The Higgs Boson and Electroweak Symmetry Breaking

1. The Minimal Standard Model

M. E. Peskin
SLAC Summer Institute, 2004

When I was a student, I was told that the goal of elementary particle physics was to learn the basic laws of the strong and weak interactions.

Today, this is a solved problem.

These laws are explained by the “Standard Model”:

The strong, weak, and electromagnetic interactions are described by a Yang-Mills gauge theory with the gauge group $SU(3) \times SU(2) \times U(1)$.

But, in science, the solution to every problem leads to new questions at a deeper level.

To describe the weak interactions in a Yang-Mills theory, the gauge symmetry must be spontaneously broken.

We know that this happens, but we do not know why.

My personal view is that this is the greatest of the Great Puzzles.

It is the key to further progress in microscopic physics.

You are likely to solve this problem during your careers, from discoveries at the next generation of accelerators.

In these lectures, I will give an introduction to the problem of Electroweak Symmetry Breaking (EWSB):

1. The Minimal Standard Model

I will discuss the properties of the Higgs boson in the simplest version of the Standard Model. I hope this will be a useful starting point for analyzing experiments on the Higgs boson and EWSB.

2. Models of Electroweak Symmetry Breaking

I will present three models that might explain EWSB. I hope this will help you to appreciate the variety of possible approaches. Eventually, experiment will decide which approach is correct.

Elements of the SU(2) x U(1) electroweak theory (Glashow, Salam, Weinberg)

add to the known quarks, leptons, bosons one scalar field φ with

$$I = \frac{1}{2} \quad Y = +\frac{1}{2}$$

The Lagrangian for φ is

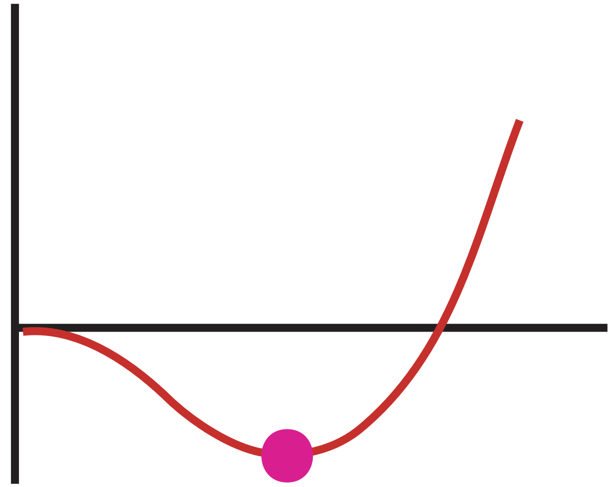
$$\mathcal{L} = |D_\mu \varphi|^2 - V(|\varphi|) - \frac{1}{4}(F^a_{\mu\nu})^2 - \frac{1}{4}(G_{\mu\nu})^2$$

+ (coupling to quarks and leptons)

Assume $V(|\varphi|)$ is such that $\langle \varphi \rangle$ is nonzero:

e.g., $V(|\varphi|) = \mu^2 |\varphi|^2 + \lambda |\varphi|^4$

with $\mu^2 < 0$



The field φ has the general structure:

$$\varphi = \begin{pmatrix} \pi^+ \\ (v + h + i\pi^0)/\sqrt{2} \end{pmatrix}$$

$\pi^\pm$, π^0 are Goldstone bosons

$$\varphi(\pi^+, \pi^0) = e^{-i\alpha^a(x)\tau^a} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

In the theory with global symmetry, they are massless.
In the theory with gauge symmetry, they are gauge degrees of freedom, and become part of W, Z

h(x) is the Higgs boson field. It corresponds to

$$v \to v + h(x)$$

$$|D_\mu\varphi|^2$$

$$= \begin{pmatrix} 0 & v/\sqrt{2} \end{pmatrix} \left| \frac{g}{\sqrt{2}} W^+\sigma^+ + \frac{g}{\sqrt{2}} W^-\sigma^- + \frac{g}{2} W^0\sigma^3 + \frac{g'}{2} B \right|^2 \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

$$= \frac{v^2}{4}[g^2 W^+ W^- + \frac{1}{2}(-gW^0 + g'B)^2]$$

The boson mass eigenstates are

$$Z = \cos\theta_w W^3 - \sin\theta_w B$$

$$A = \sin\theta_w W^3 + \cos\theta_w B$$

where

$$\cos\theta_w = \frac{g}{\sqrt{g^2 + g'^2}} \qquad \sin\theta_w = \frac{g'}{\sqrt{g^2 + g'^2}}$$

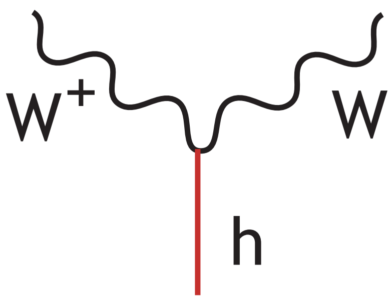
Then

$$m_W^2 = \frac{g^2}{4} v^2 \qquad m_Z^2 = \frac{g^2 + g'^2}{4} v^2$$

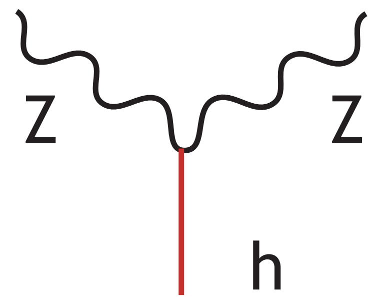
Notice the nontrivial relation $m_W/m_Z = \cos\theta_w$

With this orientation, it is straightforward to work out the couplings of the Higgs boson.

Since in Higgs appears from $v \to v + h$,
its W, Z vertices are:



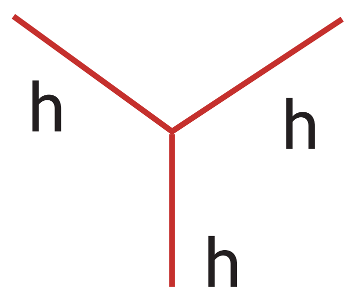
$$= 2i\frac{m_W^2}{v}g_{\mu\nu}$$



$$= 2i\frac{m_Z^2}{v}g_{\mu\nu}$$

The potential above gives $m_h = \sqrt{2|\mu^2|} = \sqrt{\lambda/2}v$

and the vertex



$$= -3i\frac{m_h^2}{v}$$

The Higgs couples to fermions through scalar and pseudoscalar operators: $\overline{f}_L f_R \ , \ \overline{f}_R f_L$

These are the operators used to build mass terms. But since left- and right-handed fermions have different SU(2)xU(1) quantum numbers, it is not possible to build such terms without the Higgs field. Using

$$L = \begin{pmatrix} \nu \\ e^- \end{pmatrix}_L \quad Q = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad e_R \quad u_R \quad d_R$$

we can form

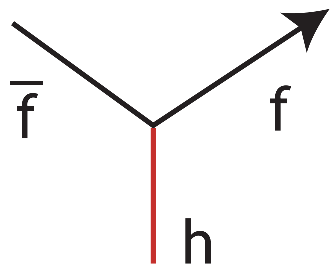
$$\mathcal{L} = -\lambda_e \overline{e}_R \varphi^\dagger \cdot L - \lambda_d \overline{d}_R \varphi^\dagger \cdot Q - \lambda_e \overline{u}_R \varphi_\alpha \epsilon_{\alpha\beta} Q_\beta$$

$$Y = +1 - \frac{1}{2} - \frac{1}{2} \qquad\qquad Y = -\frac{2}{3} + \frac{1}{2} + \frac{1}{6}$$

put $\langle \varphi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$, then $m_f = \dfrac{\lambda_f}{\sqrt{2}} v$

In the Standard Model, fermion masses arise only through EWSB.

Using $v \to v + h$ we find the Higgs coupling to fermions:



$$= -i\frac{m_f}{v}$$

If the fermion mass matrix is diagonal, the Higgs coupling is also flavor-diagonal.

Here is a direct argument. Start from the most general Lagrangian with flavor-mixing:

$$\mathcal{L} = -\lambda_e^{ij}\overline{e}_R^i \varphi^\dagger L^j + \cdots$$

We can represent any complex matrix as a product of unitary and real diagonal matrices: $\lambda_e = V_{eR} D_e V_{eL}^\dagger$

Now transform $e_R \to V_{eR} e_R \ , \ e_L \to V_{eL} e_L$

This removes flavor violation in the Higgs couplings.

Can these unitary transformations show up elsewhere in the theory ?

For leptons, if $\lambda_\nu = 0$, we can rotate $L \to V_{eL} L$

This completely removes V_{eL} from the theory. Then, e.g. ν_e is defined to be the neutrino emitted in β-decay. If neutrinos have mass, ν_e might not be a mass eigenstate.

For quarks, V_{dL} , V_{uL} can still appear in weak boson couplings:

$$Z_\mu \overline{u}_L \gamma^\mu u_L \to Z_\mu \overline{u}_L \gamma^\mu (V_{uL}^\dagger V_{uL}) u_L$$

$$= 1$$

$$W_\mu^+ \overline{u}_L \gamma^\mu d_L \to W_\mu^+ \overline{u}_L \gamma^\mu (V_{uL}^\dagger V_{dL}) d_L$$

$$= V_{CKM}$$

So, the only flavor-violating interactions in the Minimal Standard Model are hadronic weak interactions with CKM mixing.

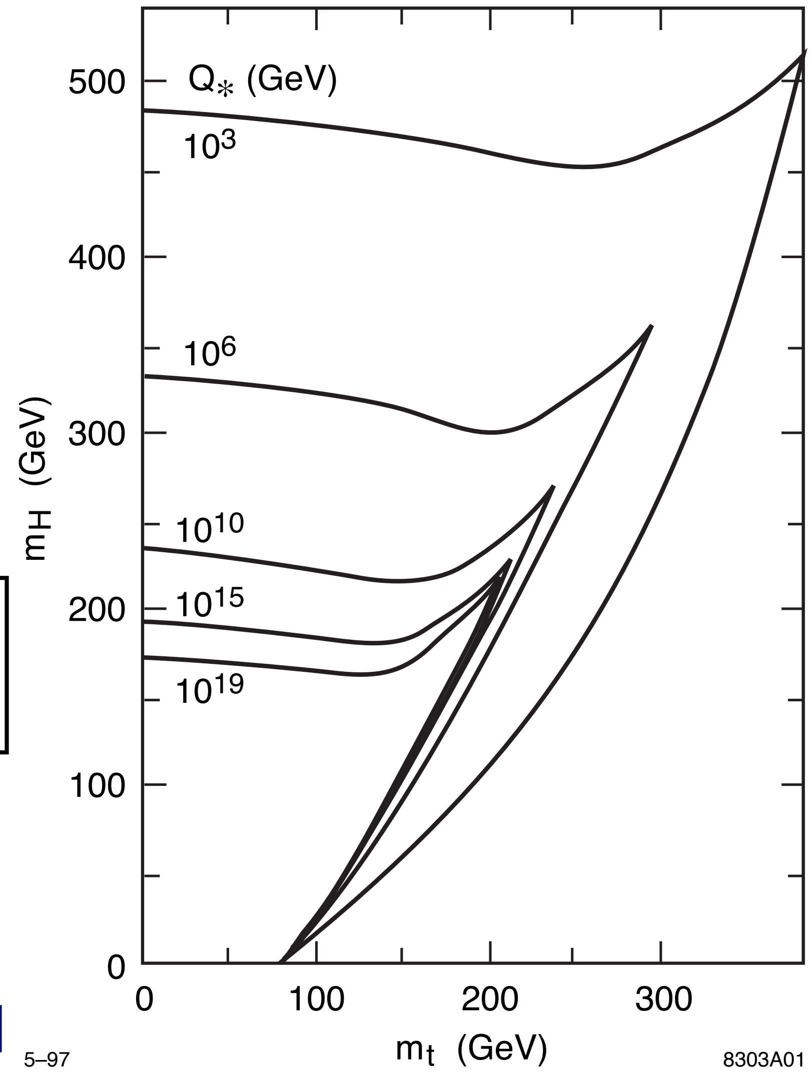
What constraints do we have on the Higgs mass and the coupling of the MSM ?

Renormalization group evolution implies that the MSM can be correct up to high energies only in a limited range of parameters.

$$\frac{d}{d\log Q}\lambda = \frac{3}{2\pi^2}\left[\lambda^2 - \frac{1}{32}\lambda_t^4 + \cdots\right]$$

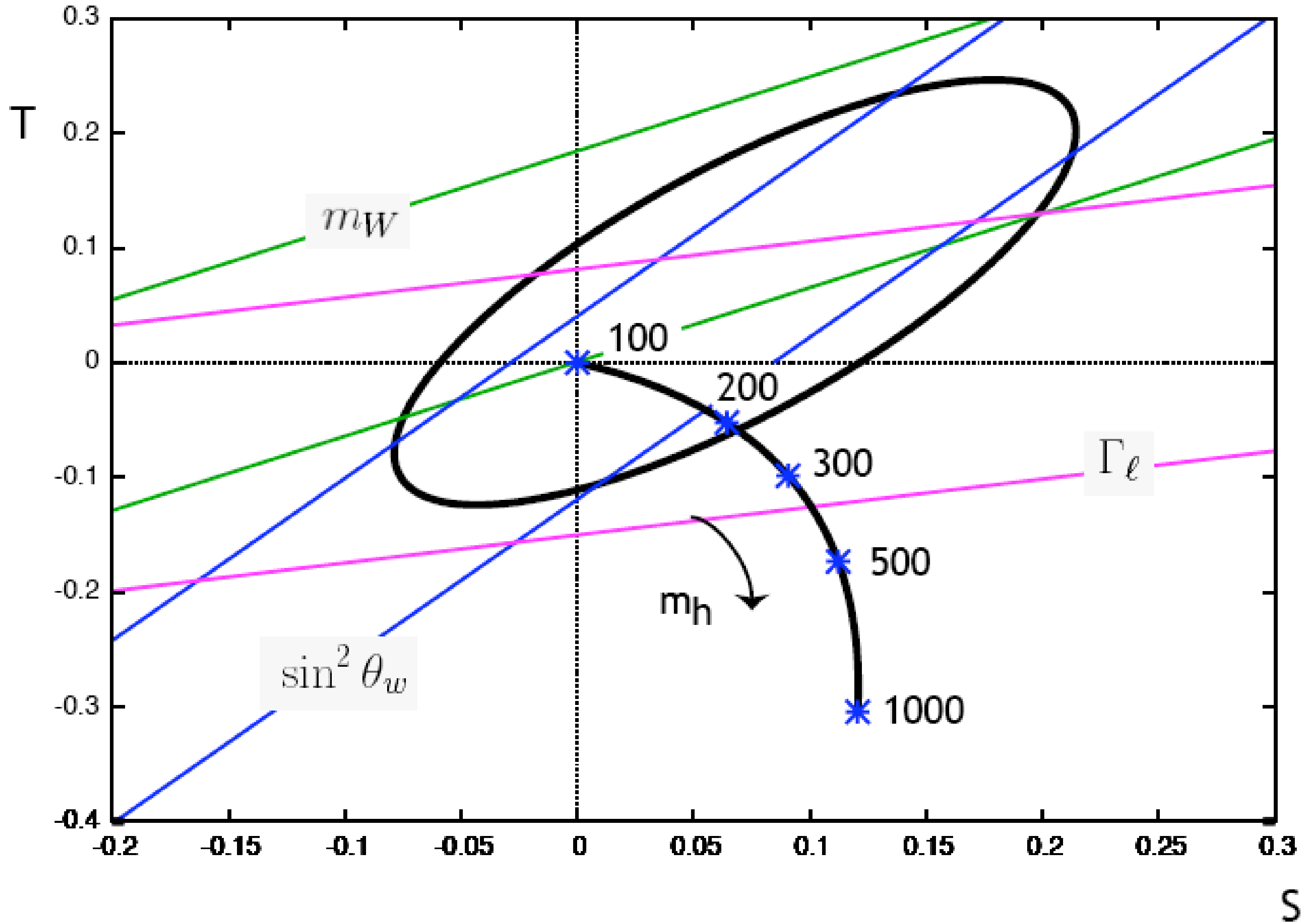
So for small λ_t, $\lambda \to \infty$ at high Q;
for large λ_t, $\lambda \to -\infty$

So, only if m_h is between about 140 and 180 GeV can the MSM be the whole story.



Lindner

Fits to precision electroweak data also constrain the Higgs boson mass in the MSM. Marciano will discuss this in his lectures.



LEP EWWG: $m_h < 240$ GeV (95% conf.)

From the couplings of the Higgs boson, we can determine the Higgs decay modes. The decay pattern of the Higgs has a quite intricate variation with energy.

First, $h \to f\overline{f}$

$$\Gamma(h \to f\overline{f}) = \frac{N_c \lambda_f^2}{16\pi} m_h = \frac{N_c \alpha}{8 s_w^2} \frac{m_f^2}{m_W^2} m_h$$

The heaviest species dominate. But, be careful; masses should be evaluted at a common scale. At $Q \sim 2m_t$ the $\overline{MS}$ masses are

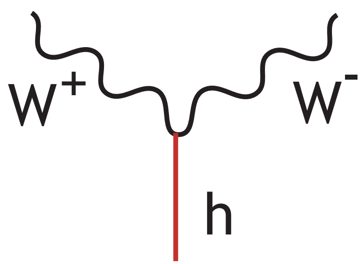
τ: 1.7 c: 0.5 b: 2.7 t: 160 GeV

Next, $h \to W^+W^-$, Z^0Z^0

$$\Gamma(h \to WW) = \frac{\alpha}{16 s_w^2} \frac{m_h^3}{m_W^2} (1 - \frac{4m_W^2}{m_h^2})^{1/2} \left[1 - 4\frac{m_W^2}{m_h^2} + 12 \frac{m_W^4}{m_h^4} \right]$$

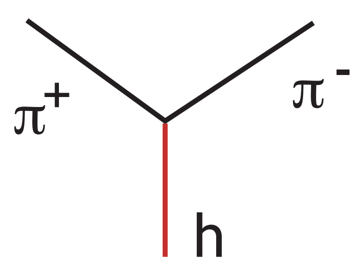
$$\Gamma(h \to ZZ) = \frac{\alpha}{32 s_w^2} \frac{m_h^3}{m_W^2} (1 - \frac{4m_Z^2}{m_h^2})^{1/2} \left[1 - 4\frac{m_Z^2}{m_h^2} + 12 \frac{m_Z^4}{m_h^4} \right]$$

It seems odd that this should go as m_h^3/m_W^2
To understand this, note that the decay is dominated by helicity = 0 W, Z bosons, for which



$$= \quad 2i\frac{m_W^2}{v}\epsilon_+^* \cdot \epsilon_-^* \sim 2i\frac{m_W^2}{v}\frac{2E_+E_-}{m_W^2} \sim i\frac{m_h^2}{v}$$

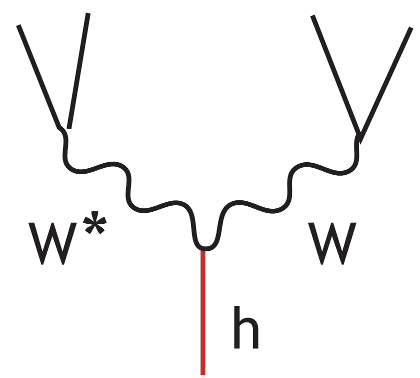
The helicity 0 weak bosons arose as eaten Goldstone bosons; and indeed



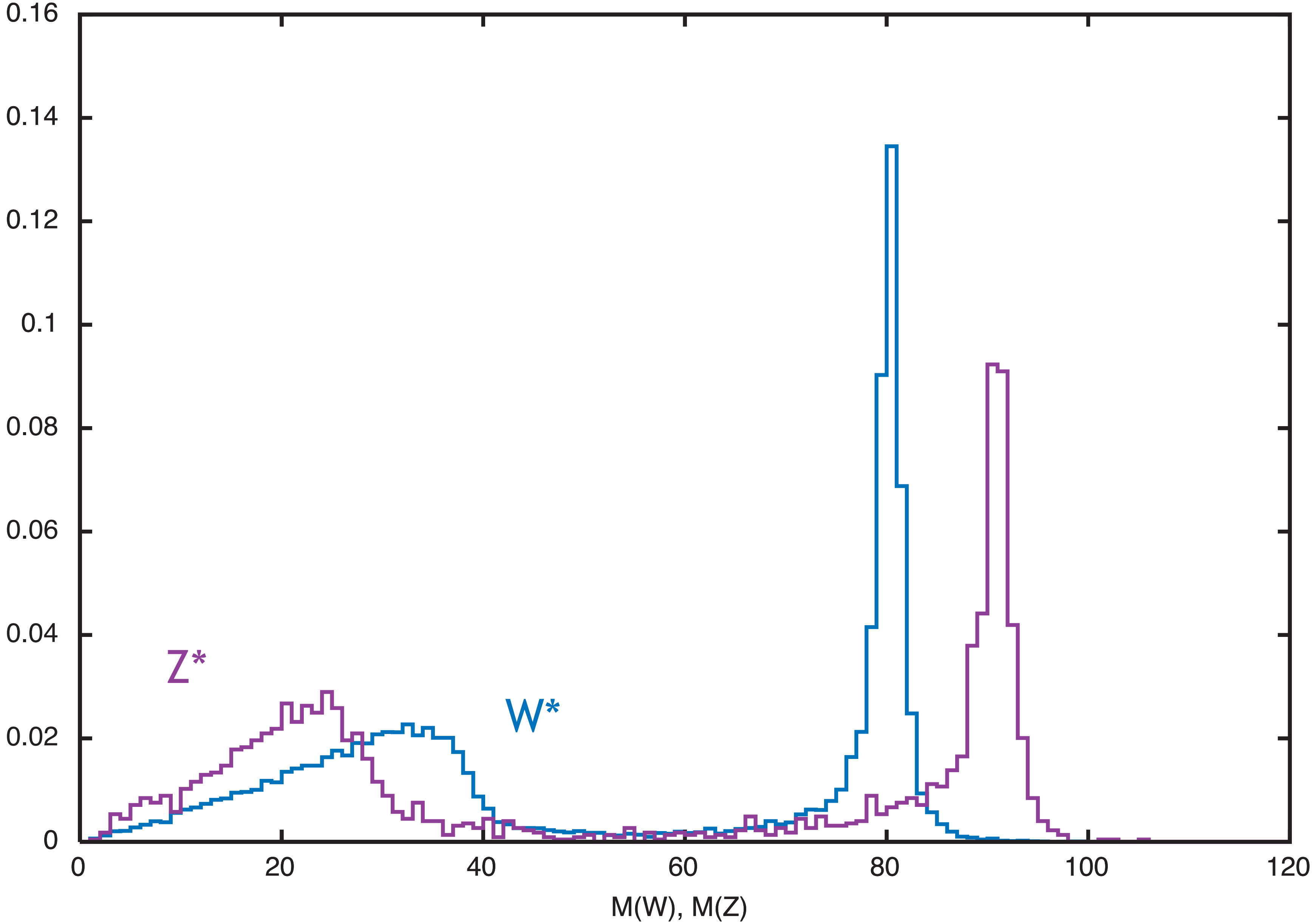
$$= \quad i\frac{\lambda v}{2} = i\frac{m_h^2}{v}$$

For $m_h > 2m_W$, $h \to W^+W^-$ is the dominant mode.

For $m_h < 2m_W$, the mode $h \to b\bar{b}$ is still suppressed, so the modes $h \to WW^*$, $h \to ZZ^*$ can compete with it.

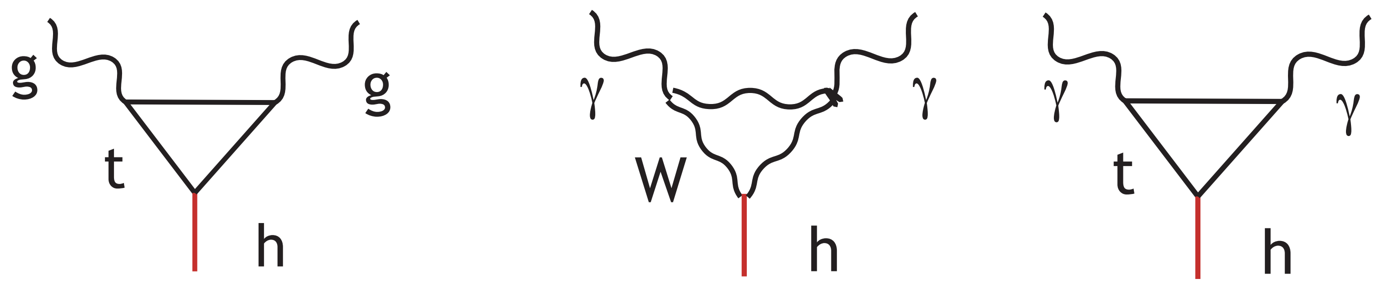


2-jet mass distributions in h→WW*, ZZ* decays
m(h) = 120 GeV



$$h \to gg \ , \ h \to \gamma\gamma \ , \ h \to \gamma Z^0$$

These processes occur through loop diagrams involving heavy particles.



$$\Gamma(h \to gg) = \frac{\alpha\alpha_s^2}{144\pi^2 s_w^2}\frac{m_h^3}{m_W^2} \cdot 2$$

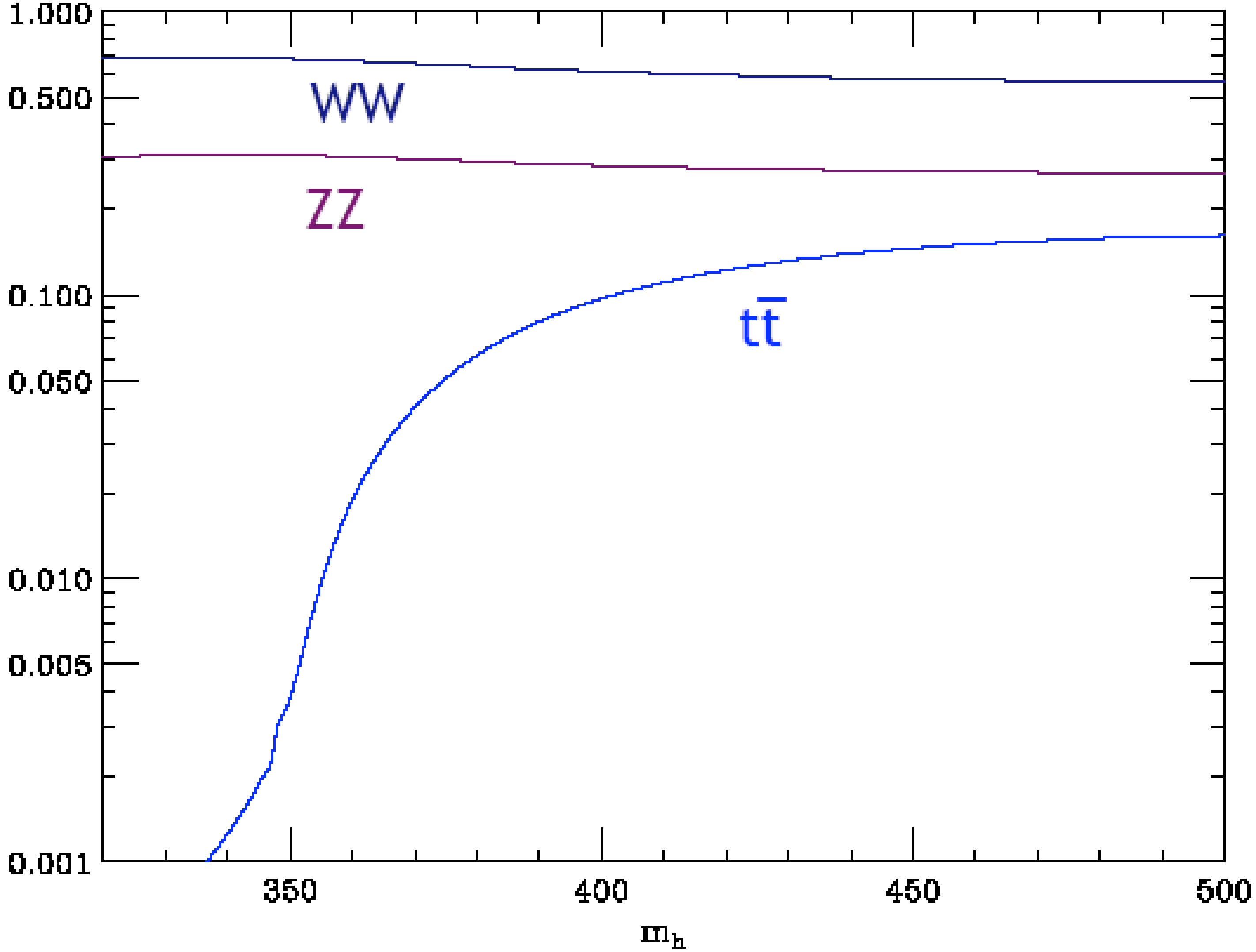
$$m_h \ll 2m_W$$

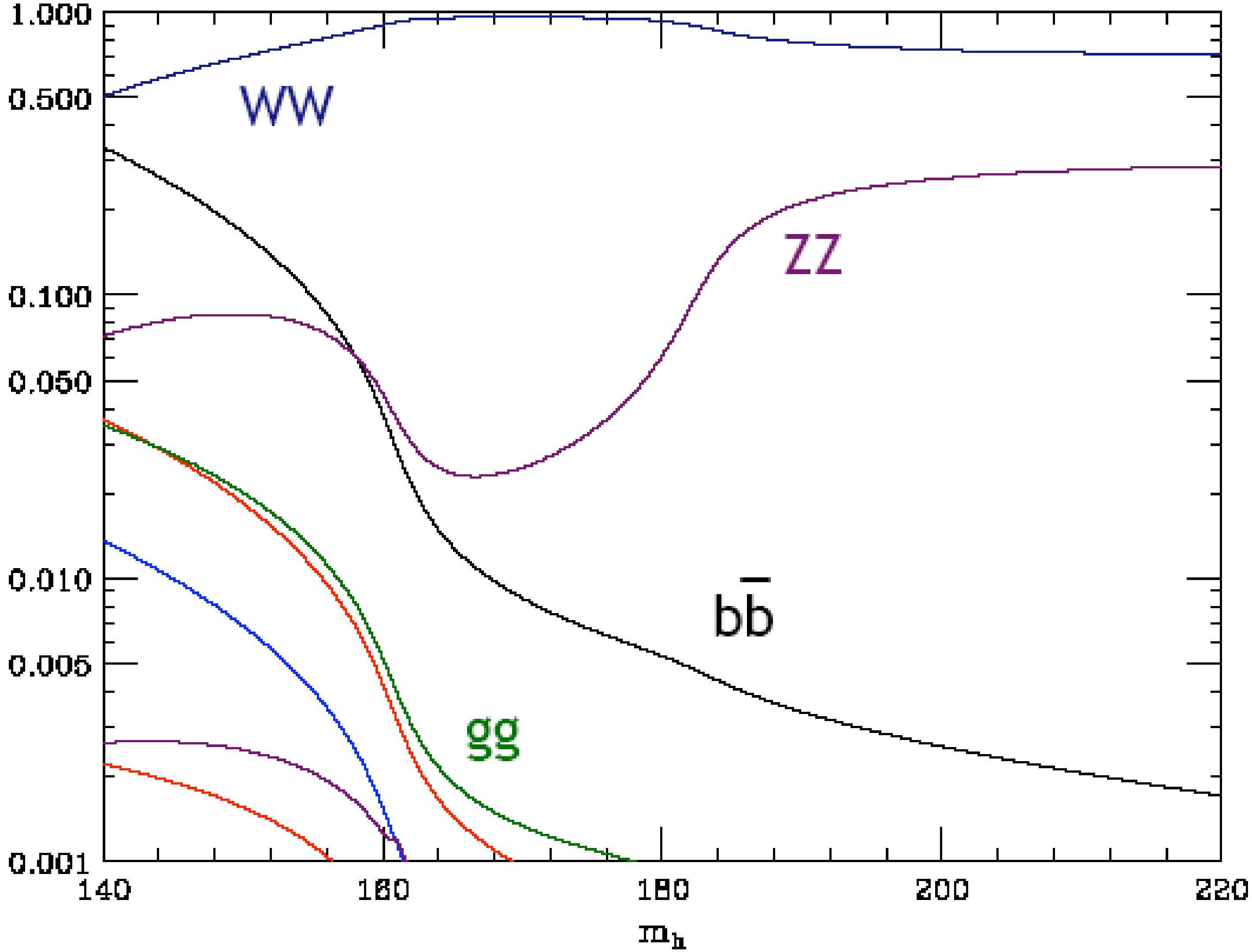
$$\Gamma(h \to \gamma\gamma) = \frac{\alpha^3}{144\pi^2 s_w^2}\left|\frac{21}{4} - 3 \cdot (\frac{2}{3})^2\right|^2$$

Notice that these decays measure sum rules over the spectrum of heavy particles with QCD/electroweak interactions that obtain mass from the Higgs boson.

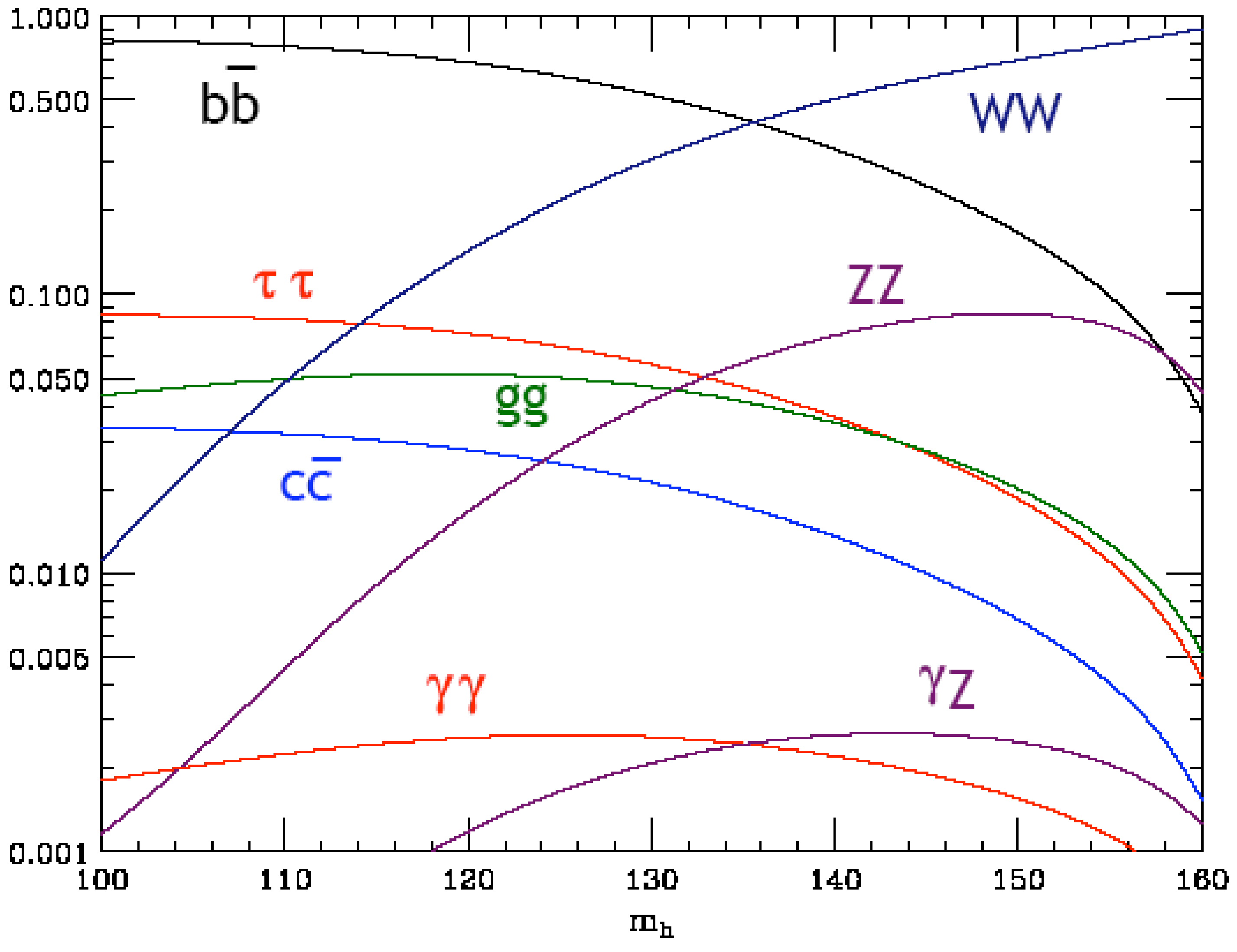
Now put all of the pieces together:

here are the branching fractions for a heavy, intermediate, and light Higgs boson in the MSM


1.000
0.500
0.100
0.050
0.010
0.005
0.001
350
400
450
500
m_h
WW
ZZ
$t\bar{t}$

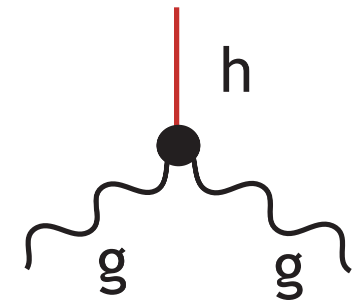


1.000
0.500
0.100
0.050
0.010
0.005
0.001
140
160
180
200
220
m_h
WW
ZZ
$b\bar{b}$
gg



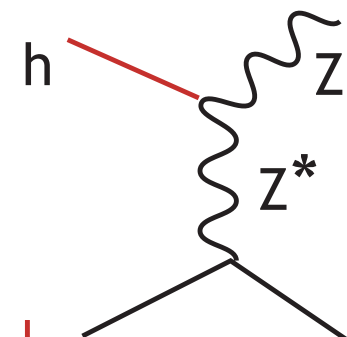
1.000
0.500
0.100
0.050
0.010
0.005
0.001
100
110
120
130
140
150
160
m_h
$b\bar{b}$
WW
$\tau\tau$
ZZ
gg
$c\bar{c}$
$\gamma\gamma$
γZ

Inverting the decay modes, we find many processes by which the Higgs boson can be created and studied. Three of the most important are:



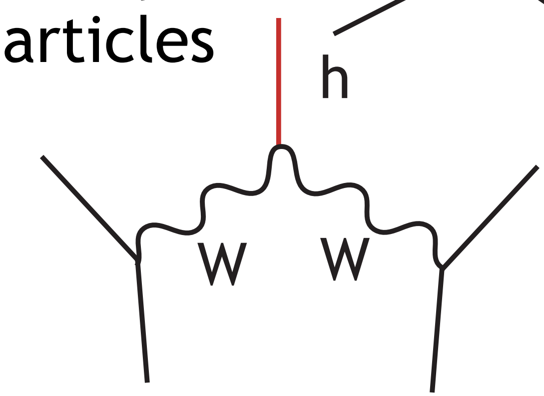
$$gg \to h$$

"gg fusion", the process with the largest cross section for Higgs production at the LHC



$$e^+e^- \to Zh \ , \ q\bar{q} \to Wh \ , \ Zh$$

"Higgs-strahlung"; this process measures the h-Z and h-W couplings, which are present only because h generates the mass of these particles



$$W^+W^- \to h$$

"WW fusion", with W radiated from initial state q or e

These processes will be described in the afternoon lectures; for a detailed treatment, see **The Higgs Hunter's Guide**, by Dawson, Gunion, Haber, and Kane.

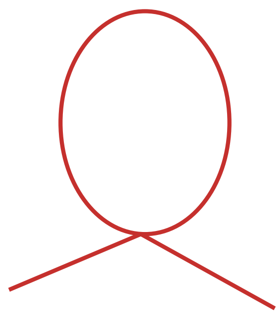
The Minimal Standard Model is a perfectly consistent theory of EWSB, but there is something troubling about it.

You might ask, why is electroweak symmetry broken in this model ?

The answer is, because $\mu^2 < 0$.

It is easy to make this problem appear worse.

For example, μ^2 is not computable even in principle in the MSM, because most of the diagrams contributing to μ^2 are quadratically ultraviolet divergent



$$\sim \frac{\lambda}{(4\pi)^2}\Lambda^2$$

If the cutoff Λ is a very high scale, e.g. $\Lambda \sim 10^{16}$ GeV it is very difficult to understand how m_h could be as small as 100 GeV.

This is called the "gauge hierarchy problem".

But isn't it enough that we do not know any particles or forces that could provide a physical mechanism for EWSB ? It is as if we wanted to explain the tides without knowing about the existence of the moon.

My cat knows that the moon is there;
it may take a $5 B accelerator to find the Higgs boson.

But conceptually, there is no difference. We have to interrogate Nature and find the missing ingredients to solve the puzzle.