How does a Full Event Generator Work?

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Outline

Goal: Learn how a full event generator, such as PYTHIA, approximates pQCD contribution in predicting event rates and distributions. What's its strength and weakness as compared to a fixed (higher) order pQCD calculation?

Reference: CTEQ School Lectures

http://www.phys.psu.edu/~cteq/

Event Generators: Introduction

- Theorists calculate S-matrix elements
  - Valid to a fixed order in perturbation theory
  - In- and Out-states are plane waves of partons
    - 2 partons in / few partons out
    - quark & gluon states are colored / quarks have fractional charge
  - Some predictions are ill-behaved
    - Inclusive predictions blur n and n+1 parton states
- Experimentalists measure Objects in detector
  - No distinction between Perturbative and Non-Perturbative
  - In- and Out-states are particles
    - “beam” on “target” / many particles out plus remainder of beam
    - particles are color neutral, integer charged
  - Observable quantities are finite
    - some of the event is “lost”, e.g. down the beam pipe

Event Generators Bridge this Gap

- **Describe the complicated Experimental Observable in terms of a chain of simpler, sequential processes**
  - **Some components are perturbative**
    - **hard scattering, parton showering, some decays, …**
  - **Others are non-perturbative and require modelling**
    - **hadronization, underlying event, k_T smearing, …**
    - **models are not just arbitrary parametrizations, but have semi-classical, physical pictures**
    - **Sometimes as important as the perturbative pieces**
  - **The Chain contains complicated integrals over probability distributions**
    - **Positive Definite**
    - **Rely heavily on Monte Carlo techniques to choose a history**
  - **Final Output is E,p,x,t of stable and quasi-stable particles**
    - **Ready to Interface with Detector Simulations**

Experimental Workhorses

- Relied upon by experiments
  - Correct for acceptance after cuts
  - Jet Energy corrections (out of cone)
  - Calorimeter response (e/h)
  - e/γ isolation
  - Behavior of Backgrounds after tight kinematic cuts
  - …
  - Planning of future facilities
- Often treated as
  - Goal of these lectures: Open Up the Box

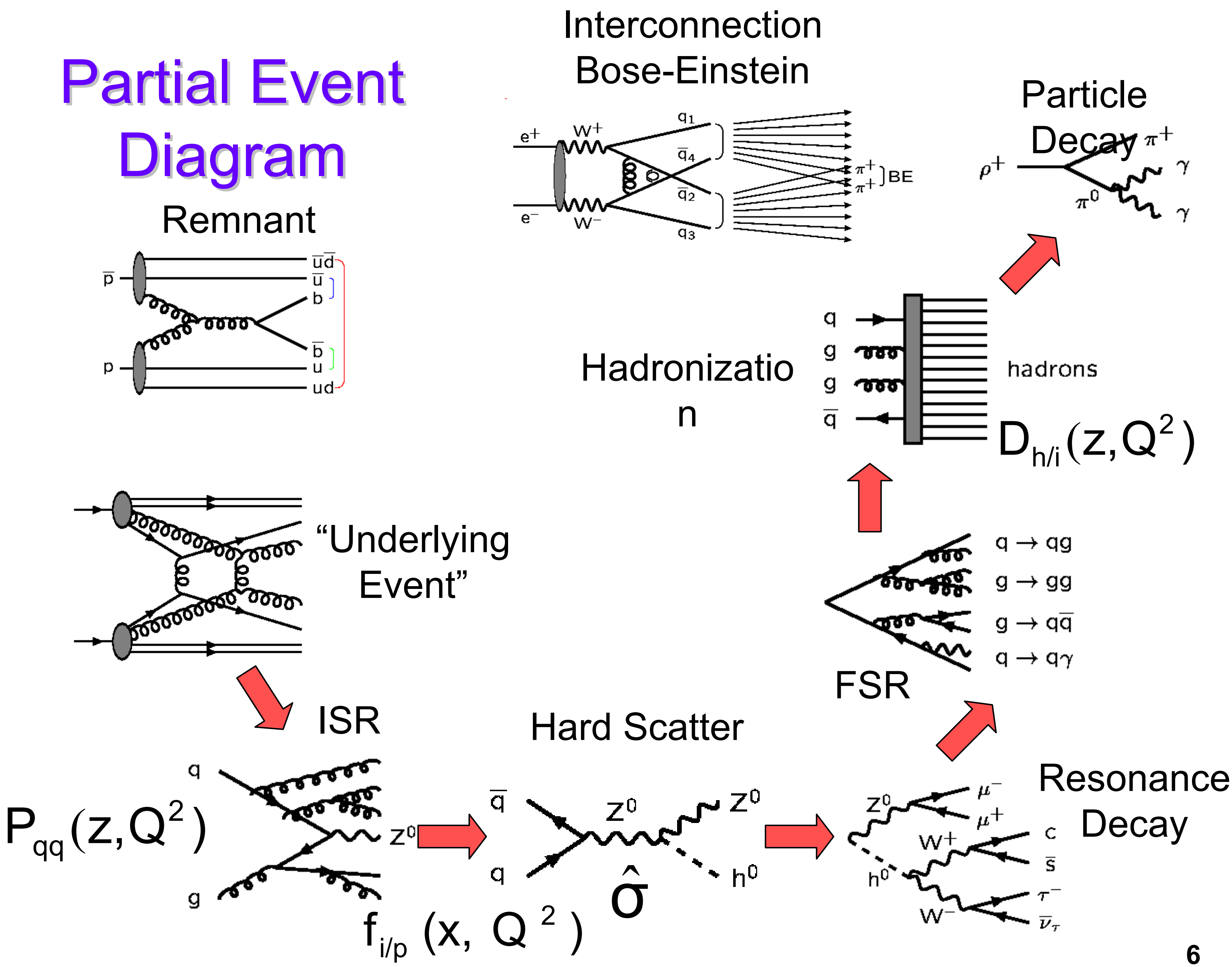
Deconstruction of an Event

- Two beam particles from bunches have a central collision
  - Partonic structure (flavor, energy) set by distribution functions
  - Possibly several collisions per crossing
- One parton from each beam may branch, e.g. $q \to qg$
  - Builds up initial-state shower
  - Includes irresolvable branchings
- Two incoming partons interact
  - Hard scattering $2 \to 1,2,\ldots$
  - Process determines character of event (energy scale, color flow,…)
- Hard scattering may produce short-lived resonances
  - color neutral W/Z/H, heavy quarks, etc. decay promptly to partons
- Outgoing partons branch
  - Builds up final-state shower

Deconstruction (cont.)

- **Other partons from same beam particles may undergo semi-hard interactions**
  - **models for "underlying event"**
- **Beam remnants propagate into final state**
  - **spin, color, and flavor structure**
- **Quarks and gluons fragment to color neutral hadrons**
  - **Normally breaks down into a set of separate color-singlets**
  - **Color rearrangement or Bose-Einstein effects may complicate the picture**
- **Many of the produced hadrons are unstable and decay further**
  - **displaced vertices/impact parameters are generated**

Partial Event Diagram



Hard Scattering

- Characterizes the rest of the event
  - Sets high energy scale
  - Fixes quantum # flow

$$2 \rightarrow 2 \text{ Scattering}$$

$$p_1 = E_{beam}(x_1, 0, 0, x_1) \qquad p_2 = E_{beam}(x_2, 0, 0, -x_2)$$

$$\hat{s} = x_1 x_2 4E_{beam}^2, \quad \tau = x_1 x_2, y = \frac{1}{2}\ln\frac{x_1}{x_2}$$

$$\hat{t} = -\frac{1}{2}\left\{ \hat{s} - m_3^2 - m_4^2 - \hat{s}\beta_{34}\cos\hat{\theta} \right\}$$

$$\hat{s} + \hat{t} + \hat{u} = m_3^2 + m_4^2$$

$$\sigma = \iiint \frac{d\tau}{\tau} dy d\hat{t}\, x_1 f(x_1, Q^2) x_2 f(x_2, Q^2) \frac{d\hat{\sigma}}{d\hat{t}}$$

$$Q^2 = K\,\hat{s},\ K \sim 1 \text{ or similar}$$

Hard Scattering: Resonance Production

$$\sigma = \iint \frac{d\tau}{\tau} dy\, x_1 f(x_1, Q^2) x_2 f(x_2, Q^2)\, \sigma(\hat{s})$$

Narrow Width Approximation:

$$\delta(\hat{s}-M^2) \rightarrow \frac{M\Gamma}{\pi} \frac{1}{(\hat{s}-M^2)^2 + (\Gamma M)^2}$$

- **Hadron Colliders**
  - **f(x)~1/x for small x, so low-mass tail enhanced & high-mass tail depleted**
- **e^+e^- Colliders**
  - **f(x)~δ(1-x), so opposite effect occurs**
- **Requires careful treatment of Γ energy dependence**

$$m\Gamma \rightarrow \hat{s}\Gamma / m$$

Other "Hard" Details

- **Running couplings**
- **Resonance production in 2→2 processes**

$$\int dm^2\ \delta(m^2 - m_R^2)\ \Rightarrow \int dm^2\ \frac{1}{\pi}\frac{m_R \Gamma_R}{(m^2 - m_R^2)^2 + m_R^2 \Gamma_R^2}$$

- **QCD Processes**
  - **Divergent: $P_T > P_0$ or $P_T^{-2} \rightarrow (P_T^2 + P_0^2)^{-1}$**
  - **Color Flow to $1/N_C$ (important later)**

$$q_{in} g_{in} \rightarrow q_{out} g_{out} \qquad |M|^2 \propto \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} - \frac{4}{9}\left(\frac{\hat{s}}{\hat{u}} + \frac{\hat{u}}{\hat{s}}\right)$$

$$\overbrace{q_{in} g_{in}}^{A}\ \frac{4}{9}\left(2\frac{\hat{u}^2}{\hat{t}^2} - \frac{\hat{u}}{\hat{s}}\right) \qquad \overbrace{q_{in} g_{out}}^{B}\ \frac{4}{9}\left(2\frac{\hat{s}^2}{\hat{t}^2} - \frac{\hat{s}}{\hat{u}}\right) \qquad \frac{1}{9}\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \Rightarrow \overbrace{\frac{1}{9}\frac{\hat{u}^2}{\hat{t}^2}}^{A} + \overbrace{\frac{1}{9}\frac{\hat{s}^2}{\hat{t}^2}}^{B}$$

Monte Carlo Event Generators

- Improving the Physics complicates the Numerics
  - Difficult Integrands
  - Many dimensions
- Well-suited to Monte Carlo methods
  - Integrands are positive definite
    - Normalize to be probability distributions
  - Hit-or-Miss
    - Test integrand to find maximum weight W_{MAX} (or just guess)
    - Calculate weight W at some random point
    - If $W > r\, W_{MAX}$, then keep it, otherwise pick new W
    - Sample enough points to keep error small
  - Can generate events like they will appear in an experiment
    - $N = \sigma[Xb]\; L[Xb^{-1}]$
- (N)NLO QCD programs are not event generators
  - Not positive definite (Cancellations between N and N+1)
  - Superior method for calculating suitable observables

The Rest

- Given a generator for the hard scattering, a full event history can be chosen
  - The rest of the event can be described by positive probability distributions
    - Cancellation between virtual and real effects are absorbed into a resolution parameter << anything observable
  - Beam particles fluctuate into partons with P=1
  - In-partons evolve from some parents with P=1
  - Out-partons evolve into daughters with P=1
  - Final state partons hadronize with P=1
- Ignores Quantum Mechanical interference between different steps
  - Essence of the Factorization Theorem

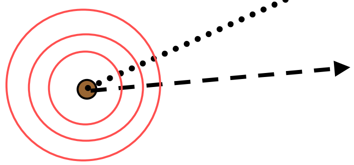
Parton Showering

- The Hard Scattering sets a high scale Q
  - $\lambda \sim 1/Q$ of probe must be small to resolve partons inside hadrons
  - Structure $f(x,Q^2)$ or fragmentation $D(x,Q^2)$ functions, couplings $\alpha_I(Q^2)$, etc. are evaluated at Q
    - Asymptotic states have a scale $Q_0 \sim 1$ GeV
  - Incoming/Outgoing partons are highly virtual
    - How do incoming partons acquire mass$^2 \sim -Q^2$
      - INITIAL STATE RADIATION (ISR)
    - How do outgoing partons approach the mass shell
      - FINAL STATE RADIATION (FSR)
  - Heuristic indication that "traditional" calculations based on a small number of Feynman diagrams are incomplete
    - EXPERIMENTALLY, additional jet structure is observed and important
  - Parton Showering Monte Carlos are an approximation to high-order, perturbative QCD, which is itself an approximation to "true QCD"

Parton Showering: More Motivation

Semi-classical description

- **Accelerated charges radiate**



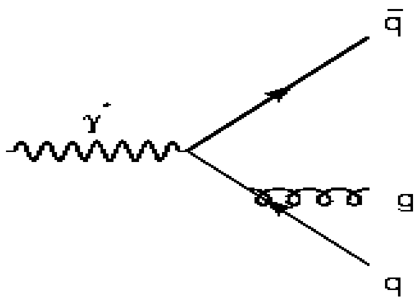
$$\vec{E} = \frac{e}{c}\frac{\vec{n}\times(\vec{n}\times\dot{\vec{\beta}})}{R} \Rightarrow \frac{dP}{d\Omega} = \frac{e^2}{4\pi c}\left|\vec{n}\times(\vec{n}\times\dot{\vec{\beta}})\right|^2$$

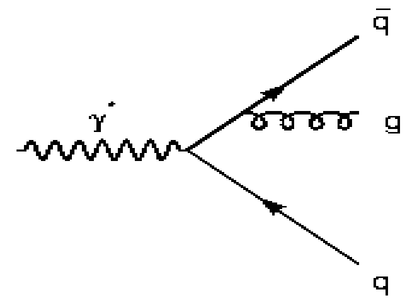
- **Color is a charge, and thus quarks also radiate**
- **Gluon itself has charge (=q-qb pair to $1/N_c$)**

Field Theory

- **Block and Nordsieck (QED)**
  - **Must include virtual and real (emission) corrections to obtain IR finite cross section**
    - **Electron is ALWAYS accompanied by photons**

Singular Behavior of NLO Matrix Elements





$$e^+e^- \to q\bar{q}g$$

$$d\sigma(q\bar{q}g) = \sigma_0 \left\{ \frac{\alpha_s}{2\pi}\frac{dt}{t}dz\left[P_{qq}(z) - \frac{t}{Q^2}\right] + \frac{\alpha_s}{2\pi}\frac{du}{u}dz\left[P_{qq}(z) - \frac{u}{Q^2}\right]\right\}$$

where $\quad s = 2p_q \bullet p_{\bar{q}},\quad t = 2p_q \bullet p_g,\quad u = 2p_{\bar{q}} \bullet p_g, z = \frac{s}{Q^2}$

also introducing $\quad P_{qq}(z) = \frac{4}{3}\frac{1+z^2}{1-z}$

Probabilistic picture relies on absence of interference in axial gauges

Note: Interference causes soft (non-leading) divergences

Leading Behavior of Emission

- Soft-collinear singularity
  - $t \to 0$ when q/g parallel, g is soft
    - pair is indistinguishable from $k = p_q + p_g$
    - $k^2 \sim E_g E_q \theta^2_{qg}$
  - z distribution given by AP splitting kernel
- Universal Result: Factorization of MASS SINGULARITIES
  - $d\sigma_{N+1} = \sigma_N \alpha_S /2\pi\ dt/t\ dz\ P(z, \phi)\ d\phi$
  - P is flavor and spin dependent [ϕ integrated out]
    - $q \to q\, g \Rightarrow C_F (1+z^2)/(1-z)$
    - $q \to g\, q \Rightarrow C_F (1+(1-z)^2)/z$
    - $g \to g\, g \Rightarrow N_C (1-z(1-z))^2/(z(1-z))$
    - $g \to q\, qb \Rightarrow T_R (z^2 + (1-z)^2)$ [no soft or collinear enhancement]
  - Ambiguity in choice of z and t
    - z corresponds to choice of n_μ in axial gauge

Sudakov Form Factor

- **Shower of resolvable emissions $q^*(p) \to q(zp) + g([1-z]p)$**
  - **RESOLVED if $z_C < z < 1 - z_C$**
- **Prob. of no resolvable emission for small δt**

$$1 - \sum_{b,c} \int_{z_-(t)}^{z_+(t)} dz \frac{\alpha_S(t)}{2\pi} P_{a\to bc}(z)\, \delta t$$

Sum over all numbers of irresolvable emissions:

$$S(t) = \exp\left\{ -\int_{t_0}^{t} dt' \sum_{b,c} \int_{z_-(t')}^{z_+(t')} dz \frac{\alpha_S(t)}{2\pi} P_{a\to bc}(z) \right\} \Leftrightarrow \Delta(t)$$

$$z_+ \sim 1 - z_C, \quad z_- \sim z_C$$

- **$\mathrm{Prob}(t_{max}, t) = S(t_{max})/S(t)$**
  - **Lends itself to Monte Carlo technique**
  - **Pick random r and solve for new t**
  - **Continue down to t_{min}**
  - **Stop shower & begin hadronization**

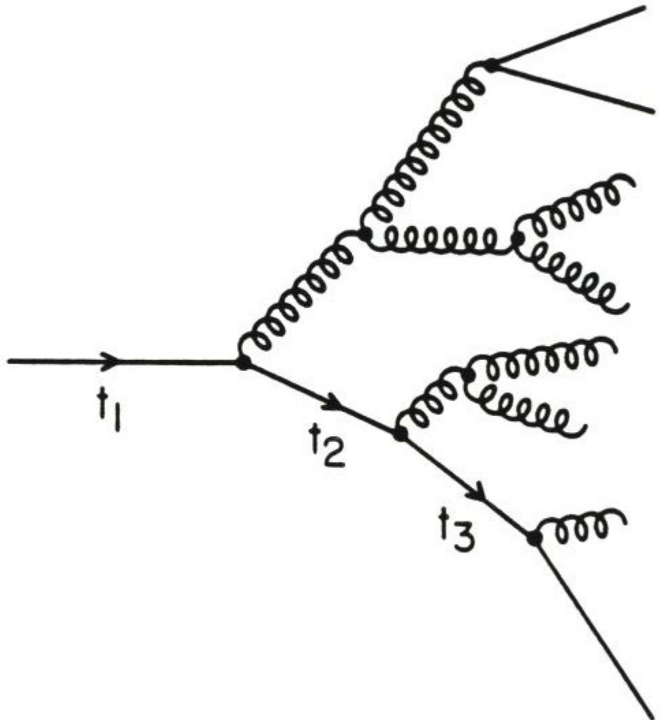
FSR is evolution of Fragmentation Function

$$D_a(x,t) = \overbrace{D_a(x,t_0)\,\Delta(t)}^{\text{NO BRANCHINGS}} + \overbrace{\int_{t_0}^{t'}\int_x^1 dt' \frac{dz}{z}\frac{\Delta(t)}{\Delta(t')}\frac{\alpha_{abc}(z,t')}{2\pi}\hat{P}_{a\to bc}(z)\,D_b(x/z,t')}^{\text{ALL PATHS WITH LAST BRANCH AT t'}}$$

- Outgoing parton from hard scattering is highly-evolved
  - Off-shell with $m^2 \sim Q^2$
  - Evolves to lower scale with Prob=1
  - Sudakov yields an explicit history of resolved emission
    - “NO” branching means NO RESOLVABLE branching
    - Contains some of the virtual pieces ignored in previous NLO example
  - End of PS naturally related to hadronization

Virtuality-Ordered PS

Highly virtual



Nearly on-shell

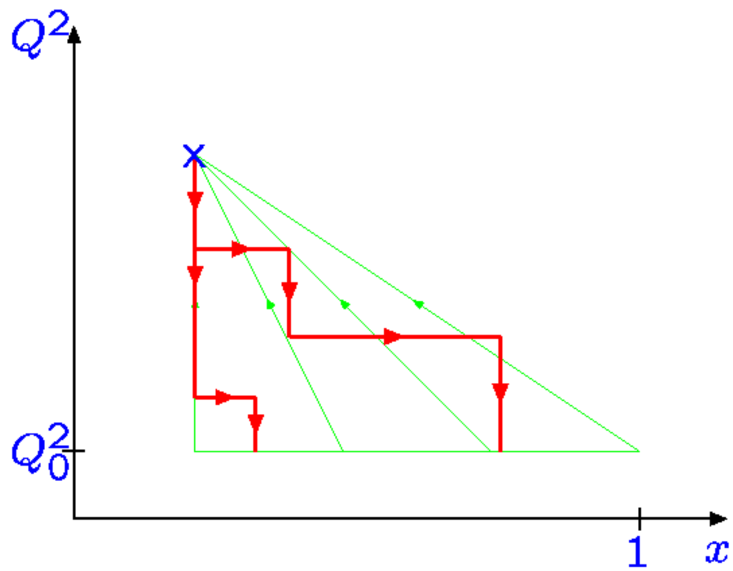
$$t_1 > t_2 > t_3$$

Initial State Radiation

- **Similar picture, but solving DGLAP for PDFs**

$$f_a(x,t') = \overbrace{f_a(x,t)\,\Delta(t')}^{\text{NOBRANCHING}} + \overbrace{\int_{t'}^{t}\int_{x}^{1} dt''\,\frac{dz}{z}\,\frac{\Delta(t')}{\Delta(t'')}\,\frac{\alpha_{abc}(z,t'')}{2\pi}\,\hat{P}_{a\to bc}(z)\, f_b(x/z,t'')}^{\text{BRANCHINGS}}$$

Increasing parton virtuality



Parent has more momentum

Forward Showering

- Showering generated by a sequence of solutions to:

$r = \Delta(t_{new})/\Delta(t_{old}) \in [0,1]$

  - $t_{start} \sim \ln Q_0^2$ $t_{final} \sim \ln Q^2$
- Each increase in t (more negative mass) requires a new branching
- This is Forward Showering
- Problems:
  - Which branch $a \rightarrow bc$ evolves further?
  - Must find x1 x2 S = Q^2
  - Reject too many configurations

Backwards Showering

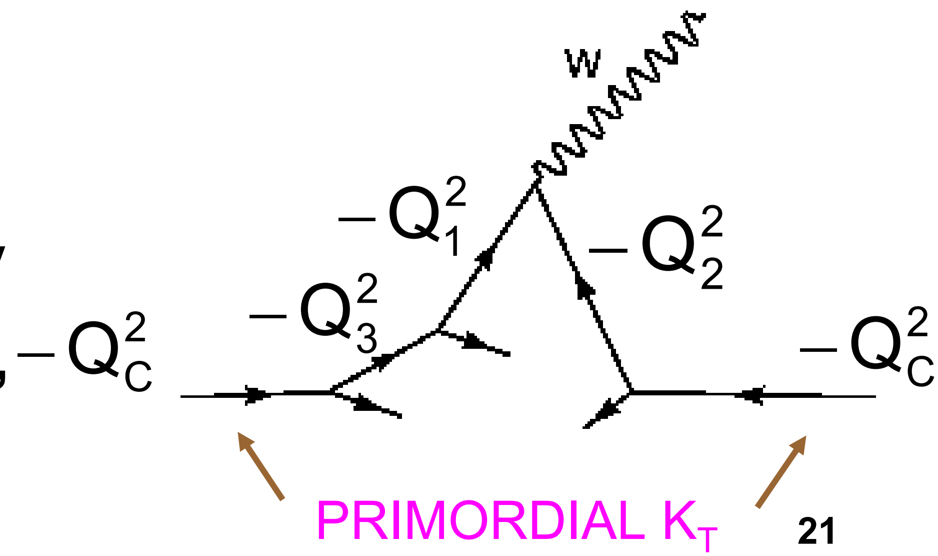
Sjostrand

$$-\ln(S) = \int_{t}^{t_{MAX}} dt' \int_{z_-}^{z_+} dz \, \frac{\alpha_{abc}(z, t')}{2\pi} \hat{P}_{a\to bc}(z) \frac{x' f_a(x', t')}{x \, f_b(x, t')}; \; x' = x/z$$

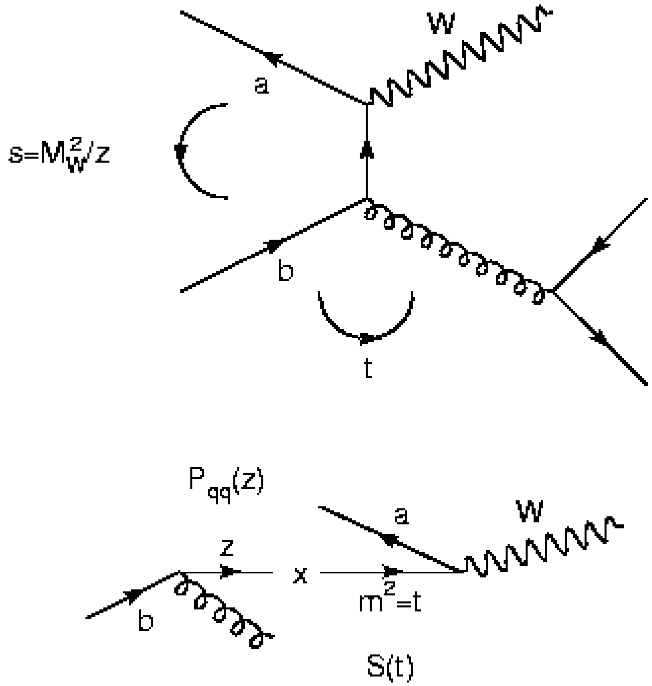
$$\frac{\Delta(t)}{f_b(x, t)} \frac{f_a(x, t')}{\Delta(t')}; \; \mathrm{Prob}(z) \propto \frac{\alpha_{abc}(z, t)}{2\pi} \frac{\hat{P}_{a\to bc}(z)}{z} f_a(x', t')$$

Marchesini/Webber

- **$-Q_0^2 > -Q_1^2 > \ldots > -Q_n^2$**
- **showering added after hard scatter with unit probability**
  - **Something happens, even if not resolvable**



Return to Diagrammatic Approach



- Compare first branch of PS to exact NLO ME
- PS does not include mass for off-shell line
  - Would seem to generate too hard of an emission
- Maximum PS virtuality ~ M_W
  - Won't generate $P_T^W > M_W$
- Fixing PS to get the correct hard limit is a field of active research
- Fixing ME to account for soft gluon emission is called Resummation

Basics of Resummation

Consider W production

At LO in pQCD, the rapidity Y and transverse momentum Q_T of W are fixed by the incoming partons.

$$\frac{d\hat{\sigma}}{dQ_T^2} \propto \delta\left(\vec{Q}_T\right)\sigma_0$$

At NLO, single gluon emission occurs with $Q_T > 0$

$$\frac{d\hat{\sigma}}{dQ_T^2} \approx \frac{\alpha_S}{Q_T^2}\ln\left(\frac{Q^2}{Q_T^2}\right)\left[c_1 + c_2\alpha_S\ln^2\left(\frac{Q^2}{Q_T^2}\right) + \ldots\right]$$

- **Cross sections at large Q_T or Q_T averaged are described well by fixed order in α_S**
- **However, some observables are sensitive to region $Q_T << Q$**
  - **For W/Z production, this is most of the data!**
- **Solution: Reorganize perturbative expansion**
  - **$\alpha^N \ln^M(Q^2/Q_T^2)$**
  - **Sums up infinite series of soft gluon emissions**

Q_T-space Formalism

- **Extension of b-space formalism**

$$\frac{d\sigma}{dQ^2 dQ_T^2 dy}(h_1 h_2 \to W + X) =$$

$$\frac{d}{dQ_T^2}\tilde{W}(Q_T, Q, x_1, x_2) + \ldots$$

$$\tilde{W} = (C \otimes f)(C \otimes f)\exp(-T)\, H(Q)$$

$$T(Q_T, Q) = \int_{Q_T^2}^{Q^2} \frac{dm}{m}\left[A \ln\left(\frac{Q^2}{m}\right) + B\right]$$

$$(C \otimes f)[x] = \int_x^1 \frac{dz}{z} C(x/z) f(z)$$

- **Looks very similar to MW PS algorithm for backwards shower**
  - **Sudakov determines P_T^W**
- **Soft gluon emissions are integrated out**
  - **Sudakov contains soft pieces not in DGLAP**
- **Total rate can be calculated to any given order in α_S**

$$\text{At LO,} \quad \frac{d\sigma}{dQ_T^2} = \sigma_0 \frac{d}{dQ_T^2}\left\{\frac{\exp\left[-\frac{1}{2}T(Q_0, Q)\right] f(x, Q_T)}{\exp\left[-\frac{1}{2}T(Q_0, Q_T)\right] f(x, Q)}\right\}$$

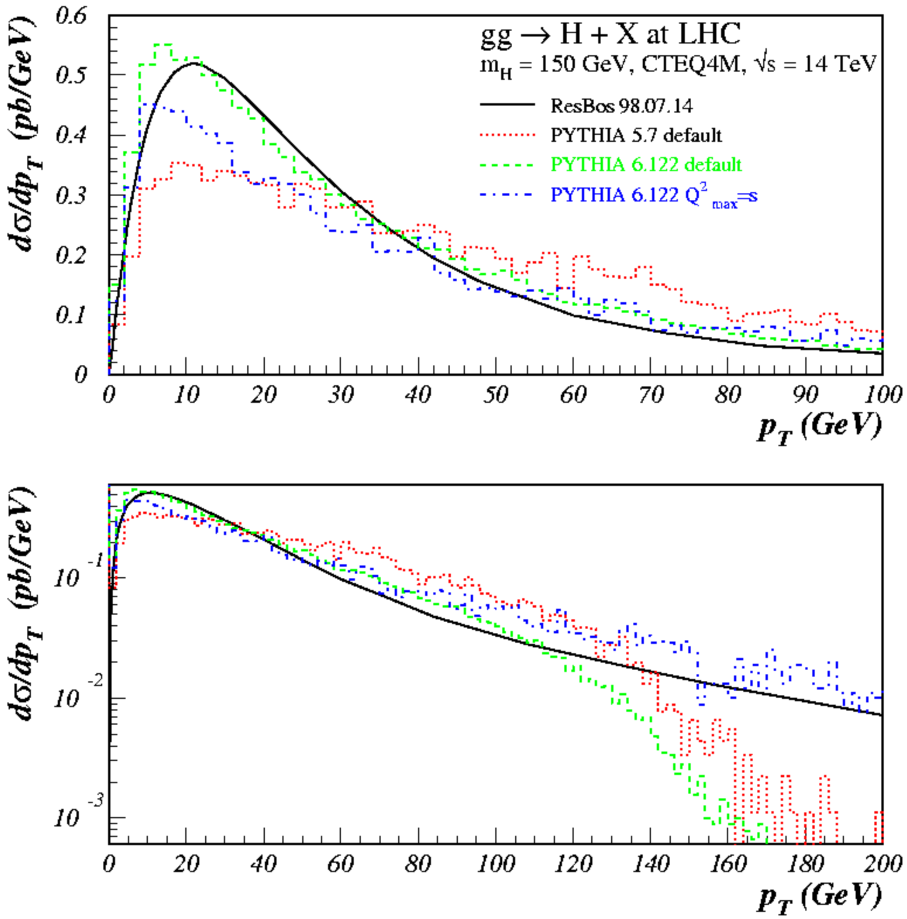
Key Features

- **Analytic Resummation**
  - **soft gluon emissions exponeniate into Sudakov form factor**
  - **k_T conserved**
  - **Total rate at (N)NLO**
    - **modified PDF's**
  - **corrections for hard emission**
  - **soft gluons are integrated out**
    - **Predicts observables for a theoretical W**
  - **Needs modelling of non-perturbative physics**
- **Parton Showering**
  - **DGLAP evolution generates a shower of partons**
    - **LL with some sub-LL**
  - **Exact soft gluon kinematics**
  - **LO event rates**
  - **underestimates single, hard emissions**
  - **Explicit history of PS**
    - **More closely related to object identified with a W**
  - **natural transition to hadronization models**
    - **Follow color flow down to small scales**

Similar physics, but different approach with different regimes of applicability

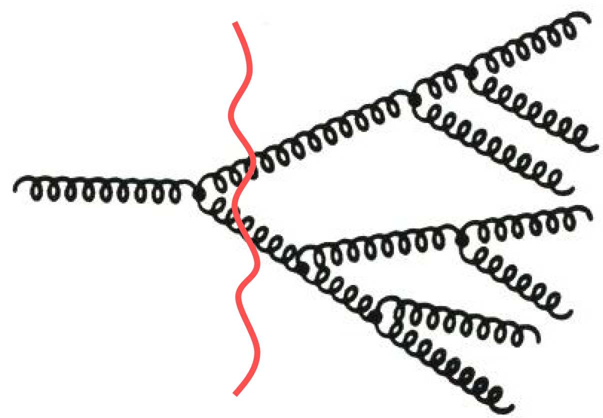
Comparison of Predictions

- Can compare for inclusive enough observables
- Demonstrates ability of analytic approach to describe the full spectrum
- Does not indicate variability of analytic prediction
  - Or (in)sensitivity to experimental cuts



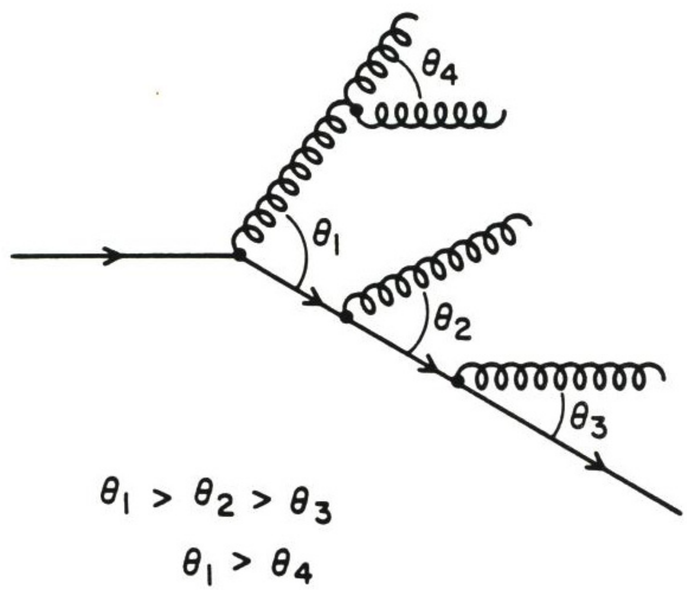
Color Coherence

- In previous discussion of PS, interference effects were ignored, but they can be relevant
  - axial gauge eliminates only collinear interference
- Add a soft gluon to a shower of N almost collinear gluons
  - incoherent emission: couple to all gluons
    - $|M(N+1)|^2 \sim N \times \alpha_S \times N_C$
  - coherent emission: soft means long wavelength
    - resolves only overall color charge (that of initial gluon)
    - $|M(N+1)|^2 \sim 1 \times \alpha_S \times N_C$



Angular-Ordered PS

- Showers should be Angular-Ordered
  - $\zeta = p_I \bullet p_J / E_I E_J = (1 - \cos\theta_{IJ}) \sim \theta_{IJ}^2/2$
  - $\zeta_1 > \zeta_2 > \zeta_3 \ldots$
- Running coupling depends on $k_T^2 \Rightarrow z(1-z)Q^2$

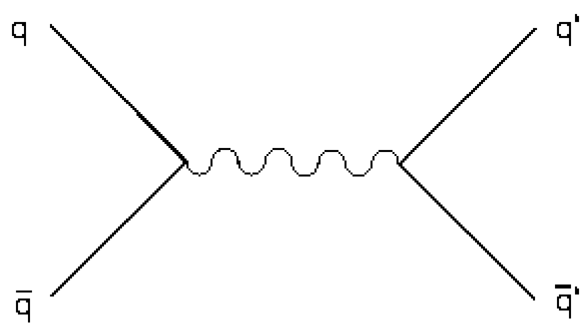


- Dead Cone
  - $Q^2 = E^2 \zeta < Q^2_{max}$
  - $Q^2_{max} = z^2 E^2$
  - $\zeta < 1$ [not 2]
  - $\theta < \pi/2$

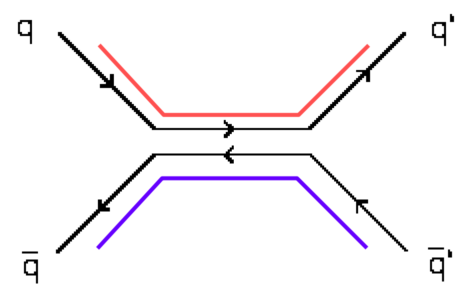
Color Coherence in Practice

- **Emission is restricted inside cones defined by the color flow**

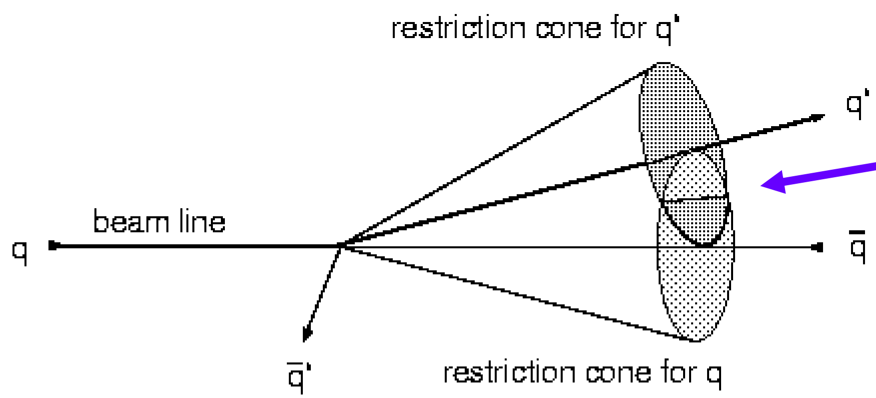
Partonic picture



Large N picture



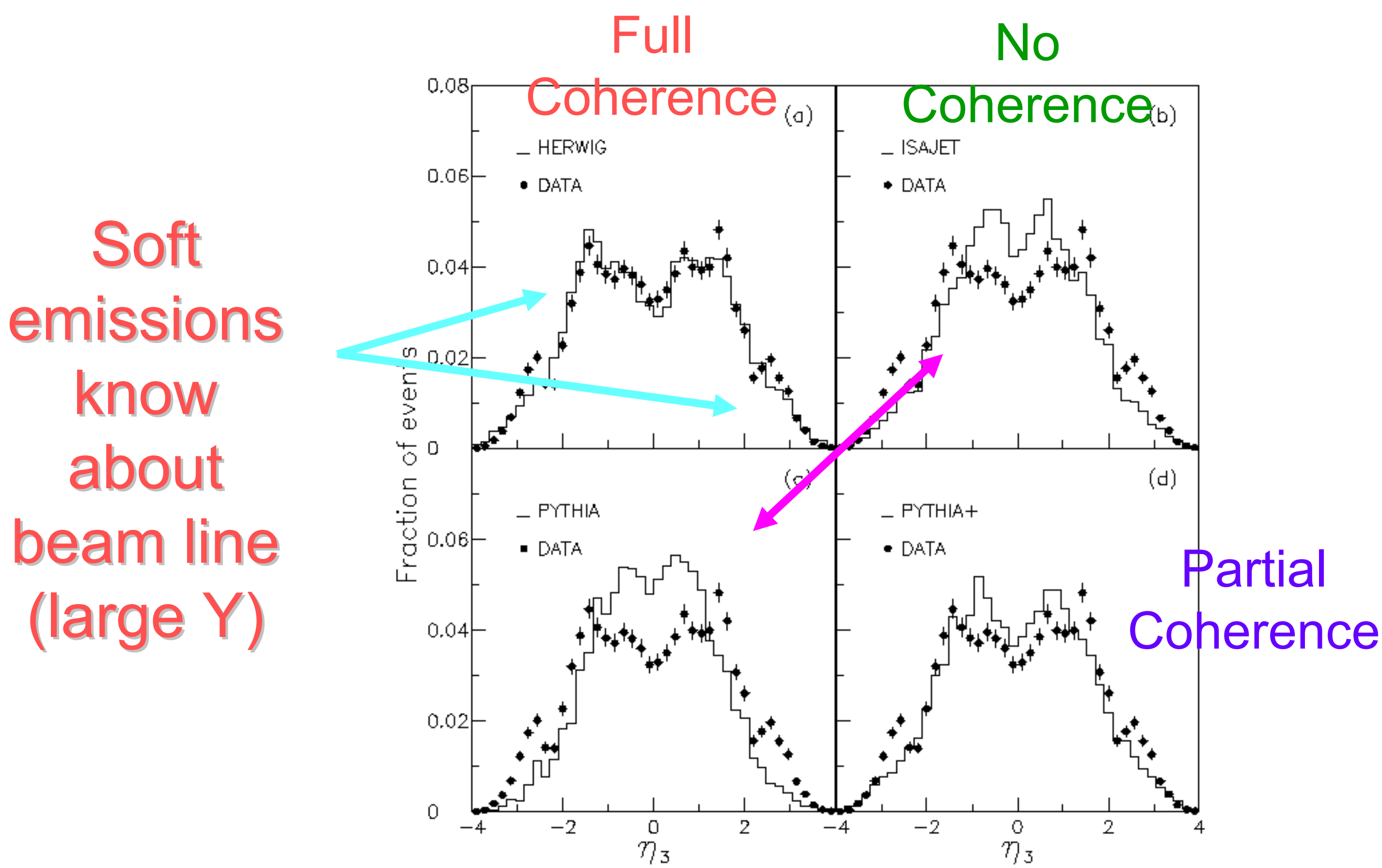
Beam line



Enhanced emission

Essential to Describe Data

- 3 Jet Distributions in Hadronic Collisions



Pseudorapidity of Gluon Jet

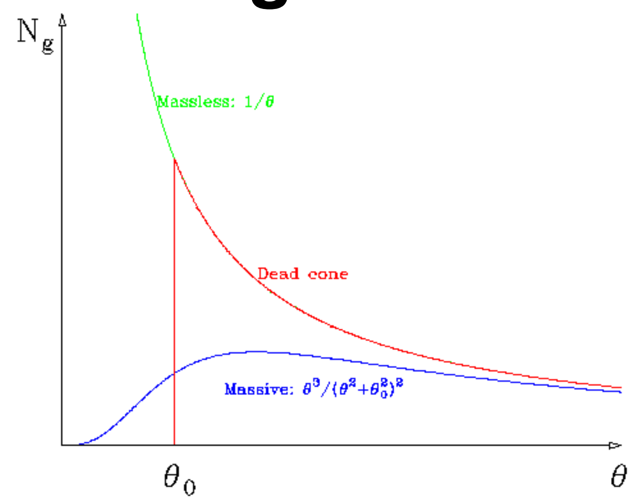
Parton Showering and Heavy Quarks

- **Heavy Quarks look like light quarks at large angles but sterile at small angles**

Eikonal expression for soft gluon emission:

$$\frac{d\sigma(q\bar{q}g)}{d\sigma(q\bar{q})} \propto -\left(\frac{p_1}{p_1 \bullet p_3} - \frac{p_2}{p_2 \bullet p_3}\right)^2 \frac{d^3p_3}{E_3}$$

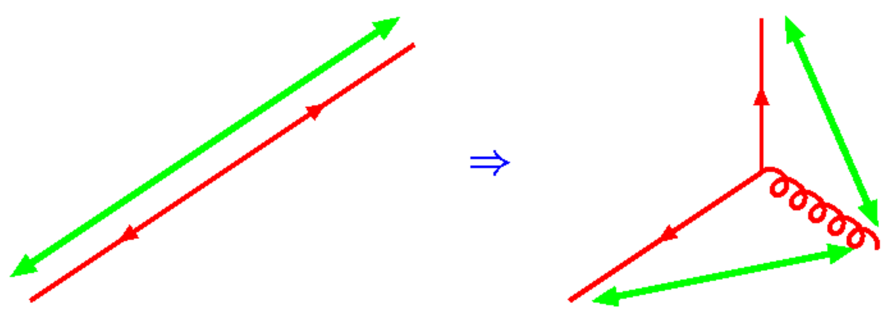
$$\Rightarrow \frac{d\sigma(x_3,\theta_{13},r=.5m_Q/E_Q)}{d\sigma(x_3,\theta_{13},0)} = \left(\frac{\theta_{13}^2}{\theta_{13}^2+4r^2}\right)^2$$



- **implemented as energy-dependent cutoff in θ-ordering**
  - **$\theta > \theta_0 = m_Q/E_Q$**
  - **Creates 'dead cone'**
- **Can be treated using $Q^2 = Q^2_{old} - M^2_{on\text{-}shell}$**

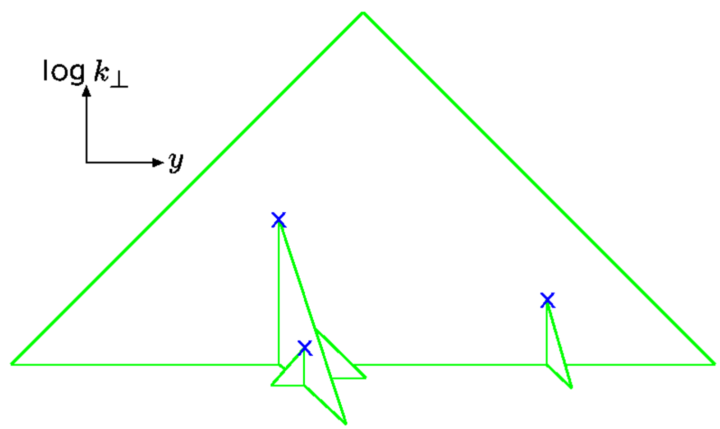
Color Dipole Model

- Conventional parton showers
  - start from collinear limit
  - modify to incorporate soft gluon coherence
- Color Dipole Model
  - start from soft limit
  - Emission of soft gluons from color-anticolor dipole is universal:
    - $d\sigma \sim \sigma_0 \, ½ \, C_A \, \alpha_S(K_T^2) \, dK_T^2/K_T^2 \, dY/2\pi$
- After emitting a gluon, color dipole is split:



Origami Diagram

- Subsequent dipoles continue to cascade
  - PS: 1 parton to 2 partons
  - CDM: one dipole (2partons) to 2 dipoles (3 partons)
- Similar to θ-ordered PS for $e^+ e^-$ annihilation



- Not suitable for ISR
- Generates more radiation at small-x

The Programs (Pyt/Isa/Wig/Aria)

- ISAJET
  - Q^2 ordering with no coherence
  - large range of hard processes
- PYTHIA
  - Q^2 ordering with veto of non-ordered emissions
  - large range of hard processes
- HERWIG
  - complete color coherence & NLO evolution for large x
  - smaller range of hard processes
- ARIADNE
  - complete color dipole model (best fit to HERA data)
  - interfaced to PYTHIA/LEPTO for hard processes

Summary

- Accelerated color charges radiate gluons
  - Gluons are also charged
  - Showers of partons develop
    - IMPORTANT effect for experiments
- Showering is a Markov process and is added to the hard scattering with P=1
  - Derived from factorization theorems of full gauge theory
  - Performed to LL and some sub-LL accuracy with exact kinematics
  - Color coherence leads to angular ordering
- Modern PS models are very sophisticated implementations of perturbative QCD
  - Still need hadronization models to connect with data

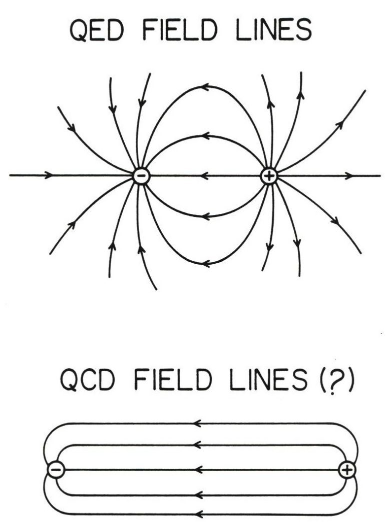
Hadronization

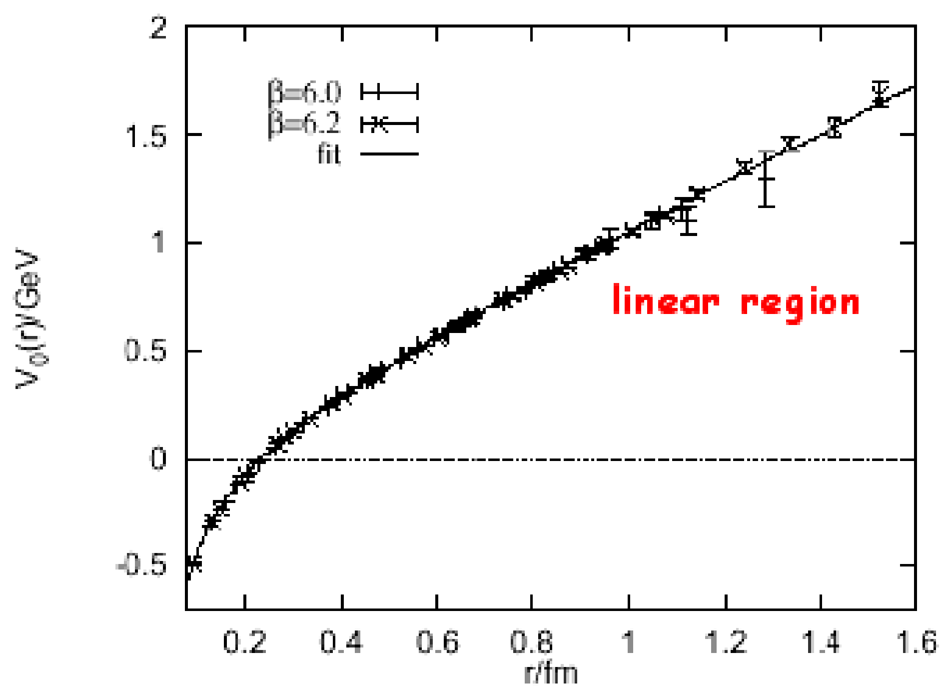
- Colored partons are not physical particles
  - Color must be neutralized
- Lattice QCD has not yet solved this difficult problem
  - It is not even clear that the QCD Lagrangian is THE fundamental one to describe this process
- Need a model of parton confinement into hadrons
  - Some Pure fits
  - Some Semi-classical models
  - Intimately tied to parton shower
  - Lots of low energy data

Confinement

- **Non-Abelian nature of QCD bundles up the field lines**
  - **“flux tube” formation**
- **Linear potential**
  - **evidenced by quarkonia spectra and lattice QCD**
  - **Reggie trajectories J∝M²**

String tension κ ~ 1





Estimate of Hadronization Effects

- **TUBE model: Jet stretches in (Rapidity,K_T)-space**
- **0<y<Y , μ=Transverse Mass (profile in K_T)**
- **Jet energy and momentum:**

$$E_{JET} = \mu \int_0^Y dy \cosh[y] = \mu \sinh[Y] = Q/2$$

$$P_{JET} = \mu \int_0^Y dy \sinh[y] = \mu (\cosh Y - 1)$$

$$M_{JET}^2 = 2 P_{JET}\mu \approx Q\mu$$

$$\text{Thrust } T = \frac{P_{JET}}{E_{JET}} \approx 1 - \frac{2\mu}{Q}$$

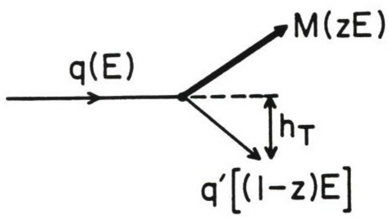
$$\langle 1-T \rangle_{pert} = .055 \quad \langle 1\text{-}T \rangle_{expt} = .068 \quad \Rightarrow \mu \sim 500 \text{ MeV}$$

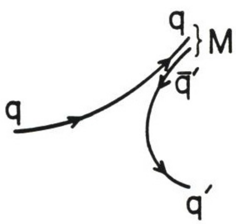
- **Quite Relevant!**
  - Non-pert effects ~ 10-20%

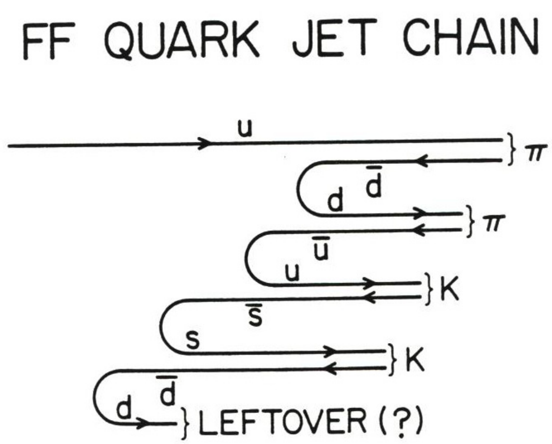
Independent Fragmentation Model

- Longitudinal momentum dependence of hadron distributions in jets approximately scale
- P_T of hadrons inside of jet is limited
- Qualitative features parametrized in Feynman-Field model
  - Does not explain or derive anything
  - Conceptual problems understood by authors
- FF Model
  - Longitudinal hadron momentum a function of $0<Z<1$
  - Transverse momentum fit to a Gaussian
  - Recursively apply $Q_A \rightarrow M(Q_A\, Qb_B) + Q_B$
    - Last remaining soft Q must be linked with other soft partons

Picture of Feynman-Field







- **Energy sharing described by a simple function f(z)**
  - **Normalized to 1**
- **Fragmentation function D(z) yields distribution of hadrons with fraction z**
  - **What is z?**
    - **0 < z < 1**
    - **E(hadron)/E(parent)**
    - **P_+=E-P_L**
    - **P_L**
  - **In practice, it doesn't matter much**

Evolution Equation

$$D(z) = f(z) + \int_0^{1-z} \frac{dz'}{z'} f(z')\, D\left(\frac{z}{1-z'}\right)$$

$$f(z) = \frac{\Gamma(a+b)}{\Gamma(a)\,\Gamma(b)} z^{a-1}(1-z)^{b-1} \quad \text{"right" endpoint behavior}$$

Mellin Transform!

$$\tilde{D}(s) = \int_0^1 \frac{dz}{z} z^s D(z) = \frac{\Gamma(a+s)\Gamma(a+b)}{\Gamma(a)\Gamma(a+b+s)} +$$

$$\tilde{D}(s)\frac{\Gamma(a+s)\Gamma(s+b)}{\Gamma(b)\Gamma(a+b+s)}$$

$$a = 1 \;\Rightarrow\; \tilde{D}(s) = b\,\mathrm{Beta}(b,s)$$

$$\frac{1}{2\pi i}\int ds\; x^{-s}\; \mathrm{Beta}(b,s) = (1-x)^{b-1}$$

$$D(z) = b\, z^{-1}(1-z)^{b-1}$$

$$\langle D(z)\rangle = \ln\left(\frac{1}{z_{cut}}\right) \Rightarrow \text{logarithmic multiplicity}$$

Extensions

- Original $f(z) = 1 - a_F + 3\, a_F\, (1\text{-}z)^2$
- Gluons included by first splitting: $dP/dz \sim z^2 + (1\text{-}z)^2$
- Heavy Quarks
  - Old-fashioned (time-order) perturbation theory
  - $Q \rightarrow (Q\bar{q})[z] + q[1\text{-}z]$

$$
\begin{aligned}
\Delta E &= E_{OUT} - E_{IN} \\
&= \sqrt{(zP)^2 + m_a^2} + \sqrt{([1-z]P)^2 + m_b^2} - \sqrt{P^2 + M^2} \\
&\approx zP\left(1 + \frac{m_a^2}{2z^2P^2}\right) + [1-z]P\left(1 + \frac{m_b^2}{2[1\text{-}z]^2P^2}\right) - P\left(1 + \frac{M^2}{2P^2}\right)
\end{aligned}
$$

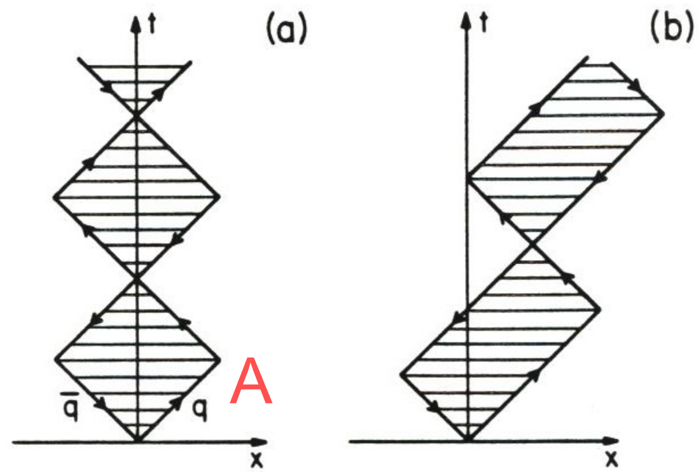
$$
D(z) \sim \frac{1}{(\Delta E)^2} = \left[1 - \frac{1}{z} - \frac{\varepsilon}{1\text{-}z}\right]^{-2}
$$

Problems with FF

- Energy is not conserved
  - Can conserve E+p but not E or p separately
- Frame dependent
  - Different multiplicity for collider vs fixed-target mode
- Naively, flavor and charge not conserved
- Space-time picture is wrong
  - Fast hadrons created early in the chain
  - Hadrons should result from color screening
- Not stable to soft or collinear gluon radiation
- Nonetheless, A Decent Parametrization

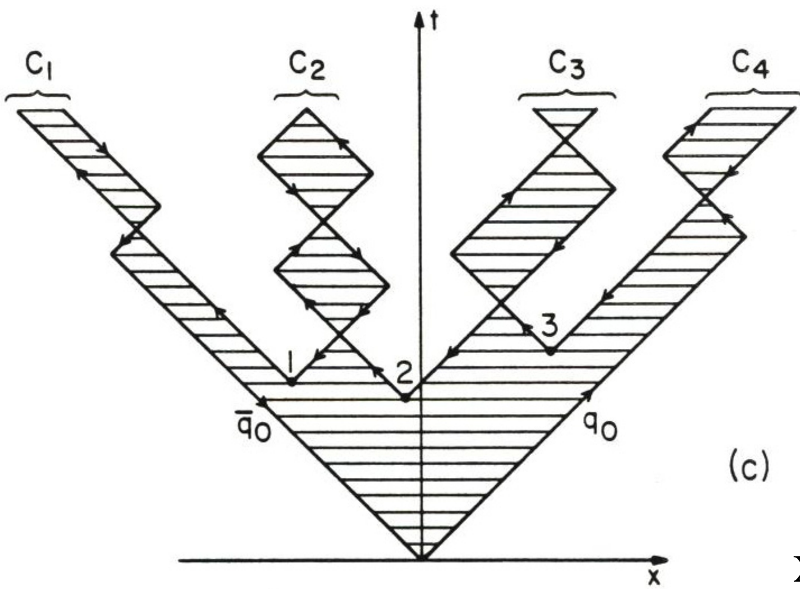
String Model of Confinement

- Color singlet Q-Qb pair with mass W bound by string with
  - dp/dt = $\pm \kappa$
    - p_0=W/2
  - $p_Q(t) = p_0 - \kappa t$
  - $p_{Qb}(t) = -(p_0 - \kappa t)$
  - $E_{STRING} = 2 \kappa t$
  - Q & Qb stop and reverse at $t=p_0/\kappa$
- Obeys area law: $W^2 = 2\kappa^2 A$
  - (a) Yo-yo mode
  - (b) Q-Qb with net momentum



The Lund String Model

- **Allow for the string to break**
- **q-qb pairs created in field via quantum mechanical tunneling**
- **d(Prob)/dx dt= $\exp(-\pi m_T^2/\kappa)$, $m_T^2 = m_\pi^2 + p_T^2$**
- **Expanding string breaks into hadrons long**



Similar f(z), different picture

- **Simple Algorithm**

$$\Delta x^+ = (x^+_{i-1} - x^+_i) = z_i x^+_{i-1}$$

$$0 < z_i < 1 \text{ chosen with prob. } f(z)$$

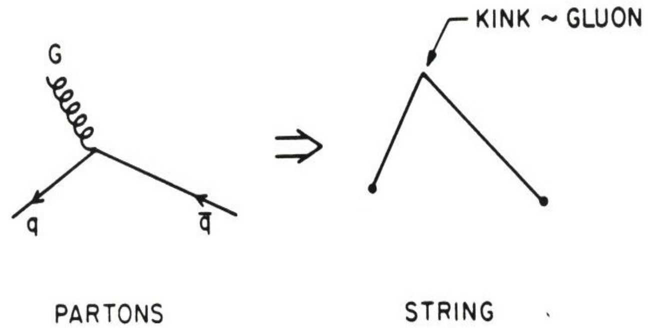
$$\Delta x^- \Delta x^+ \equiv -\frac{m_i^2}{\kappa^2}$$

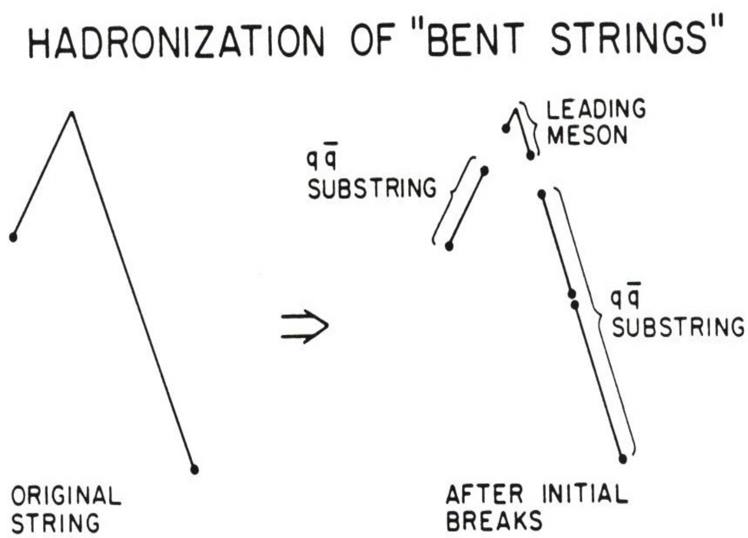
$$x_0^+ = 2E_0/\kappa; \quad x_0^- = 0 = x^+_{n+1}; \quad x^-_{n+1} = 2\overline{E}_0$$

Lund Symmetric Fragmentation Function

- String picture constrains fragmentation function:
  - Lorentz covariance
  - left-right symmetry
    - Shouldn't matter which end of the string fragments first
- $f(z) \sim z^{a-b-1} \ (1-z)^{b}$
  - a,b adjustable parameters for quarks a,b
  - Also quark masses are adjustable
- Baryon production from tunneling diquarks
  - More parameters
- SUMMARY of String Idea
  - Hadrons do not form independently from isolated quarks
  - Hadron production is a cooperative phenomenon
    - INVOLVES Q, Qb, and CONFINING FIELD

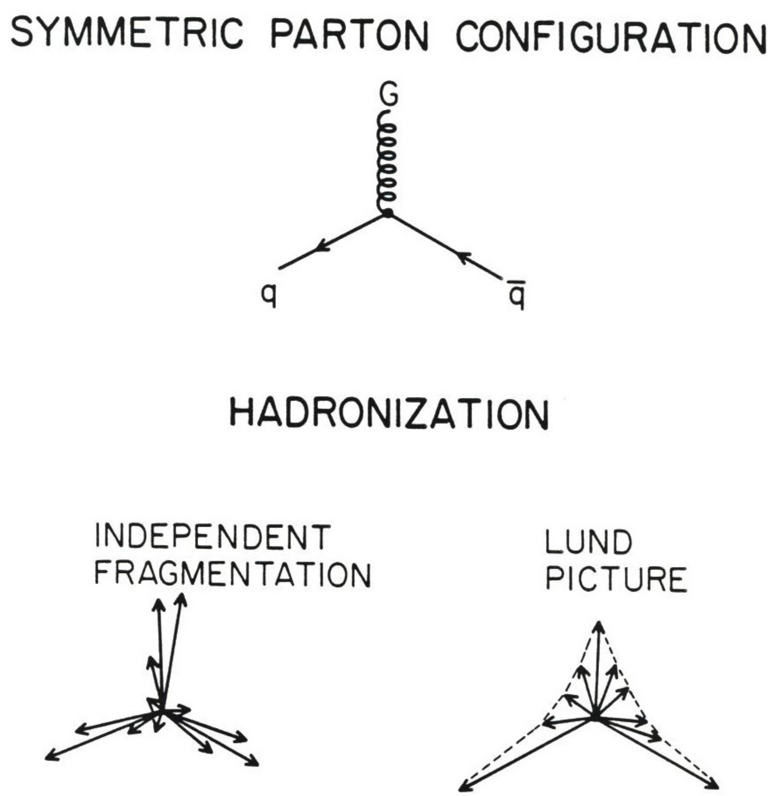
Add a Soft Gluon to String





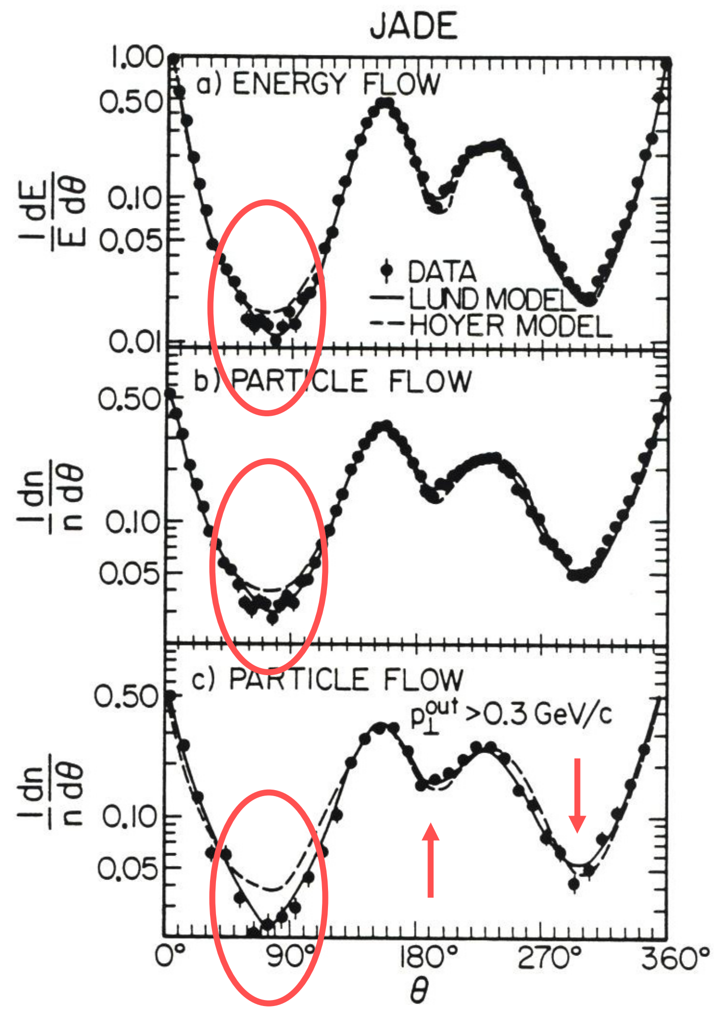
- Gluon imparts momentum to the string
- Evolves as before
- Kink always in the middle of a bit of string
  - NOTE that since gluons are generated via the PS mechanism, the hadronization process is NOT independent of how gluons are generated
    - Coherence
    - Cutoff scale

Three Jet Events



Feynman-Field

String motion

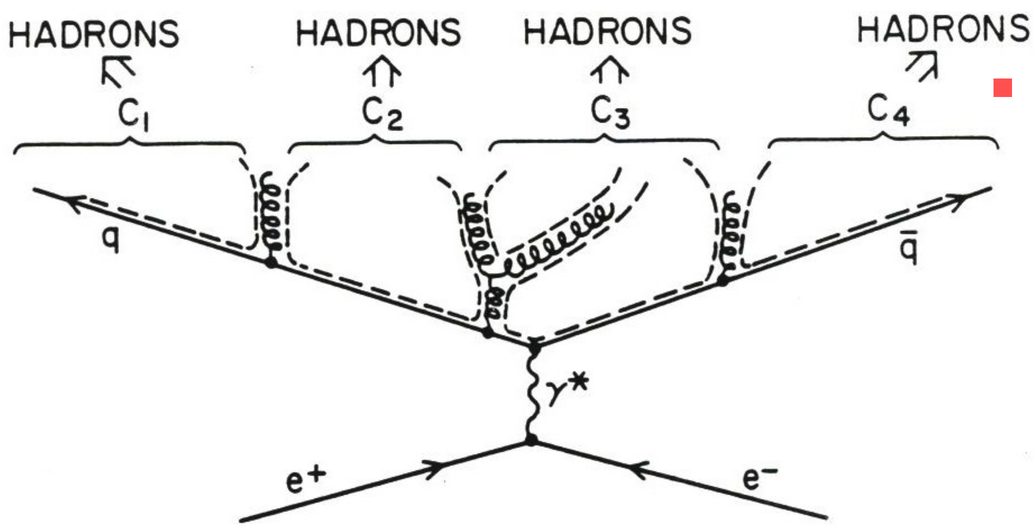


Color Screening

- Color Confinement is an experimental fact
  - In FF, confinement plays no role
  - In the String Model, confinement replaces q-qb pairs with strings [it is everything]
- Screening of color charge can arise through the emission of soft gluons
  - This may be perturbative or non-perturbative
  - Most likely, a mixture of both
- QCD-cluster models push the perturbative limit
  - Initial state is evolved into an ensemble of low-mass color singlets (screening occurs here)
  - Final state hadrons are formed from clusters using a simple parametrization or statistical model

Preconfinement

- Planar approximation
  - gluon = color-anticolor pair
- Follow color structure of parton shower
  - color-singlet pairs end up close in phase space
  - …irectly related to



- C_K Mass spectrum
  - asymptotically independent of energy & production mechanism
  - Peaked at small mass/size
  - Falls rapidly for large mass/size
  - Infrared cutoff Q_{gluon}

The Naive Cluster Model

- **Project color singlets onto continuum of high-mass mesonic resonances (=clusters).**
  - **Decay to lighter well-known resonances and stable hadrons**

$$P(C_j[w] \rightarrow H_1 + H_2) \;=\; \mathrm{P_F} \cdot \mathrm{P_S} \cdot \mathrm{P_K}$$

$$P_S = (2J_1 + 1)(2J_2 + 1)$$

$$P_F \;=\; \text{Flavor dependent}$$

$$\mathrm{P_K} \;=\; \theta(\mathrm{W\text{-}M_1} - M_2)\; \frac{\lambda^{\frac{1}{2}}(w^2, M_1^2, M_2^2)}{\mathrm{w}}$$

Can also allow for subcluster formation: C $\rightarrow$H + C'

$$\rho(M) = A\, \theta(M\text{-}M_0)\, (M\text{-}M_0)^N$$

More Refined Cluster Model

- Three Regions of Cluster Mass
- C cannot decay to 2 hadrons
  - ENERGY SHIFTED LOCALLY
- C can decay to 2 hadrons and is below FISSION threshold
  - ISOTROPIC decay into pairs of hadrons
- Heavier C undergo FISSION to small, non-fissionable clusters
  - NOT UNLIKE STRING FRAGMENTATION
  - Fission threshold becomes crucial parameter
    - 15% of primary clusters get split
    - Generates 50% of hadrons

Comparison

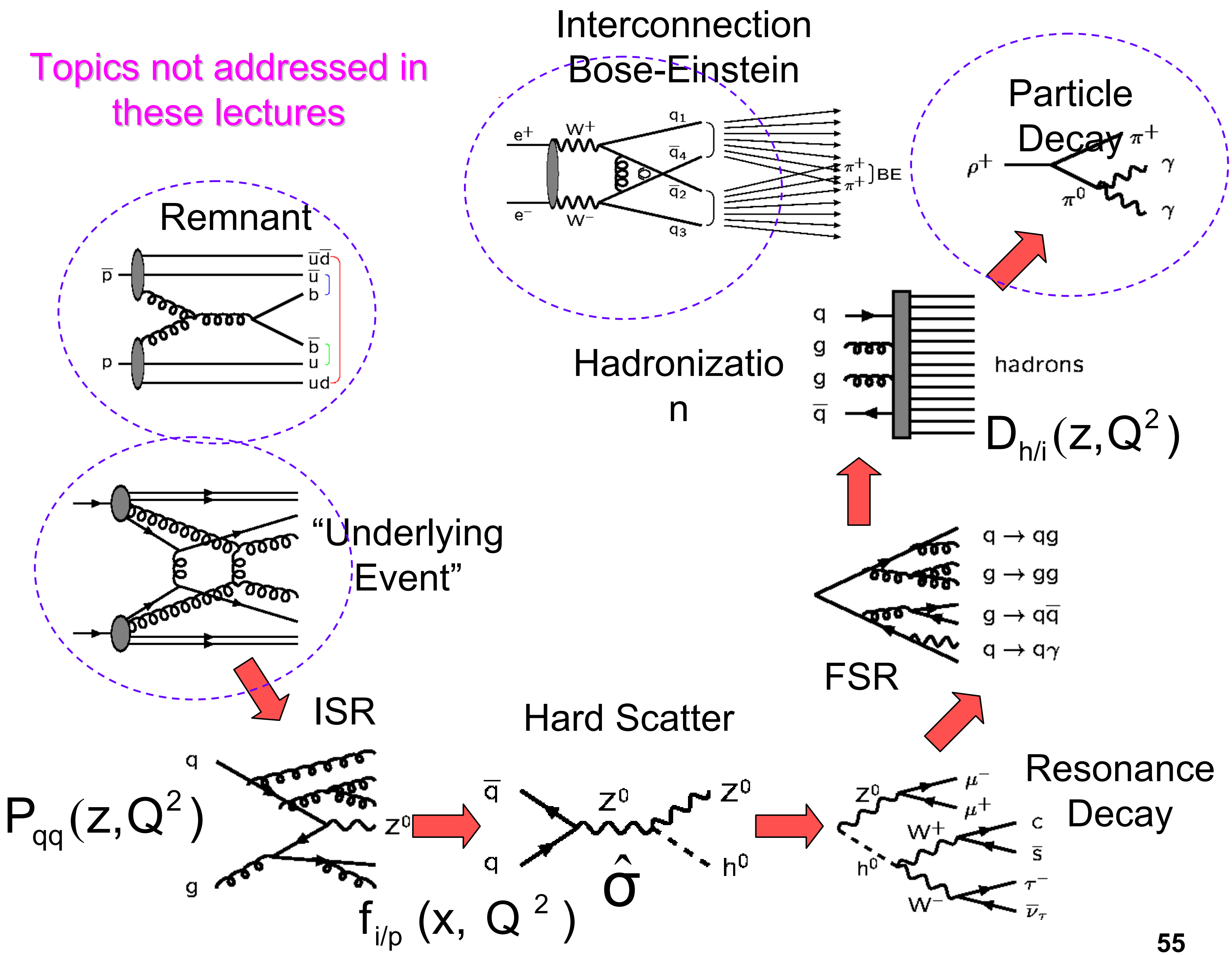
- Strings
  - PRODUCTION of HADRONS is non-perturbative, collective phenomena
  - Careful Modelling of non-perturbative dynamics
- Improving data has meant successively refining perturbative phase of evolution

- Clusters
  - PERTURBATION THEORY can be applied down to low scales if the coherence is treated correctly
  - There must be non-perturbative physics, but it should be very simple
- Improving data has meant successively making non-pert phase more stringlike

STRING model includes some non-perturbative aspect of color coherence

The Programs

- ISAJET
  - Independent fragmentation & incoherent parton showers
- JETSET (now PYTHIA)
  - THE implementation of the Lund string model
  - Excellent fit to $e^+ e^-$ data
- HERWIG
  - THE implementation of the cluster model
  - OK fit to data, but problems in several areas
    - String effect a consequence of full angular-ordering


Topics not addressed in these lectures
Interconnection
Bose-Einstein
Particle
Decay
Remnant
Hadronizatio
n
hadrons
$D_{h/i}(z,Q^2)$
"Underlying
Event"
q → qg
g → gg
g → q$\bar{q}$
q → qγ
FSR
ISR
Hard Scatter
Resonance
Decay
$P_{qq}(z,Q^2)$
$f_{i/p}(x, Q^2)$
$\hat{\sigma}$

Topics Not Covered

- *Some aspects of the event are beyond the scope of this introductory set of lectures [and my expertise], yet can be important when comparing to data and can impact new physics searches*
  - Treatment of the beam remnant may be relevant for forward jet tagging studies
    - Higgs production through WW fusion
  - Underlying event affects isolation and jet-energy corrections
    - Observing Higgs in photons or jets
  - Interconnection and Bose-Einstein effects are relevant to precision EW measurements
  - Tau leptons and b-hadrons must be decayed correctly to understand polarization effects & tagging efficiency
    - Is phase space enough?
- The objects that experimentalists observe are not the same as the output of an event generator!

Selected Topics for Real Users

- **K-factors and hard-emission corrections**
  - ***"When can I (should I) use a K-factor?"***
  - **Event rate is fixed by the hard scattering**
    - **Almost always the Born Level rate**
    - **For QCD processes, there may be a large scale dependence**
      - $\delta\alpha_S^2 / \alpha_S^2 = 2\,\delta\alpha_S/\alpha_S \Rightarrow$ **20-40% variation possible**
    - **Hard scale sets the maximum scale for PS**
    - **Correcting the event rate does not mean correcting the kinematics**
  - **Scale by a K-factor when not applying very tight cuts**
    - **E.g. Normalizing WZ production rate overestimates low P_T of the W+Z pair, but underestimates high tail**
      - **THIS IS OKAY IF YOUR CUTS DO NOT BIAS THE P_T OF THE PAIR**
      - **THIS HAS TO BE CHECKED EXPLICITLY**

Selected Topics (II)

- Much activity recently in correcting the PS to generate hard emissions
- Basically, 2 approaches (W/Z+jets)
  - Fill out the kinematic regions not accessed by the parton shower
    - FOR PYTHIA, SOME LOW K_T EMISSIONS ARE SHIFTED HIGHER
      - STILL HAVE TO APPLY K-FACTOR, BUT KINEMATICS ARE BETTER
      - MAY BE CORRECT IF SAME K-FACTOR APPLIES TO NNLO CORRECTIONS OF NLO
    - FOR HERWIG, MORE COMPLICATED SINCE SOME NEW RATE IS ADDED IN THROUGH α_S CORRECTIONS
      - AGREEMENT WITH DATA STILL REQUIRES K-FACTOR
  - Add PS to NLO calculation
    - SHOULD NOT BE TRUSTED IN CERTAIN KINEMATIC REGIONS
    - VERY SUBTLE (SHOULDN'T GENERATE PARTON EMISSIONS IN SAME KINEMATIC REGION AS EXPLICIT HARD EMISSIONS)

Selected Topics (III)

- Adding an external process (e.g. in PYTHIA)
  - Big 3 all allow for this option
  - Typically avoid complex phase space integrals which may need special treatment
  - First step is to obtain the necessary Monte Carlo
    - Requirements
      - NUMERICAL STABILITY
      - INFORMATION ABOUT INCOMING PARTONS
        - IF PDFs ARE TREATED INCLUSIVELY, USER MUST CHOOSE FLAVOR
      - INFORMATION ABOUT OUTGOING PARTONS
        - IF A JET MONTE CARLO, USER MUST IDENTIFY GLUONS OR QUARKS
      - COLOR FLOWS
        - PROJECT COLOR DECOMPOSITION INTO LARGE N LIMIT

Selected Topics (IV)

- Numerical stability is most important
  - Need unbiased (soft) parton-level cuts, but this usually means a large cross section
    - MUST SIMULATE ENOUGH EVENTS TO UNDERSTAND RARE BACKGROUNDS
- Can guess at the rest
  - Conservation of flavor
    - Probability to radiate a gluon is C_A/C_F
  - Planar approximation for color flow
    - $g \to q\ qb$
    - Draw all possible connections through Feynman diagrams
    - Pick one randomly
    - Or bias those with a certain singularity structure

Selected Topics (V)

- Second step is to define the process
  - Either need to know maximum event weight W_{MAX} and allow generator to do unweighting
  - Or set W_{MAX}=1 and only generate unweighted events
  - Or decide that weighted events are okay
    - Set W_{MAX}=1, but pass W through a common block to histogramming routine
  - Pass kinematics and quantum numbers to a common block
    - PYUPEV in PYTHIA
    - Must specifiy color and anti-color flow
      - In- gluon (1) passes its color to out-quark (3)
      - In-gluon (1) passes its anti-color to out-antiquark (4)
      - Out-quark (3) gets its color from In-gluon (1)
      - Etc.

Selected Topics (VI)

- **Specify showering pairs and scales**
  - **Even a single out-going QCD parton must have a partner to conserve energy-momentum**
    - **$qg \rightarrow q\gamma\gamma$ requires some choice of $(q\gamma)$ pairing**
  - **Should not set scales harder than typical scale of new process**
  - **If scale is too low, then not enough gluon radiation to screen color charge**
  - **MOST SUBTLE PART OF WHOLE PROCEDURE**

Overall Summary

- Event Generators accumulate our knowledge and intuition about the Standard Model into one package
  - Apply perturbation theory whenever possible
    - hard scattering, parton showering, decays
  - Rely on models or parametrizations when present calculational methods fail
    - hadronization, underlying event, beam remnants
- Out of the box, they give reliable estimates of the full, complicated structure of an event
  - Sophisticated users will find more flexibility & applications
    - And will avoid easy mistakes
  - Understanding the output will lead to a broad understanding of the Standard Model (and physics beyond)