: $\eta = -\ln(tg(\theta/2))$
  - Relative isolation: $\Delta R = (\Delta \phi^2 + \Delta \eta^2)^{1/2}$



Jet "Definitions" - Algorithms at CDF

- Cone algorithms (JetClu, Midpoint)
  - Cluster objects based on their proximity in $y(\eta)$-ϕ space
  - Starting from seeds (calorimeter towers/particles above threshold), find stable cones (p_T-weighted centroid = geometric center).
  - Seeds have been necessary for speed, but source of infrared unsafety.
  - In Run II QCD studies, often use "Midpoint" algorithm, i.e. look for stable cones from middle points between stable cones → Infrared safety restored up to NNLO.
  - Stable cones sometime overlaps → merge cones when overlap > 75%

Infrared unsafety:
soft parton emission changes jet clustering



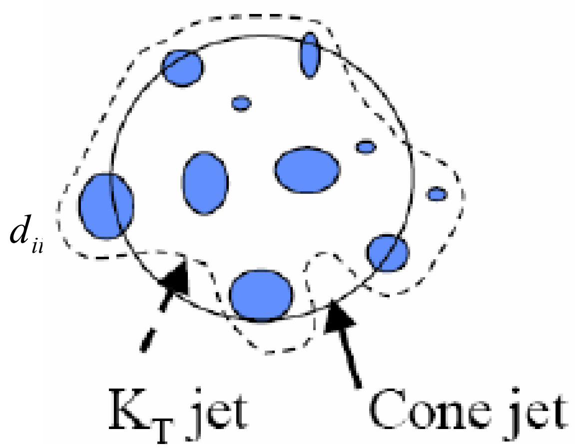
N.B., Recently a new version of seedless algorithm (SIScone) became available which is fast enough for practical use.

Jet "Definitions" - Algorithms at CDF

k_T algorithm

- Cluster objects in order of increasing their relative transverse momentum (k_T)
  - $d_{ii} = p^2_{T,i}, \quad d_{ij} = \min(p^2_{T,i}, p^2_{T,j}) \dfrac{\Delta R^2}{D^2}$

  until all objects become part of jets
- D parameter controls merging termination and characterizes size of resulting jets



- No issue of splitting/merging. Infrared and collinear safe to all orders of QCD.
- Every object assigned to a jet: concerns about vacuuming up too many particles.
- Successful at LEP & HERA, but relatively new at the hadron colliders
  - More difficult environment (underlying event, multiple $p\bar{p}$ interactions…)

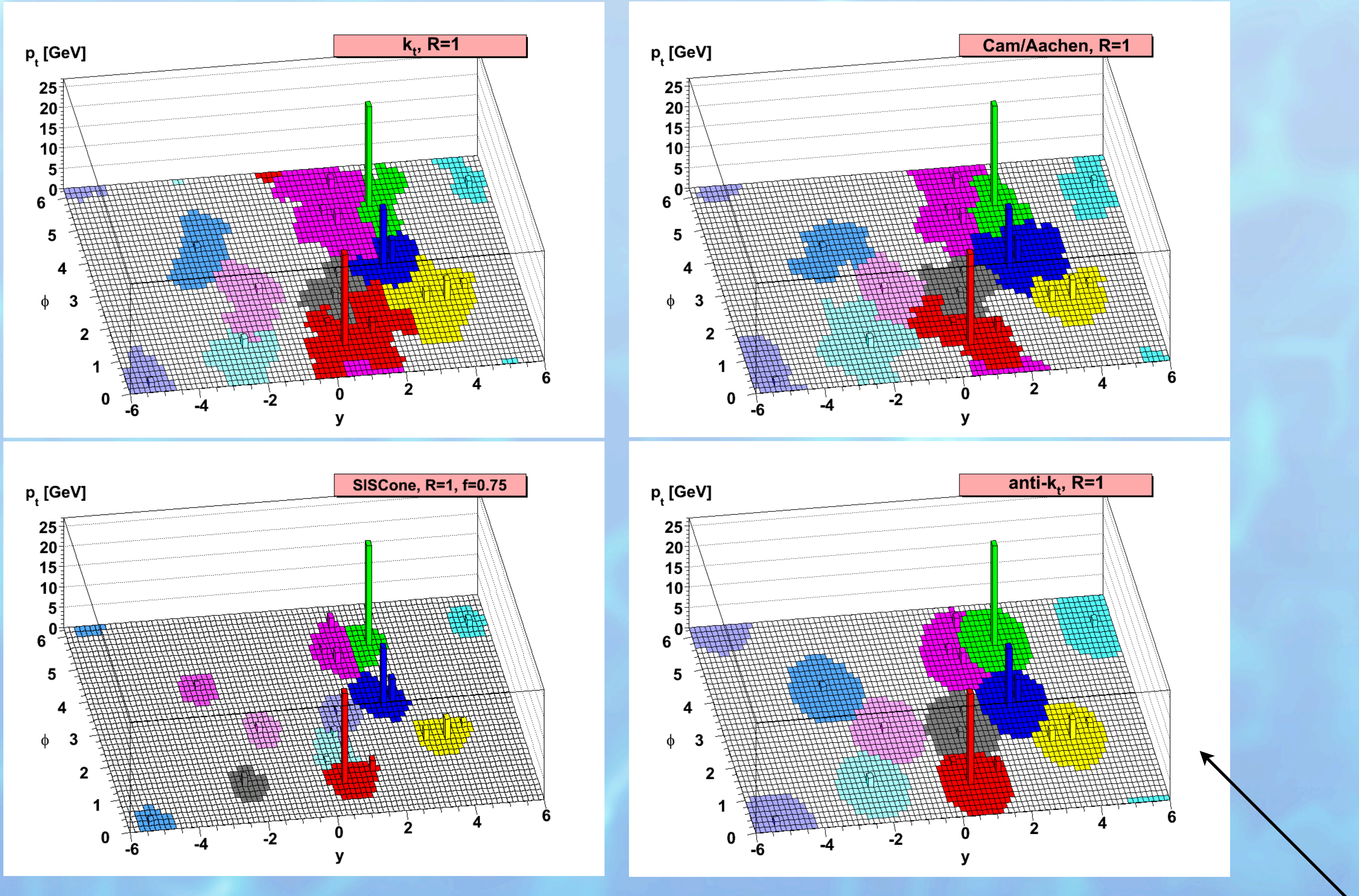
Other clustering algorithm

- p=1
  - the regular k_T jet algorithm
- p=0
  - Cambridge-Aachen algorithm
- p=-1
  - anti-k_T jet algorithm →
  - Cacciari, Salam, Soyez '08
  - also P-A Delsart '07
  - soft particles will first cluster with hard particles before clustering among themselves
  - no split/merge
  - leads mostly to constant area hard jets

$$d_{ij} = \min\left(p_{T,i}^{2p}, p_{T,j}^{2p}\right)\frac{\Delta R_{ij}^2}{D^2}$$

$$d_{ii} = p_{T,i}^{2p}$$

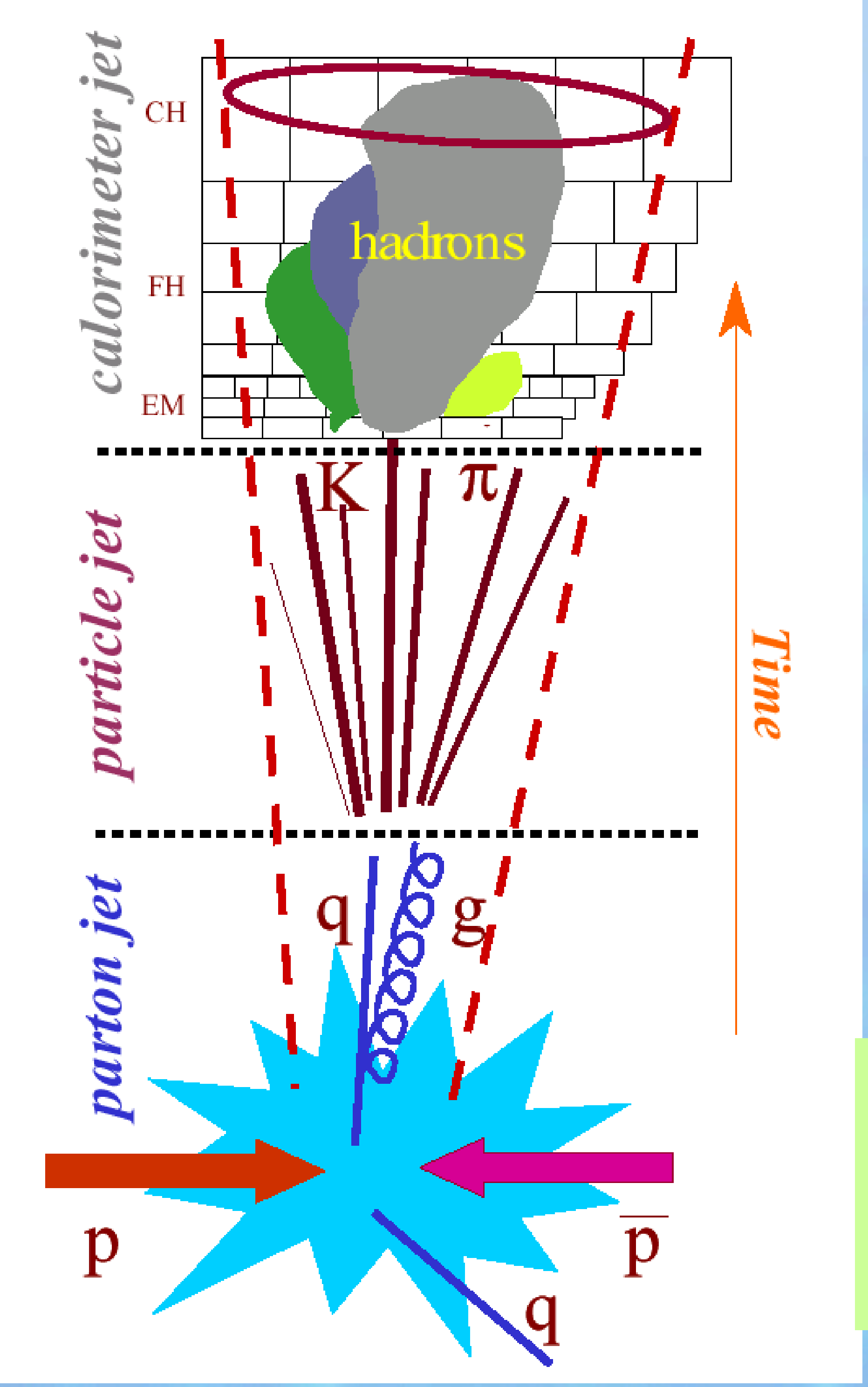
- #1 algorithm for ATLAS, CMS



Anti-Kt jet clustering algorithm

arXiv: 0802.1189
Cacciari, Salam, Soyez

Jet Finding

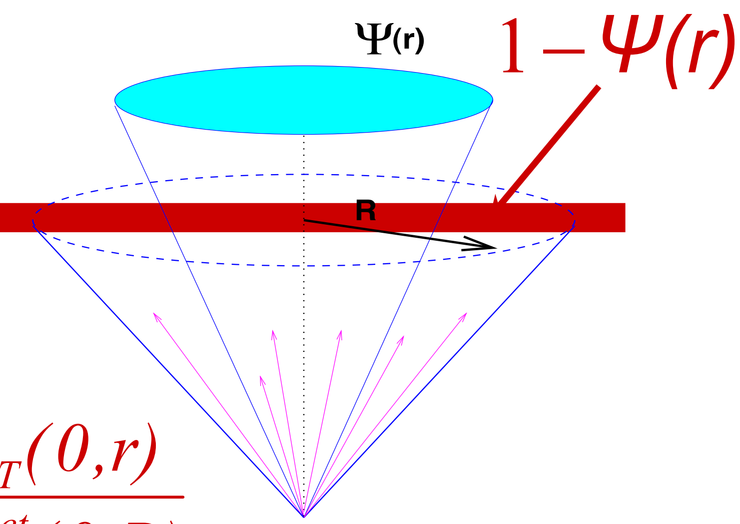


- **Calorimeter jet (cone)**
  - jet is a collection of energy deposits with a given cone *R*: $R = \sqrt{\Delta\varphi^2 + \Delta\eta^2}$
  - cone direction maximizes the total E_T of the jet
  - various clustering algorithms

→ correct for finite energy resolution
→ subtract underlying event
→ add out of cone energy

- **Particle jet**
  - a spread of particles running roughly in the same direction as the parton after hadronization

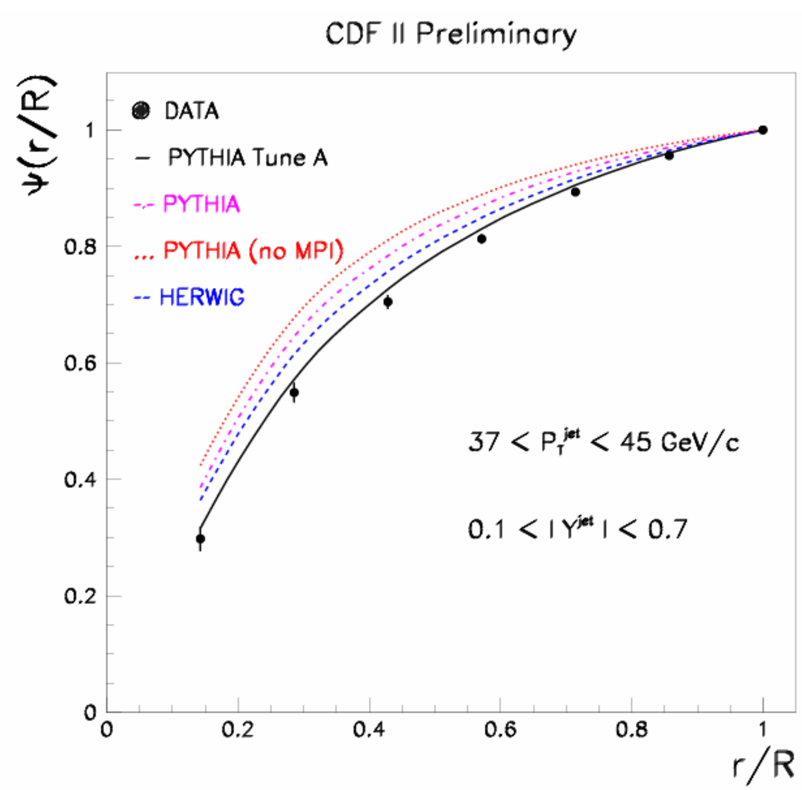
Jet Fragmentation Studies

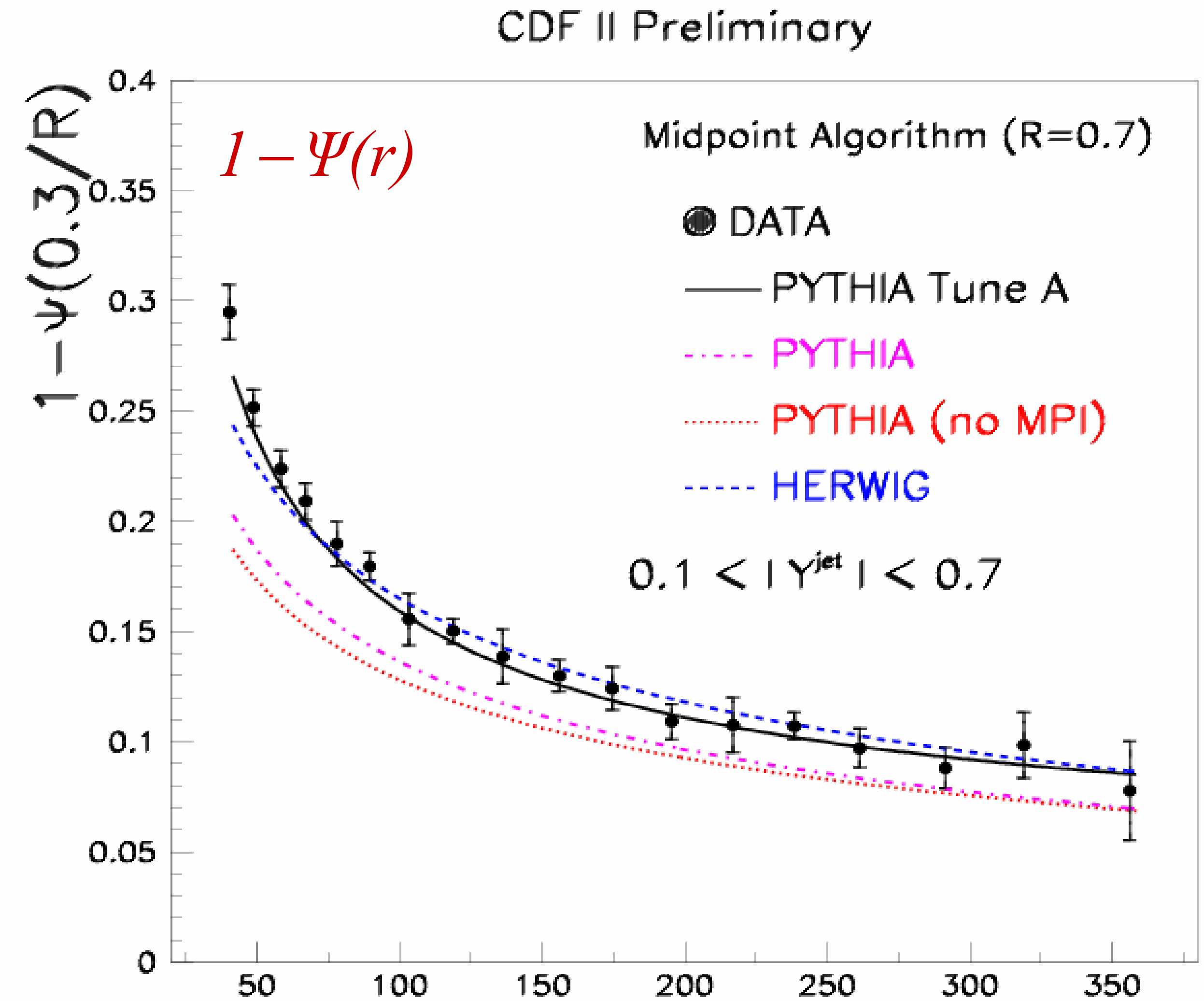


Need to simulate jets properly: particle composition, multiplicity, momentum distribution etc

$$\Psi(r) = \frac{1}{N_{jets}} \Sigma_{jets} \frac{p_T(0,r)}{p_T^{jet}(0,R)}$$

e.g. **2 hadrons with $p_T = 50$ GeV/c**
$\neq$ 20 hadrons with $p_T = 5$ GeV/c
due to calorimeter non-linearity





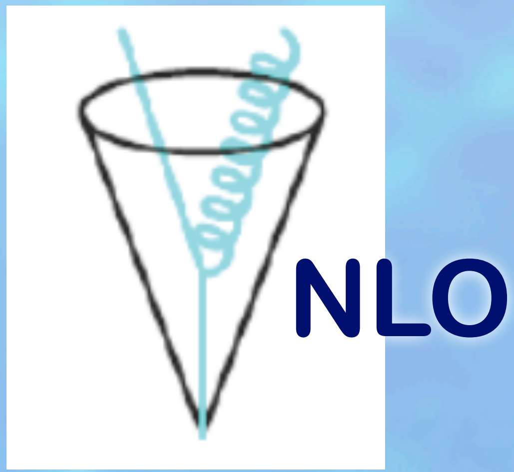
Tuned MC, PYTHIA Tune A (enhanced ISR + MPI), describes the data

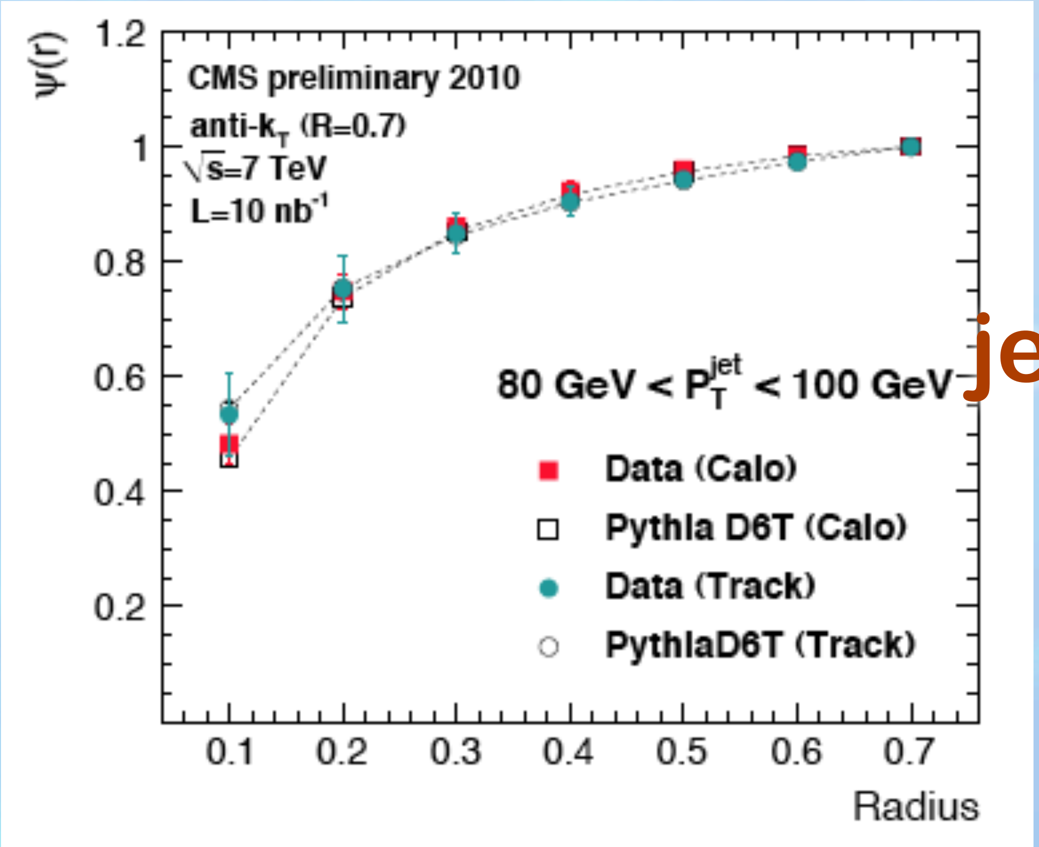
 We know how to model the jet fragmentation reasonably well !!

Various Theoretical Predictions

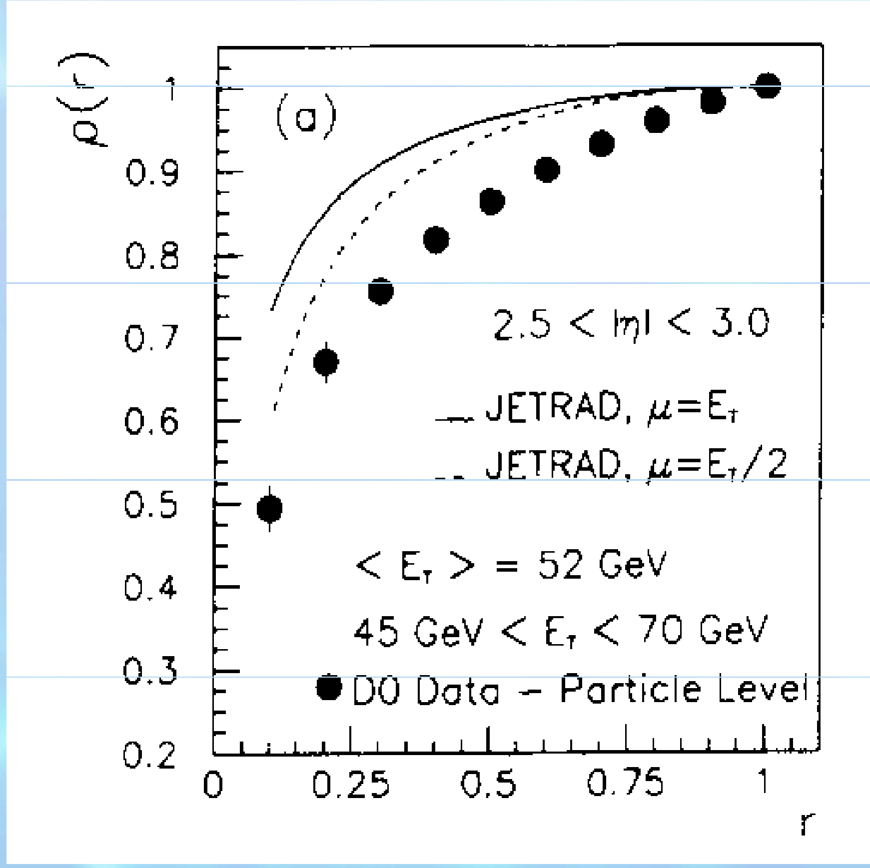
Various Theoretical Predictions

- **Event Generators**: leading log radiations, hadronization, underlying events, etc.
- **Fixed order QCD calculation**: finite number of soft/collinear radiations
- **Resummation**: all order soft/collinear radiations

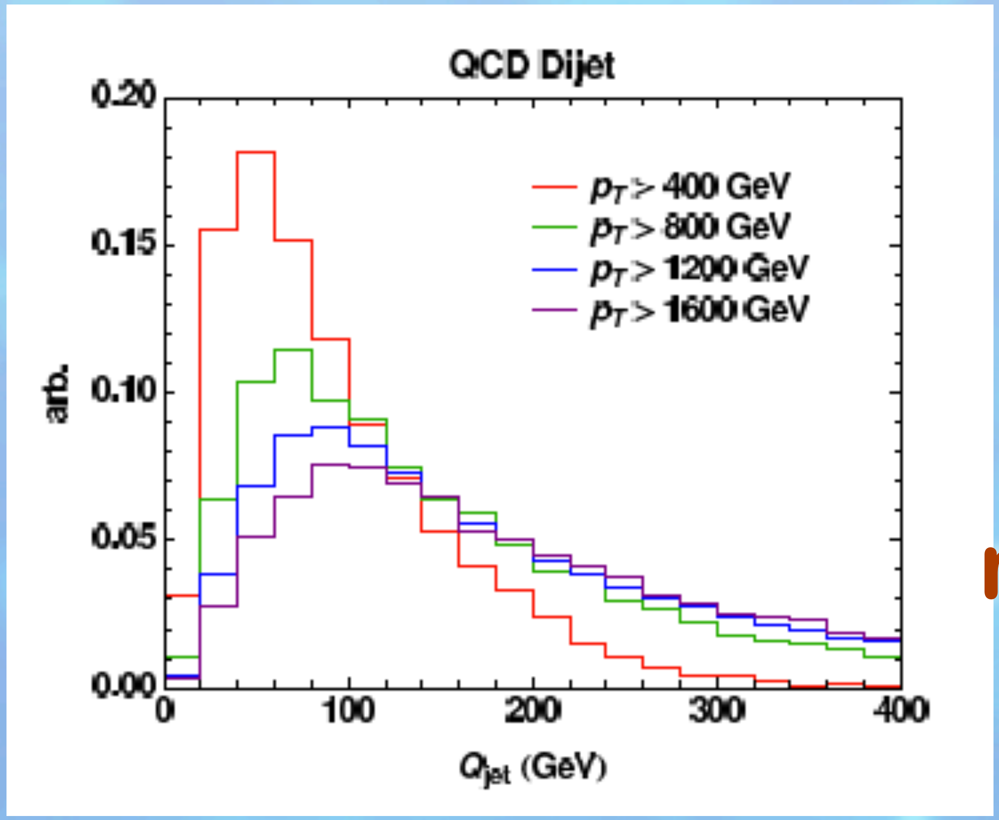




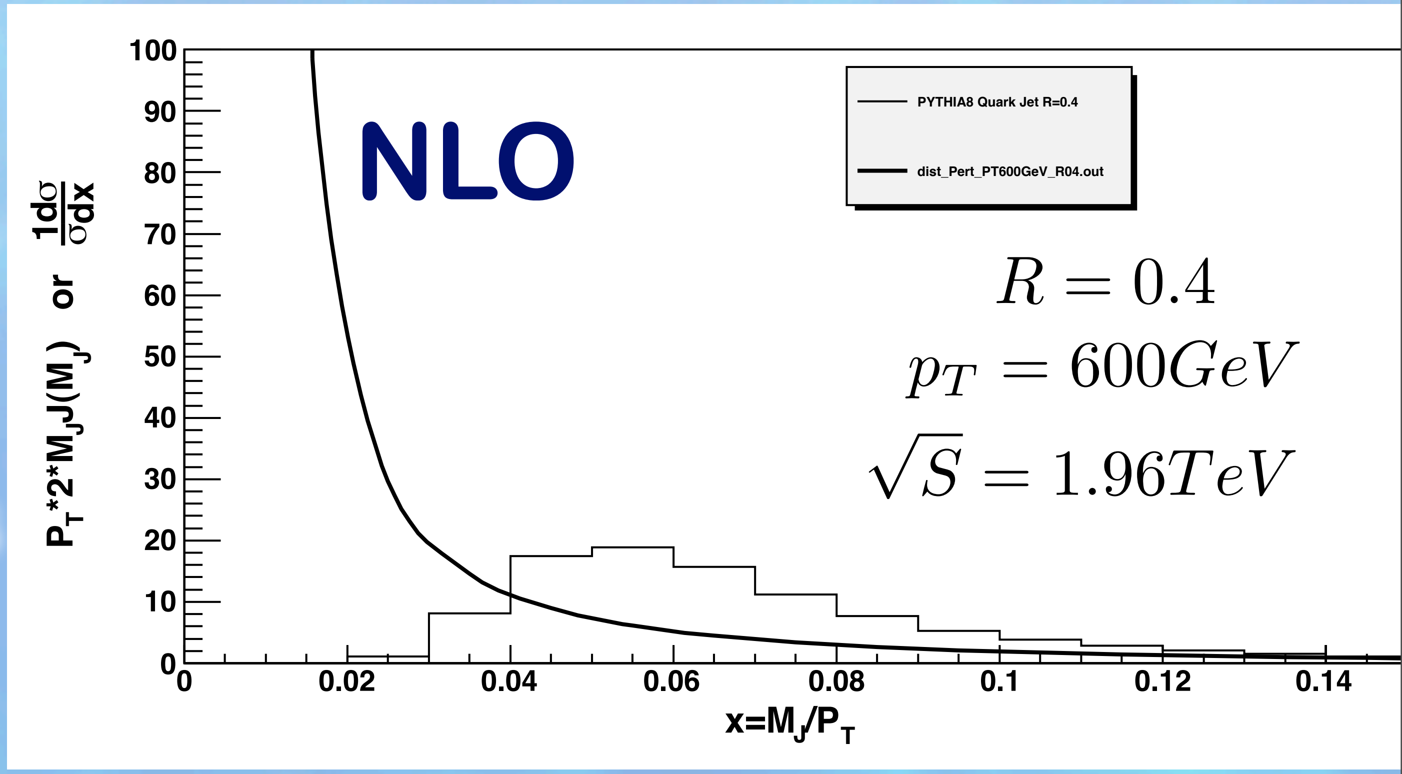
jet energy profile



D0 Collaboration/Physics Letters B 357(1995) 500-508



jet mass



Thaler & Wang, arxiv:0806.0023

Our resummation results

- At the first time that **pQCD resummation approach** is established to investigate jets.

- Improve predictions on Jet **energy profile** and jet **mass distribution** to describe CDF and CMS data.

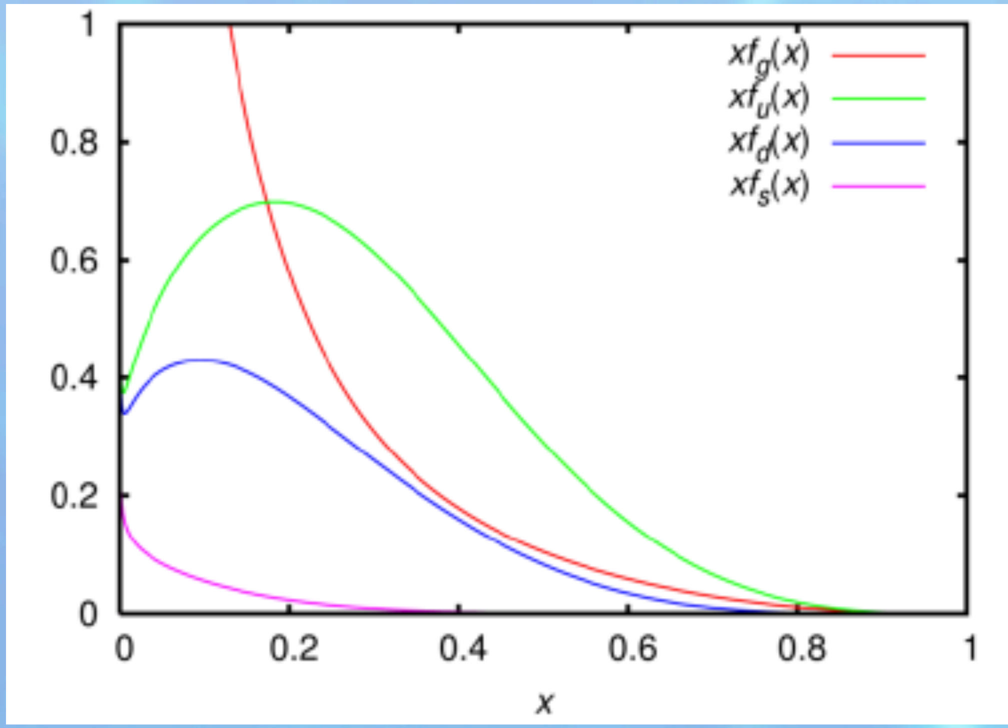
Factorization Theorem

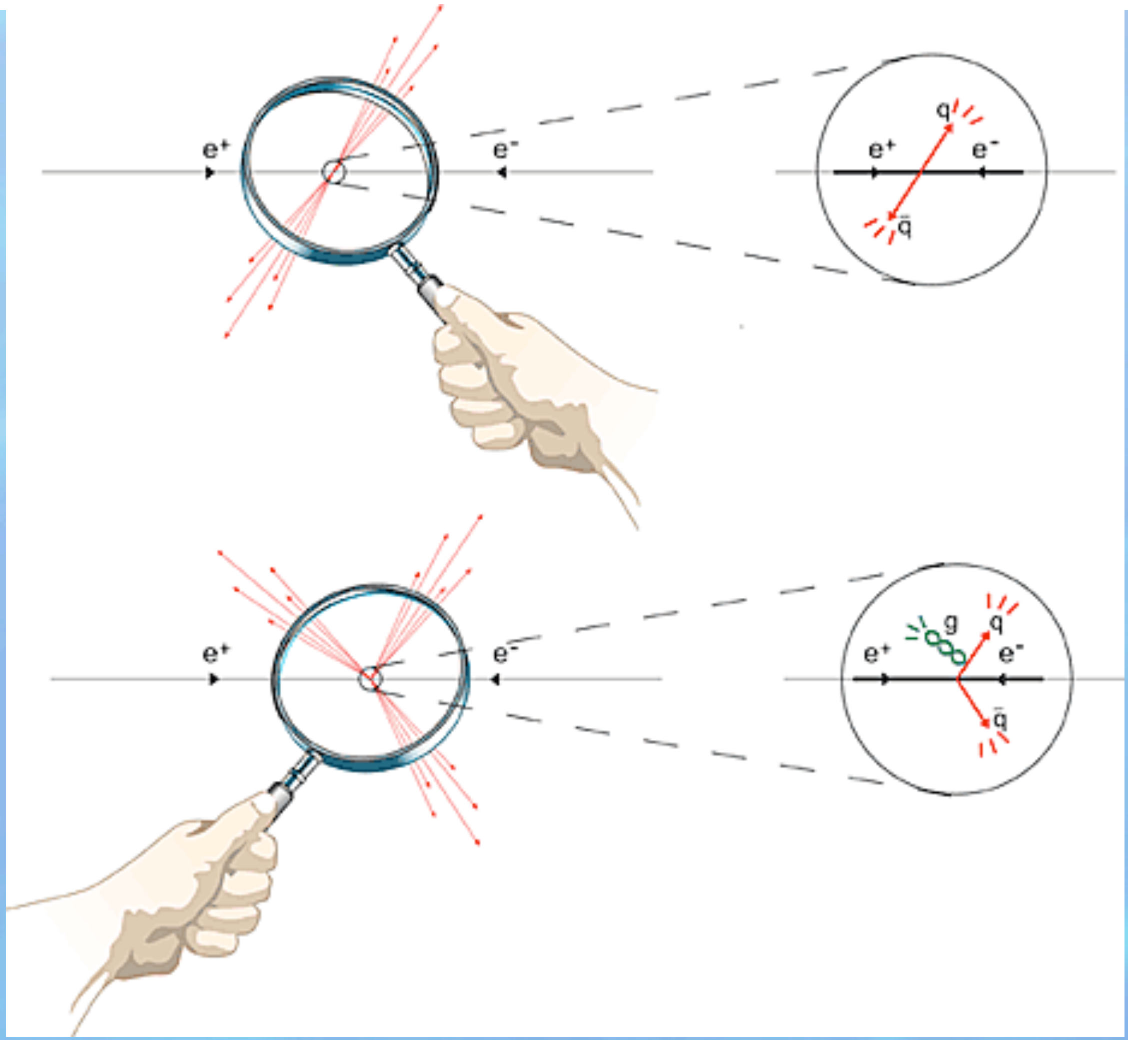
$$\sigma_{hh'} = \sum_{i,j} \int_0^1 dx_1 dx_2\, \phi_{i/h}(x, Q^2)\, H_{ij}\left(\frac{Q^2}{x_1 x_2 S}\right) \phi_{j/h'}(x_2, Q^2)$$

Nonperturbative, but universal, hence measurable

Infrared safe (IRS), calculable in pQCD

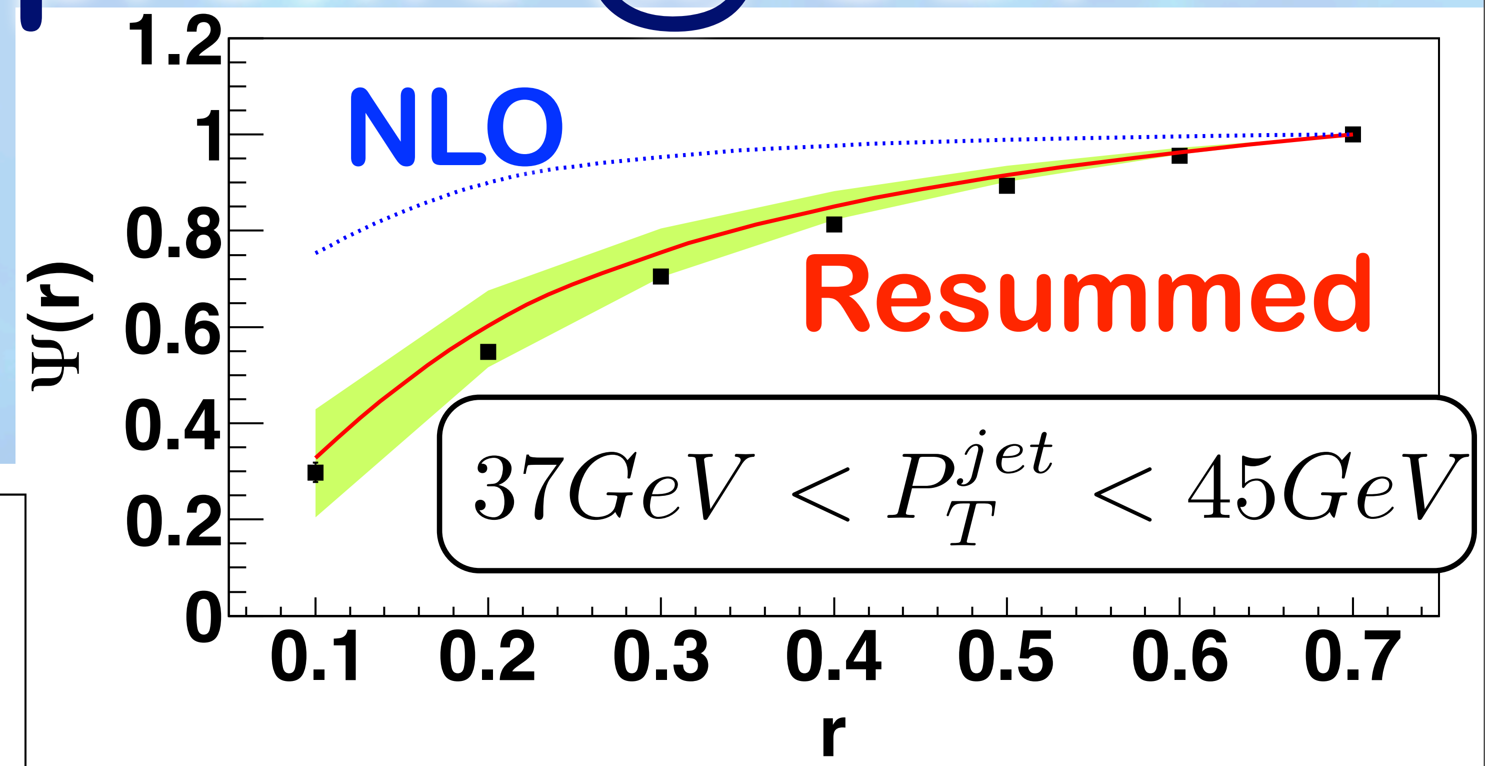
CTEQ
MSTW
NNPDF

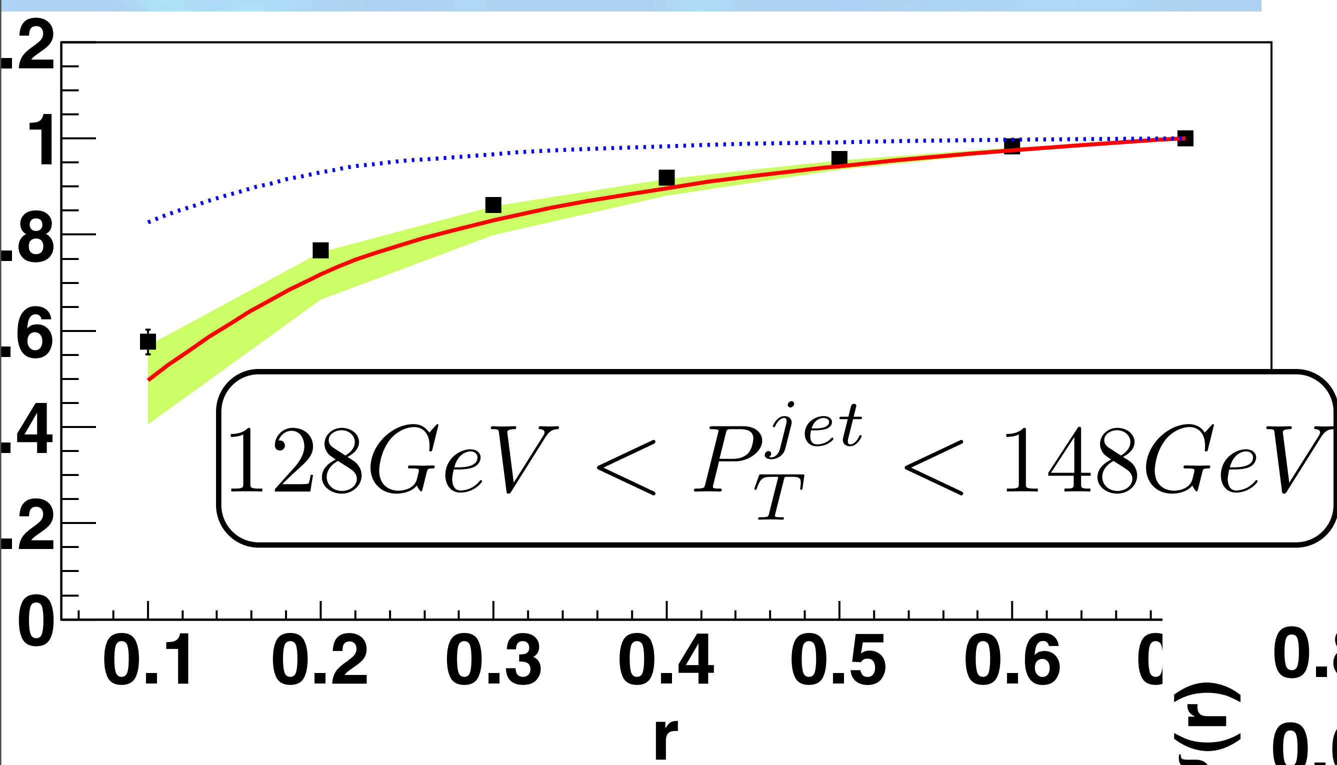


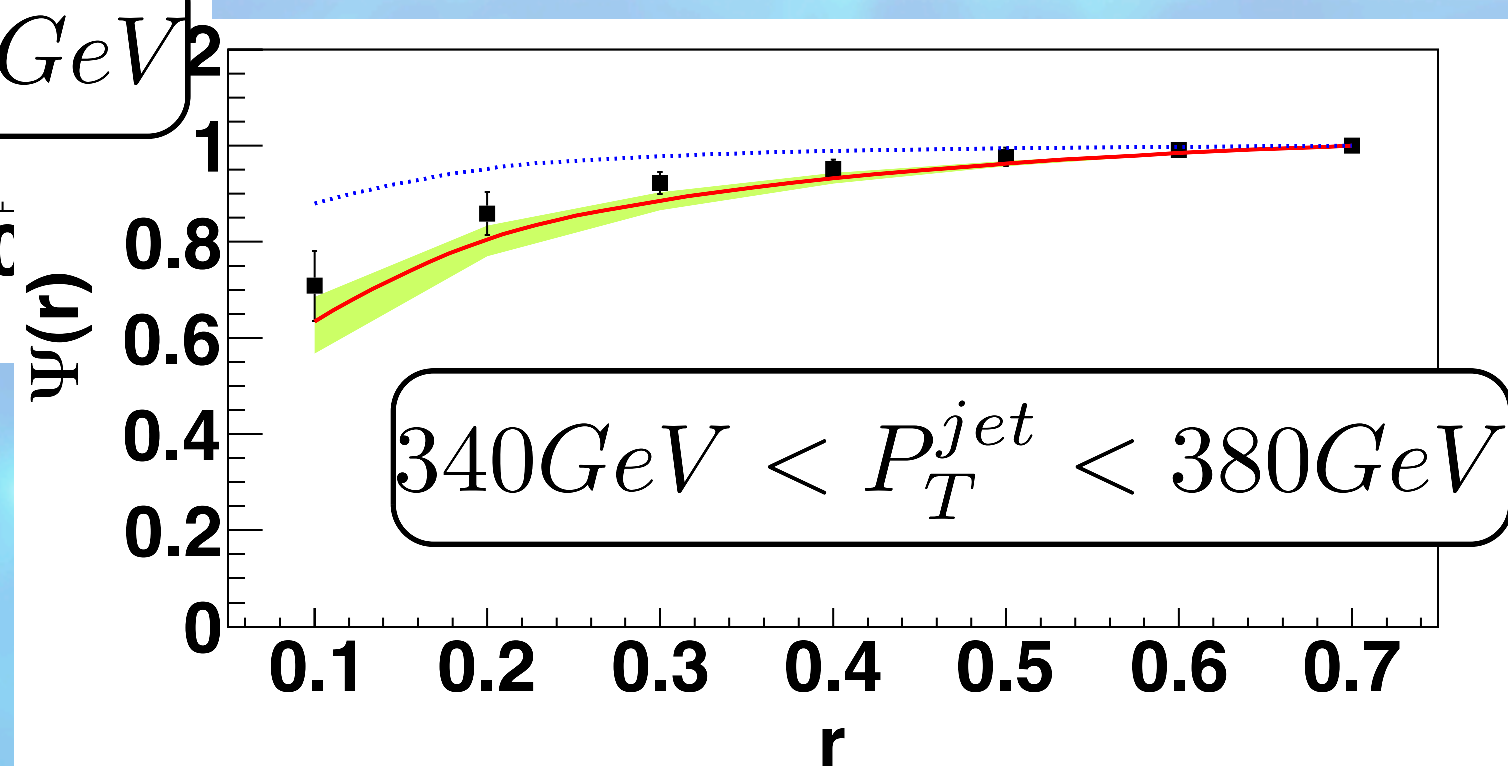


Jet energy profile @ CDF

$$\Psi_q(r) \equiv \frac{\bar{J}^E_q(1, P_T, \nu^2_{\rm fi}, R, r)}{\bar{J}^E_q(1, P_T, \nu^2_{\rm in}, R, R)},$$

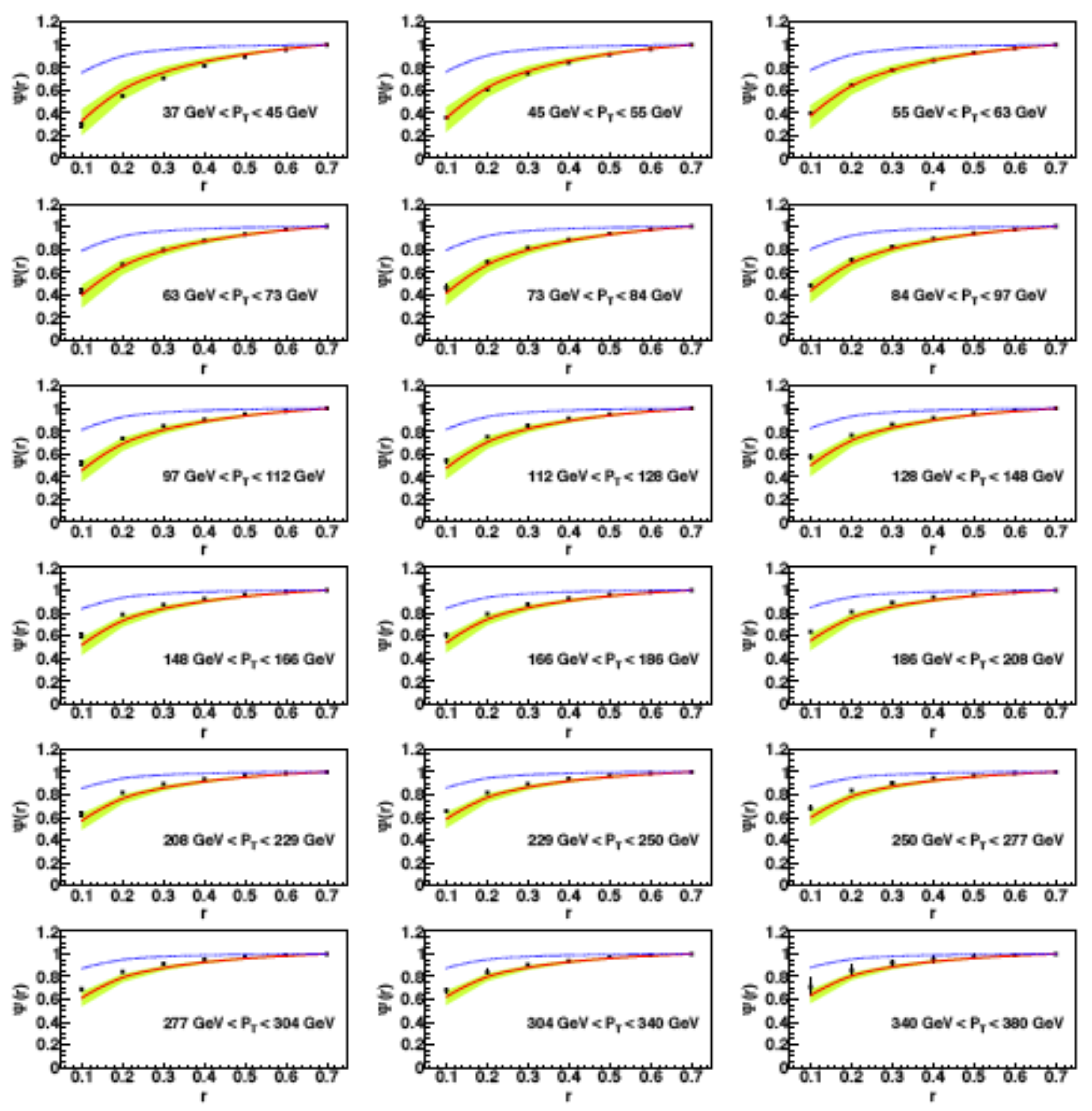




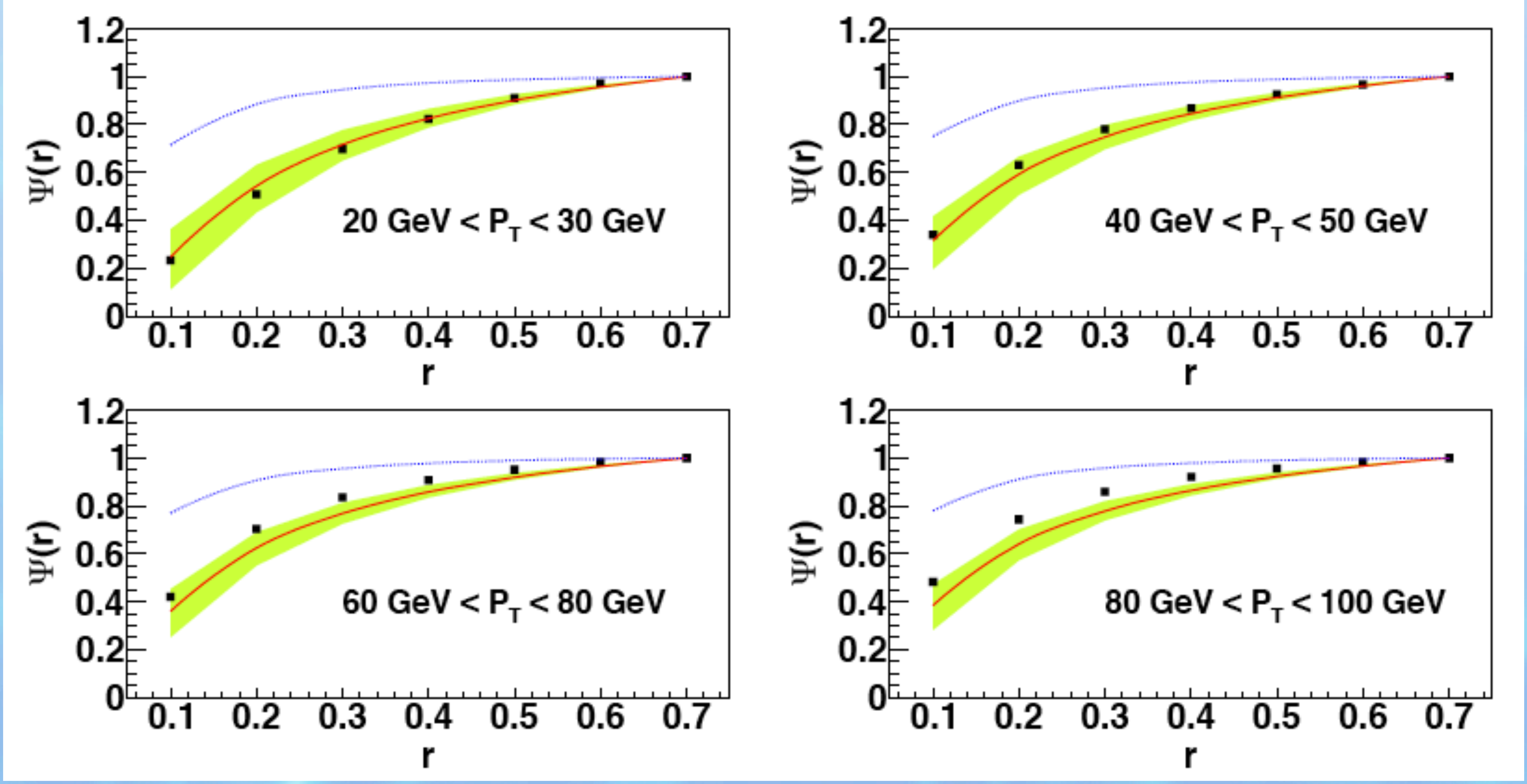


CDF data
PRD71(2005)112002

Gluon jet dominates in low pT region.


Ψ(r)
r
37 GeV < P_T < 45 GeV
45 GeV < P_T < 55 GeV
55 GeV < P_T < 63 GeV
63 GeV < P_T < 73 GeV
73 GeV < P_T < 84 GeV
84 GeV < P_T < 97 GeV
97 GeV < P_T < 112 GeV
112 GeV < P_T < 128 GeV
128 GeV < P_T < 148 GeV
148 GeV < P_T < 166 GeV
166 GeV < P_T < 186 GeV
186 GeV < P_T < 208 GeV
208 GeV < P_T < 229 GeV
229 GeV < P_T < 250 GeV
250 GeV < P_T < 277 GeV
277 GeV < P_T < 304 GeV
304 GeV < P_T < 340 GeV
340 GeV < P_T < 380 GeV

Jet energy profile @ CMS



Predicted by perturbative resummation calculation, and non-perturbative physics input is not needed.

Dependence on pT@ LHC